PHY127 FS2023

Prof. Ben Kilminster

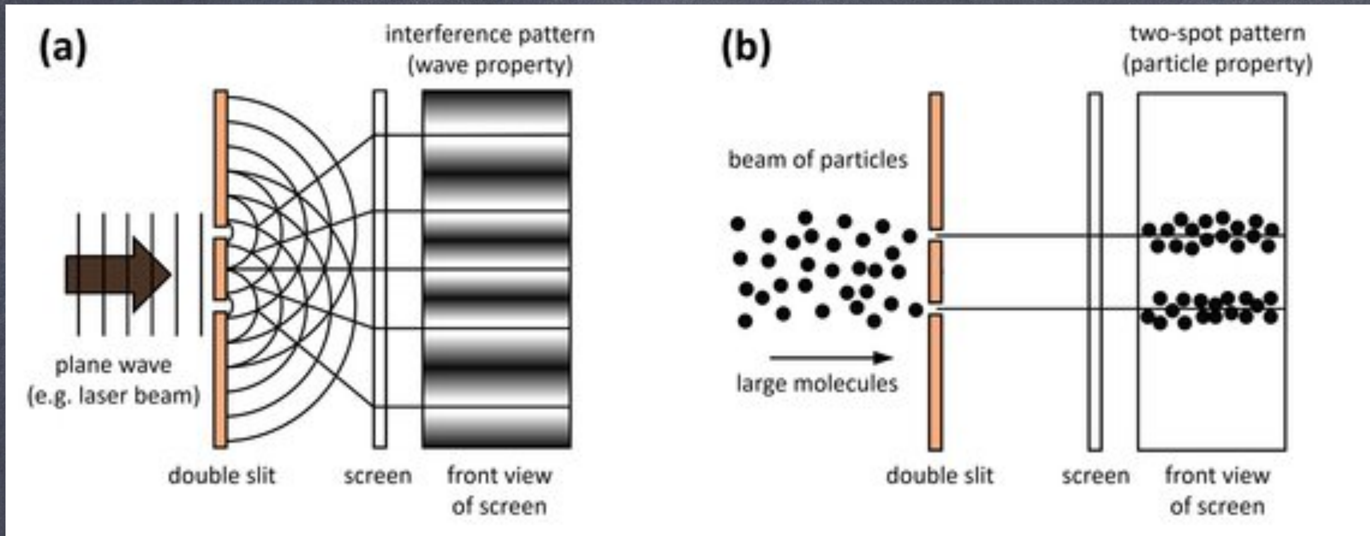
Lecture 4

March 17, 2023

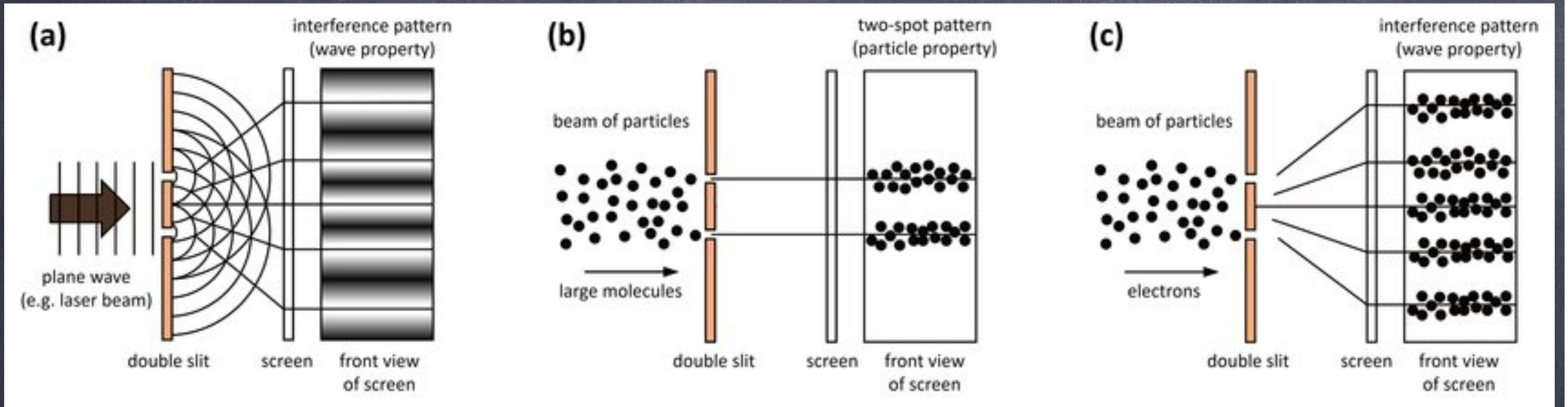
	Unanswered	Right	Wrong
Sam Elliot in "The Big Lebowski	5	15	18
Robert Redford, "The Sundance Kid	7	6	25
Mandy Patinkin, "The Princess Bride"	5	12	21
Will Ferrell, "Anchorman	7	10	21
Clark Gable, "Gone with the Wind	7	6	25
Sacha Baron Cohen, "Borat	8	12	18

