Differential decay rates of CP even and odd Higgs bosons to massive quarks at NNLO in QCD

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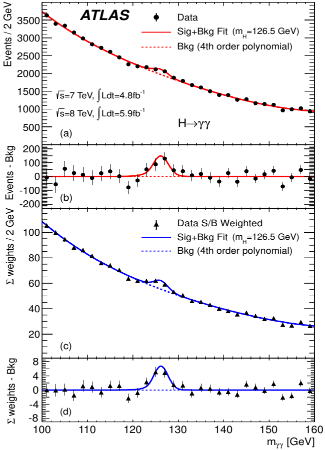
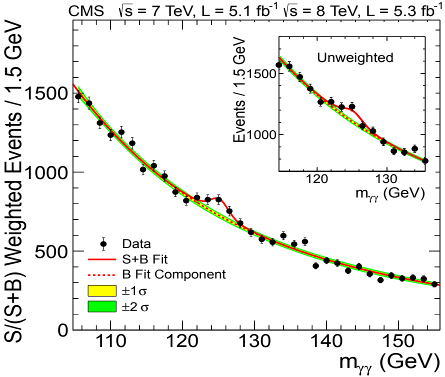
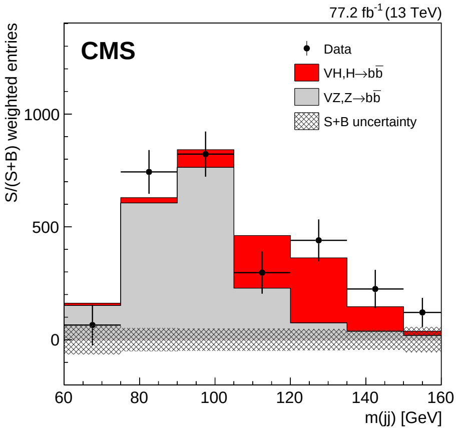
based on JHEP 1807 (2018) 159 **[**arXiv:1805.06658**]**

The discovery of the $h(125)$ at the LHC (2012)





$h \to b\bar{b}$ is the most probable decay channel of the $h(125)$, and finally observed recently at the LHC (in the VH-events)!



The measured signal strength $\mu = 1.04 \pm 0.20$

Many Beyond Standard Model extensions predict heavy scalars and pseudo-scalars coupled to heavy quarks...

- The two-Higgs-doublet model (2HDM)

  $h_1,\ h_2,\ A_0,\ H^{\pm}$

- The Minimal Supersymmetric SM (MSSM)

- Models of (strong) dynamical electroweak symmetry breaking, e.g. Techicolor, $\cdots$

- $\cdots\cdots$

Much work done previously on neutral scalar bosons decay into quarks.

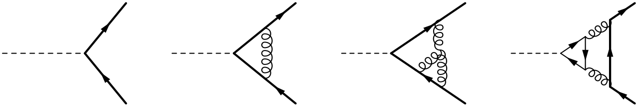
- ***inclusive***:
  - known up to N^4LO for CP-even Higgs into massless quarks;
    [Baikov,Chetyrkin,Kuhn,06; Davies,Steinhauser,Wellmann,17; Herzog,Ruijl,Ueda,Vermaseren,Vogt,17]
  - $NNLO$ corrections for CP-even/odd Higgs in power expansion of $\frac{m_Q}{m_h}$;
    [Surguladze,94; Chetyrkin,Kwiatkowski,96; Chetyrkin,Kniehl,Steinhauser,97; Larin,Ritbergen,Vermaseren,95;Harlander,Steinhauser,97; Chetyrki
    Harlander,Steinhauser,97,98]
  - ......
- ***differential***:
  - CP-even Higgs decay into massless quarks at $NNLO$
    [Anastasiou, Herzog, Lazopoulos, 2012; Duca, Duhr, Somogyi, Tramontano, Trocsanyi, 2015]
  - The Higgs decay into massless b quarks at N^3LO
    [Mondini, Schiavi, Williams, 2019]

Focus of the talk

We consider the decay of a neutral Higgs boson h of arbitrary *CP* to a massive quark antiquark pair at NNLO order in perturbative QCD, i.e. $h \to Q\bar{Q}X \ \ (Q = t, b)$, at the *fully differential* level (using the *antenna subtraction method*).

- BSM heavy CP-even/odd Higgs boson decay into $t\bar{t}$ pair:
  inclusive decay width as a function of m_h, and $M_{t\bar{t}}$ distributions, etc
- the SM $h(125)$ Higgs boson decay into a massive $b\bar{b}$ pair:
  inclusive decay width, 2-jet, 3-jet, and 4-jet decay width,
  and the energy distribution of the leading jet for two-jet events.

Power counting the m_b **dependence** of $h \to b\bar{b}$



$$\Gamma_{h\to b\bar{b}} \sim y_b^2\, C_0 + \alpha_s y_b^2\, C_1 + \alpha_s^2 \left(y_b^2\, C_2^{[b]} + y_b\, y_t\, C_2^{[b,t]} \right) + \mathcal{O}\left(\alpha_s^3\right)$$

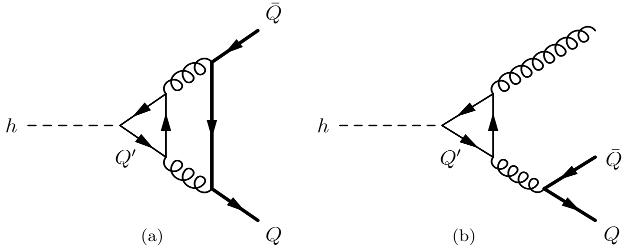
with $y_b \sim m_b$.

In the region $m_b \ll m_h$, upon pulling out the overall m_b^2 :

- $\ln[m_b]$ factor $\Rightarrow$ $\overline{\mathrm{MS}}$-*running* $\bar{y}_b$
- leading constant terms $\sim \mathcal{O}(m_b^0)$ (no $1/m_b$!)
  - the main bulk captured by the "$y_b \neq 0$ but $m_b = 0$" approximation;
  - the top-triangle loop-induced contributions (e.g. the right-most diagram);
- power suppressed terms $\sim \left(\frac{m_b^2}{m_h^2}\right)^N$ with $N \geq 1$.

Top-Triangle diagrams absent in massless b-quark approximation

These amplitudes are ultraviolet- and infrared-finite (directly calculable in 4-dimension)



- These contributions can **not** be consistently added in a massless computation at NNLO [Caola, Luisoni, Melnikov, Rontsch, 17].
- **The full exact analytic results of these top-yukawa contributions were recently calculated** [Primo, Sasso, Somogyi, Tramontano, 18].

The *hybrid* UV-renormalization in *pQCD* with $n_f + 1$ quarks:

- External-fields (and their masses): **on-shell scheme**

- α_s: $\overline{\text{MS}}$ scheme (no *decoupling* term)

- The Yukawa-coupling $\bar{y}_Q$: $\overline{\text{MS}}$ **scheme**

  (to absorb all intermediate large $\ln[\frac{m_Q}{m_h}]$ for small m_Q)

The $h \to Q\bar{Q}$ decay rate in $m_Q \ll m_h$ with **on-shell** y_Q is long known to contain large $\ln[\frac{m_Q}{m_h}]$ from on-shell y_Q renormmalization! [Braaten, Leveille, 1980]

IR-subtraction: $0 = \int d\sigma^{\mathcal{S}} - \int d\sigma^{\mathcal{S}}$ **into** $\sigma_{\textbf{NLO}} = d\sigma^{\mathcal{R}} + d\sigma^{\mathcal{V}}$.

The $d\sigma^{\mathcal{S}}$ in the *antenna* subtraction method [Kosower, 1998; Gehrmann-De Ridder, Gehrmann, Glover, 2005] are constructed according to the *universal IR-factorization formulae* of *color-ordered* partial QCD-amplitudes,

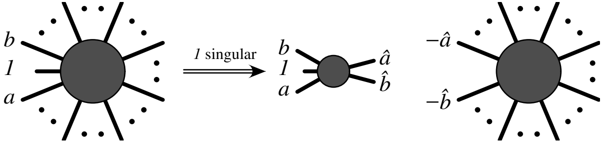


Figure: The antenna factorization of a *color-ordered* partial amplitude (PRD 71, 045016)