Question

Prof.K's new mustache is most like :

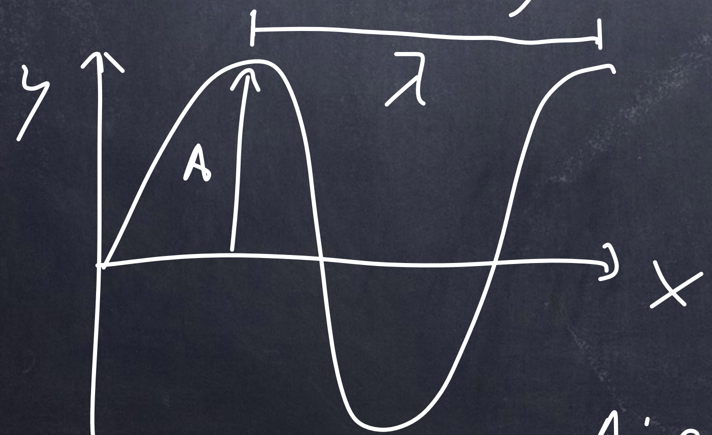

(a)
interference pattern
(wave property)
plane wave
(e.g. laser beam)
double slit
screen
front view
of screen
(b)
two-spot pattern
(particle property)
beam of particles
large molecules
double slit
screen
front view
of screen

waves particles photons



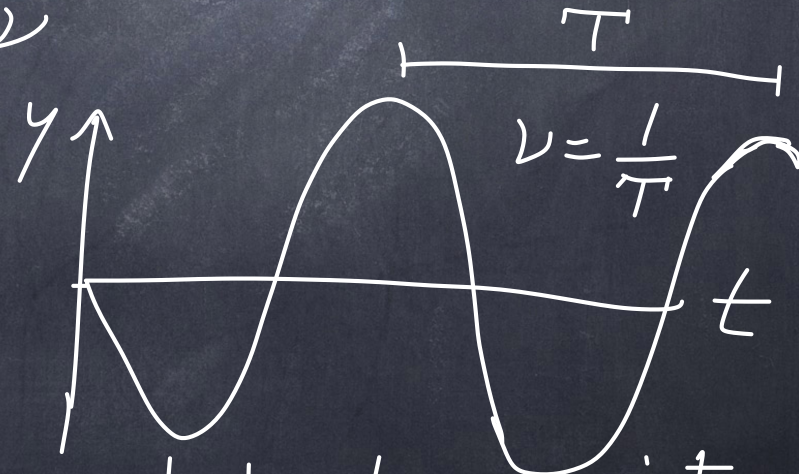
particles are waves and energy is quantized.
Quantization can be thought of as standing waves. Today we make standing waves. We will see, touch, hear standing waves → let's us understand quantized energy levels of an atom.