At NLO:

$$d\sigma^{\mathcal{S}}_{\text{NLO}} \propto \int_{d\Phi_{n+1}} \sum \mathbf{A}_{a1b}(p_a, p_1, p_b) \otimes |\mathcal{M}^{\mathcal{H}}_n(\cdots, P_{\hat{a}}, P_{\hat{b}}, \cdots, p_{n+1})|^2$$

$$= \int_{d\Phi_n} \sum \mathcal{A}_{a1b} \otimes |\mathcal{M}^{\mathcal{H}}_n(\cdots, P_{\hat{a}}, P_{\hat{b}}, \cdots, p_{n+1})|^2$$

where the $\mathbf{A}_{a1b}$ is the *antenna-function*, with its integrated counterpart $\mathcal{A}_{a1b}$.

Organizations of ingredients for $h \to Q\bar{Q} + X$ $(Q = t, b)$

Schematically the **NNLO** corrections with *antenna* IR subtraction terms: [Bernreuther, Bogner, Dekkers, 11/13]

$$
\begin{aligned}
d\sigma_{\mathrm{NNLO}} &= \int_{d\Phi_4} \left(d\sigma^{RR}_{\mathrm{NNLO}} - d\sigma^{S}_{\mathrm{NNLO}} \right) \\
&+ \int_{d\Phi_3} \left(d\sigma^{RV}_{\mathrm{NNLO}} - d\sigma^{T}_{\mathrm{NNLO}} \right) \\
&+ \int_{d\Phi_2} d\sigma^{VV}_{\mathrm{NNLO}} + \int_{d\Phi_3} d\sigma^{T}_{\mathrm{NNLO}} + \int_{d\Phi_4} d\sigma^{S}_{\mathrm{NNLO}}
\end{aligned}
$$

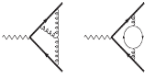
- RR: Tree-level double real radiation correction: $h \to Q\bar{Q}gg$, $Q\bar{Q}q\bar{q}$ and $Q\bar{Q}Q\bar{Q}$

implicit IR-singularity removed by $d\sigma^{S}_{\mathrm{NNLO}}$.

- RV: One-loop correction to $h \to Q\bar{Q}g$

explicit and implicit IR-singularity removed by $d\sigma^{T}_{\mathrm{NNLO}}$

- VV: Two-loop corrections to $h \to Q\bar{Q}$ [Bernreuther,Bonciani,Gehrmann,Heinesch,Mastrolia,Remiddi, 2005]



explicit IR-poles removed by $\int_{d\Phi_3} d\sigma^{T}_{\mathrm{NNLO}} + \int_{d\Phi_4} d\sigma^{S}_{\mathrm{NNLO}}$

Decay widths in terms of on-shell and $\overline{\text{MS}}$ Yukawa-couplings

The *differential* decay width of a Higgs boson with a generic CP into **unpolarized** $Q\bar{Q}$:

$$d\Gamma^{Q\bar{Q}} = a_Q^2 d\Gamma_S^{Q\bar{Q}} + b_Q^2 d\Gamma_P^{Q\bar{Q}}$$

with the "*reduced*" Yukawa-couplings a_Q and b_Q as in $-y_Q h\left[a_Q \bar{Q}Q + b_Q \bar{Q} i\gamma_5 Q\right]$ (where $y_Q = \frac{m_Q}{v_{vev}}$).

Expanded to order α_s^2:

$$\begin{aligned} d\Gamma^{Q\bar{Q}} &= y_Q^2\left[d\hat{\Gamma}_0^{Q\bar{Q}} + \frac{\alpha_s(\mu)}{\pi} d\hat{\Gamma}_1^{Q\bar{Q}} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 d\hat{\Gamma}_2^{Q\bar{Q}}\right] \\ &\equiv y_Q^2\, d\hat{\Gamma}_0^{Q\bar{Q}}\left[1 + \frac{\alpha_s(\mu)}{\pi} d\gamma^{Q\bar{Q}_1} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 d\gamma_2^{Q\bar{Q}}\right] \end{aligned} \tag{1}$$

The on-shell and $\overline{\text{MS}}$ Yukawa-couplings are related by

$$y_Q^2 = \bar{y}_Q^2(\mu)\left[1 + \mathbf{r_1}(m_Q,\mu)\frac{\alpha_s(\mu)}{\pi} + \mathbf{r_2}(m_Q,\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\right] \tag{2}$$

Inserting (2) into (1) and **re-expanding** to order α_s^2

$$d\overline{\Gamma}^{Q\bar{Q}} = \bar{y}_Q^2(\mu) d\hat{\Gamma}_0^{Q\bar{Q}}\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(d\gamma_1^{Q\bar{Q}} + \mathbf{r_1}\right) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2\left(d\gamma_2^{Q\bar{Q}} + \mathbf{r_1} d\gamma_1^{Q\bar{Q}} + \mathbf{r_2}\right)\right].$$

We work in 6-flavor QCD:

Standard-Model inputs

$$m_t^{on} = 173.34 \text{ GeV, corresponding to } \overline{m}_t(\mu = m_t) = 163.46 \text{ GeV;}$$
$$\alpha_s^{(5)}(m_Z) = 0.118; \qquad \mathbf{G}_F = 1.166379 * 10^{-5} \frac{1}{\text{GeV}^2}$$

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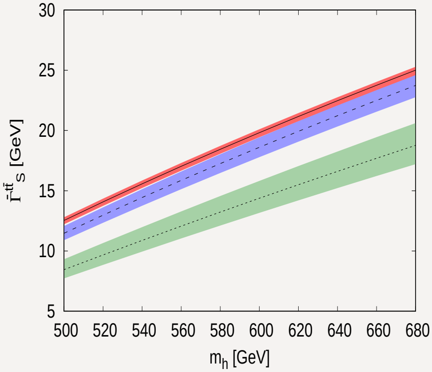
The $t\bar{t}$ inclusive decay widths at NNLO QCD, $\overline{\Gamma}_X^{t\bar{t}}$ and $\Gamma_X^{t\bar{t}}$ $(X = S, P)$

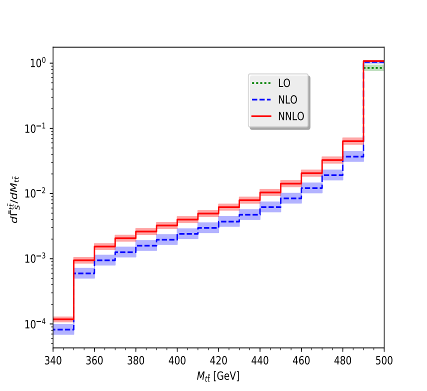
m_h [GeV]	$\overline{\Gamma}_S^{t\bar{t}}$ [GeV]	$\Gamma_S^{t\bar{t}}$ [GeV]	$\overline{\Gamma}_P^{t\bar{t}}$ [GeV]	$\Gamma_P^{t\bar{t}}$ [GeV]
500	$12.529^{+0.265}_{-0.314}$	$12.955^{+0.037}_{-0.046}$	$22.392^{+0.283}_{-0.411}$	$22.931^{+0.030}_{-0.062}$
680	$25.007^{+0.285}_{-0.408}$	$25.647^{+0.075}_{-0.101}$	$32.188^{+0.214}_{-0.397}$	$32.784^{+0.185}_{-0.225}$

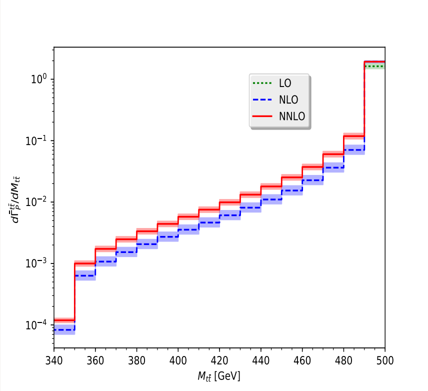
These NNLO QCD results for inclusive $t\bar{t}$-decay widths (exact in m_t) **agree** with the large m_h approximation result (to 4-th order in $(m_t/m_h)^2$) in [Harlander, Steinhauser, 1997].

The decay width into $t\bar{t}$ of scalars/pseudo-scalars at LO, NLO, and NNLO in α_s as a function of m_h.



Distribution $d\overline{\Gamma}_X^{t\bar{t}}/dM_{t\bar{t}}$ of the $t\bar{t}$ invariant mass with $m_h = 500$ GeV.