To describe a sine wave moving with velocity $\mathbf{v}$, where $v = \lambda \nu$



A: amplitude

if we freeze a wave at time $t=0$



if we look at a point $x=0$

The wave travels a distance λ in a time $T = \frac{1}{\nu}$
the velocity is then $v = \frac{\lambda}{T} = \lambda\nu$

The formula for the wave is called the wave function:

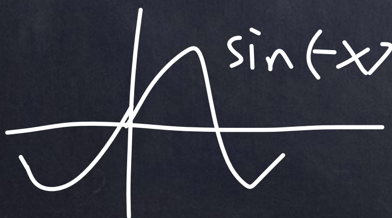
$$y(x,t) = A\sin(kx - \omega t)$$

k: wave number $\quad k = \frac{2\pi}{\lambda}$

ω: angular frequency $\quad \omega = 2\pi\nu = \frac{2\pi}{T}$

If $t=0$, $y(x, t=0) = A\sin kx = A\sin\frac{2\pi x}{\lambda}$

If $x=0$, $y(t=0,t) = A\sin(-\omega t) = -A\sin\omega t = -A\sin\frac{2\pi t}{T}$



$\sin(-x) = -\sin(x)$

waves on a string:

depends on tension (F) (force), + the mass density $\mu = \frac{\text{mass}}{\text{length}}$

Using Newton's Laws, $\Sigma F = ma$, we can derive the wave equation to explain how waves move on a string (see script for physics 1, chapter 3)

wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad v = \sqrt{\frac{F}{\mu}}$$

The symbol $\frac{\partial}{\partial x}$ means the partial derivative with respect to x.

y: The height of the wave

x: the distance along the string

v: velocity of the wave

t: the time

$y = y(x, t)$! this means the amplitude (y) depends on x & t.

A solution to the wave equation is

$$y = A \sin(kx - \omega t)$$

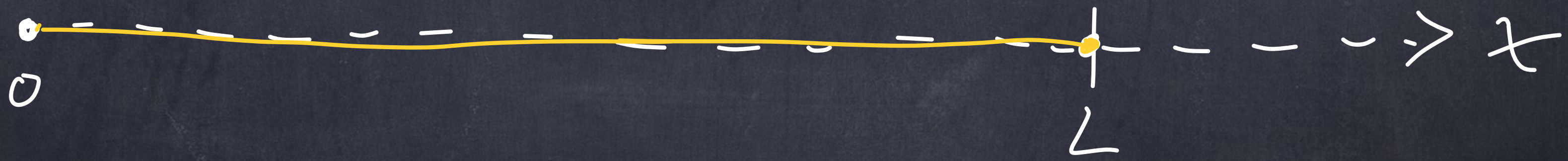
Standing waves:

A formula for a standing wave is

$$y(x,t) = A \cos \omega t \sin kx \quad \Big] \text{ one solution.}$$

At a time $t=0$ $\Rightarrow$ $y(x, t=0) = A \sin kx$

string



string is fixed at $x=0$ and $x=L$

The general wave function that is a solution to the wave equation for a standing wave at time $t=0$ is:

$$y(x) = A \sin\left(\frac{2\pi x}{\lambda}\right)$$

To find the solutions, we substitute different λ as a function of L

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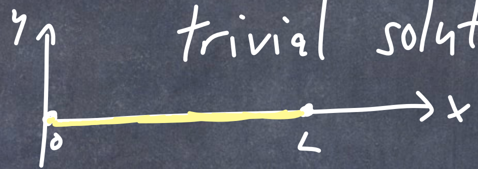
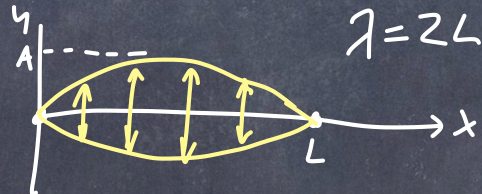
trivial solution (not moving)