We work in a "5-flavor" QCD:

Standard-Model inputs

$$m_h = 125.09 \text{ GeV}; \qquad \overline{m}_b(\mu = \overline{m}_b) = 4.18 \text{ GeV};$$
$$\alpha_s^{(5)}(m_Z) = 0.118; \qquad \mathbf{G}_F = 1.166379 * 10^{-5} \frac{1}{\text{GeV}^2}$$

From the 5-flavor QCD two-loop running-mass formula, it reads $m_b^{on} = 4.78$ GeV and $\overline{m}_b(\mu = m_h) = 2.80$ GeV, and hence $\overline{y}_b(\mu) = \frac{\overline{m}_b(\mu)}{v_{vev}} = 0.01137$.

We represent our result for inclusive decay width using $\overline{\text{MS}}$ Yukawa-coupling $\overline{y}_b$:

$$\overline{\Gamma}_{NNLO}^{b\bar{b}} = \overline{\Gamma}_{LO}^{b\bar{b}} \left[1 + \mathbf{g_1} \frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2} \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] ,$$

where

$$\overline{\Gamma}_{LO}^{b\bar{b}} = \overline{y}_b^2(\mu) \hat{\Gamma}_0^{b\bar{b}}, \quad \mathbf{g_1} = \gamma_1^{b\bar{b}} + r_1, \quad \mathbf{g_2} = \gamma_2^{b\bar{b}} + r_1 \gamma_1^{b\bar{b}} + r_2 \,.$$

SM $h(125) \to b\bar{b} + X$: the inclusive decay width

For the QCD-correction coefficients $\mathbf{g_1}$, $\mathbf{g_2}$ defined in

$$\overline{\Gamma}^{b\bar{b}}_{NNLO} = \overline{\Gamma}^{b\bar{b}}_{LO} \left[1 + \mathbf{g_1} \frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2} \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] ,$$

we obtain

	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
$\mathbf{g_1}$	3.024	5.796	8.569
$\mathbf{g_2}$	3.685	37.371	86.112
$\overline{\Gamma}^{b\bar{b}}_{LO}$ [MeV]	2.153	1.910	1.717
$\overline{\Gamma}^{b\bar{b}}_{NLO}$ [MeV]	2.413	2.307	2.196
$\overline{\Gamma}^{b\bar{b}}_{NNLO}$ [MeV]	2.425	2.399	2.353

The known results for massless b quarks ($\mu = m_h$):

$$\mathbf{g_1}(m_b = 0) = 5.6666 \text{ and } \mathbf{g_2}(m_b = 0) = 29.1467$$

[Gorishnii, Kataev, Larin, Surguladze, 1990; K. G. Chetyrkin, 1996]

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we obtain

$1 + \mathbf{g_1}\alpha_s + \mathbf{g_2}\alpha_s^2 + \cdots$	total value	components
massive (α_s^2)	1.2560	$1 + 0.20789 + 0.04808$
massless (α_s^4)	1.2413	$1 + 0.203242 + 0.0374917 + 0.001927 + (-0.001366)$

$n_f = 5, \quad \mu = m_h$ [Baikov,Chetyrkin,Kuhn,06]

QCD correction factors: on-shell V.S. $\overline{\textbf{MS}}$:

$$\overline{\Gamma}^{b\bar{b}}_{NNLO} = \overline{\Gamma}^{b\bar{b}}_{LO}\left[1 + \mathbf{g_1}\frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2}\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\right],$$

where

$$\overline{\Gamma}^{b\bar{b}}_{LO} = \overline{y}_b^2(\mu)\hat{\Gamma}_0^{b\bar{b}}, \quad \mathbf{g_1} = \gamma_1^{b\bar{b}} + r_1, \quad \mathbf{g_2} = \gamma_2^{b\bar{b}} + r_1\gamma_1^{b\bar{b}} + r_2\,.$$

$y^{\overline{MS}}_b$	$\mu = m_h$
$\mathbf{g_1}$	5.796
$\mathbf{g_2}$	37.371
$\overline{\Gamma}^{b\bar{b}}_{LO}$ [MeV]	1.910
$\overline{\Gamma}^{b\bar{b}}_{NLO}$ [MeV]	2.307
$\overline{\Gamma}^{b\bar{b}}_{NNLO}$ [MeV]	2.399

$y_b^{\text{on-shell}}$	$\mu = m_h$
$\left\{\gamma_1^{b\bar{b}},\ r_1\right\}$	$\{-9.93,\ +15.73\}$
$\left\{\gamma_2^{b\bar{b}},\ r_2\right\}$	$\{-113.2,\ +306.7\}$
$\Gamma^{b\bar{b}}_{LO}$ [MeV]	5.578
$\Gamma^{b\bar{b}}_{NLO}$ [MeV]	3.592
$\Gamma^{b\bar{b}}_{NNLO}$ [MeV]	2.772

$h \to b\bar{b}$ decay width with $m_b = 0.5$ GeV

The QCD-correction coefficients $\mathbf{g_1}$, $\mathbf{g_2}$

$$\overline{\Gamma}^{b\bar{b}}_{NNLO} = \overline{\Gamma}^{b\bar{b}}_{LO} \left[1 + \mathbf{g_1} \frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2} \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right].$$

With $m_b = 0.5$ GeV, and excluding the **top-quark triangle loop diagrams** (which contribute $g_{2,t} = 6.898$), we obtain:

$$\mathbf{g_1}(m_b = 0.5\text{GeV}) = 5.6685, \text{ and } \mathbf{g_2}(m_b = 0.5\text{GeV}) = 29.187.$$

The known results for massless b quarks ($\mu = m_h$):

$$\mathbf{g_1}(m_b = 0) = 5.6666 \text{ and } \mathbf{g_2}(m_b = 0) = 29.1467$$

[Gorishnii, Kataev, Larin, Surguladze, 1990; K. G. Chetyrkin, 1996]

A note on top-quark loop induced contributions at NNLO

Recently the exact results of y_t-dependent $\mathcal{O}(\alpha_s^2)$ corrections to $h \to b\bar{b}$ are known analytically [Primo, Sasso, Somogyi, Tramontano, 18]

$$\mathbf{d} = 100\left(1 - \frac{\Gamma_{y_t}^{Approx}}{\Gamma_{y_t}^{Exact}}\right)$$

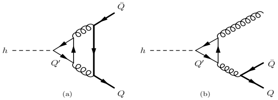


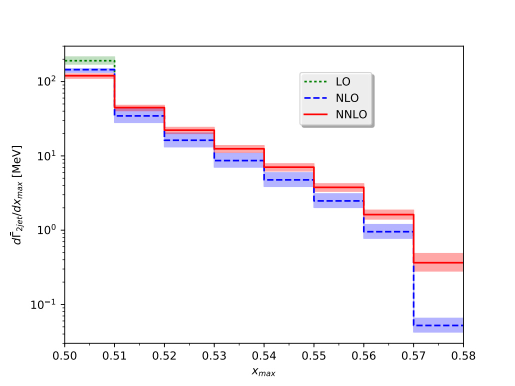
Table: The discrepancy $\mathbf{d}$ between the exact analytic result and the approximate formula [Chetyrkin,Kwiatkowski,96] (with $m_b = 4.92$ GeV)

m_t \ m_h	20	75	125	180
100	2.123	0.075	1.025	6.704
125	2.329	0.011	0.335	2.107
175	2.452	-0.019	0.018	0.355
250	2.566	-0.024	-0.055	-0.035
350	2.656	-0.023	-0.069	-0.113

The impact of these y_t contributions **at differential level** is small, typically below 5% [Primo, Sasso, Somogyi, Tramontano, 18].

SM $h(125) \to b\bar{b} + X$: the x_{max} distribution

The distribution of the energy of the leading jet in two-jet events is defined w.r.t $x_{max} = \max(E_{j_1}/m_h, E_{j_2}/m_h)$ using *Durham jet-algorithm* with $y_{cut} = 0.05$.



Similar distributions were presented before for *massless* b quarks [Anastasiou, Herzog, Lazopoulos, 2012] (JADE, with $y_{cut} = 0.1$) and in [Duca, Duhr, Somogyi, Tramontano, Trocsanyi, 2015] (Durham, with $y_{cut} = 0.05$).

Summary and Outlook

- ☑ A set up is presented for calculating the fully ***differential*** decay width of a *scalar* and *pseudo-scalar* to a **massive** $Q\bar{Q}$ pair at NNLO in α_s, which can be used to compute any *infrared-safe* (differential) observable in these decays.

- ☑ The set-up is applied to the decays of heavy scalars and pseudo-scalars to $t\bar{t}+X$, and to the decay of the Standard-Model $h(125)$ Higgs boson to massive $b,\ \bar{b}$ quarks. As a check, inclusive decay rates known before are recovered.

- ☒ We expect that this set up should be useful for having a more precise (and consistent) theoretical description of the α_s^2 QCD corrections to the production of the Higgs boson in association with a massive vector boson at LHC, $\mathbf{pp} \to \mathbf{V}(W/Z) + \mathrm{H}(b\bar{b})$.

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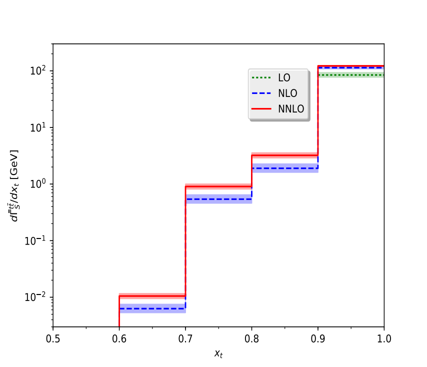
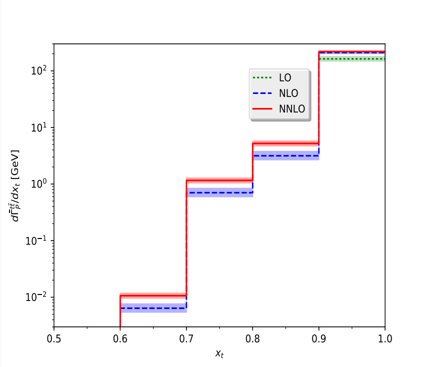
THANK YOU

Backup-Slides

Decays of BSM scalars/pseudo-scalars into $t\bar{t}$: the x_t distribution

The normalized top-quark energy $x_t = 2E_t/m_h$ (in the rest frame of the Higgs boson) $d\bar{\Gamma}_X^{t\bar{t}}/dx_t$ for a scalar/pseudo-scalar Higgs boson at LO, NLO, and NNLO QCD.





$h(125) \to b\bar{b} + X$: different jet rates

The n-jet rates can be represented, in analogy to the inclusive decay width, to order α_s^2 as follows:

$$\bar{\Gamma}_{2\,\text{jet}}^{b\bar{b}} = \bar{\Gamma}_{LO}^{b\bar{b}} \left[1 + g_1(2\,\text{jet}) \frac{\alpha_s^{(5)}}{\pi} + g_2(2\,\text{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] ,$$

$$\bar{\Gamma}_{3\,\text{jet}}^{b\bar{b}} = \bar{\Gamma}_{LO}^{b\bar{b}} \left[g_1(3\,\text{jet}) \frac{\alpha_s^{(5)}}{\pi} + g_2(3\,\text{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] ,$$

$$\bar{\Gamma}_{4\,\text{jet}}^{b\bar{b}} = \bar{\Gamma}_{LO}^{b\bar{b}} \times g_2(4\,\text{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 .$$

Table: The coefficients $g_i(\text{n}\,\text{jet})$ defined above and computed with the Durham algorithm using $y_{cut} = 0.01$ and $y_{cut} = 0.05$ for three renormalization scales μ.

	$y_{cut} = 0.01$			$y_{cut} = 0.05$		
	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
$g_1(2\,\text{jet})$	-5.055	-2.283	0.490	0.291	3.063	5.836
$g_2(2\,\text{jet})$	-56.351	-66.532	-61.658	-19.496	-0.650	33.250
$g_1(3\,\text{jet})$	8.079	8.079	8.079	2.733	2.733	2.733
$g_2(3\,\text{jet})$	36.873	80.741	124.609	22.256	37.096	51.937
$g_2(4\,\text{jet})$	23.163	23.163	23.163	0.926	0.926	0.926