0 bumps



$y(x) = 0$

1 bump



$\lambda = 2L$

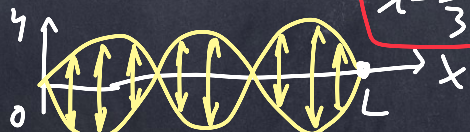
$y(x) = A\sin\left(\frac{\pi x}{L}\right)$

2 bumps

$\lambda = L$

$y(x) = A\sin\left(\frac{2\pi x}{L}\right)$

3 bumps



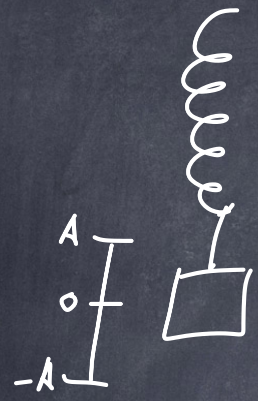
$\boxed{\lambda = \frac{2}{3}L}$

$y(x) = A\sin\left(\frac{3\pi x}{L}\right)$

In general, the solutions are $y(x) = A\sin\left(\frac{n\pi x}{L}\right)$ for $n = 0, 1, 2, 3, \cdots$

n = number of bumps

Energy transmitted by a wave



A simple harmonic oscillator has an energy of $\epsilon = \frac{1}{2}kA^2$

energy

A: amplitude
k: spring constant

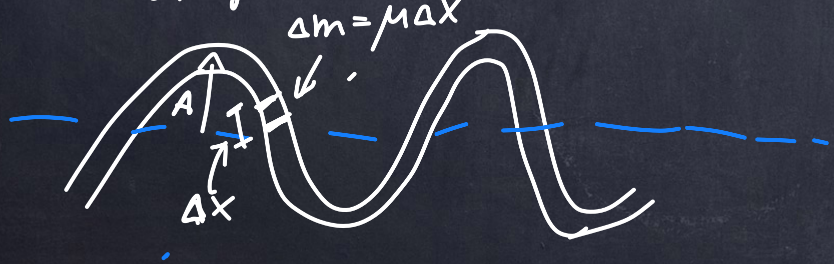
The energy depends on the amplitude squared.

The spring constant is related to the angular frequency by $k = m\omega^2$

m: mass of object
ω: angular frequency.

A string oscillating up and down is also a simple harmonic oscillator. Here the energy depends on $m\omega^2$

$$\epsilon = \frac{1}{2} m\omega^2 A^2$$



A small part of string carries an energy:

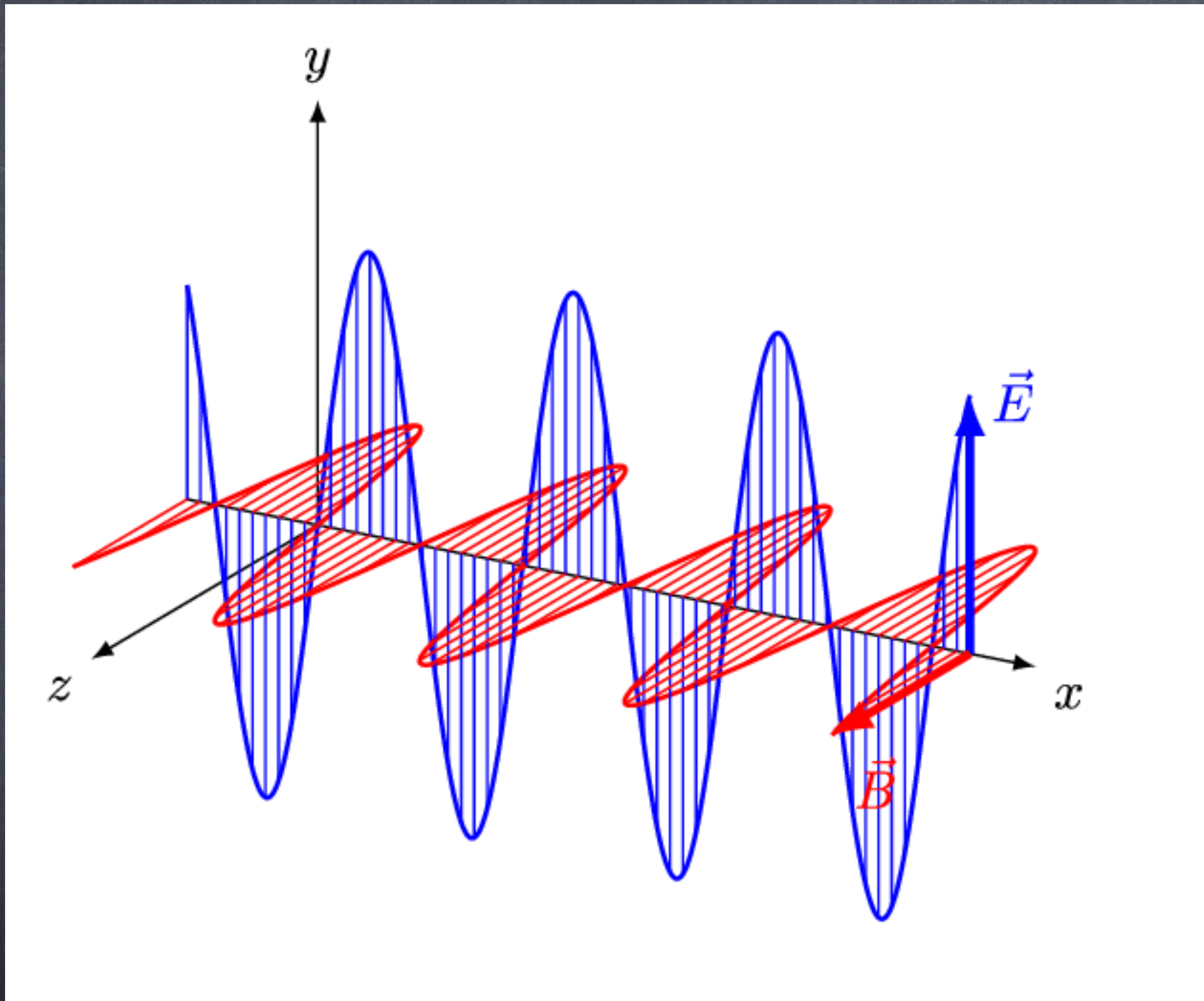
$$\Delta\epsilon = \frac{1}{2}\Delta m \omega^2 A^2 = \frac{1}{2}\mu\omega^2 A^2 \Delta x$$

Power transmitted is energy per time:

$$P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\mu\omega^2 A^2 \underbrace{\frac{\Delta x}{\Delta t}}_{v} = \frac{1}{2}\mu\omega^2 A^2 v$$

Both the energy and power are proportional to the amplitude <u>squared</u>.

An electromagnetic wave :



$\bar{E}$: $\hat{y}$

$\bar{B}$: $\hat{z}$

v : $\hat{x}$

For an EM wave: $\underset{\text{height}}{y(x)} \xrightarrow{\text{amplitude}} \underset{\text{electric field}}{\bar{E}(x)}$

wave functions: $E_y = \underbrace{E_{y_0}}_{\text{maximum amplitude}} \sin(kx - \omega t)$

$$B_z = B_{z_0} \sin(kx - \omega t)$$

simple proportionality: $E = cB$

The energy density of the EM wave is

$$\eta_E = \frac{1}{2}\varepsilon_0 E^2$$

η: energy per unit volume

$$\eta_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E^2 \leftarrow \left\{ c^2 = \frac{1}{\varepsilon_0 \mu_0} \ \& \ E = cB \right.$$

The total energy density is

$$\eta = \eta_E + \eta_B = \varepsilon_0 E^2$$

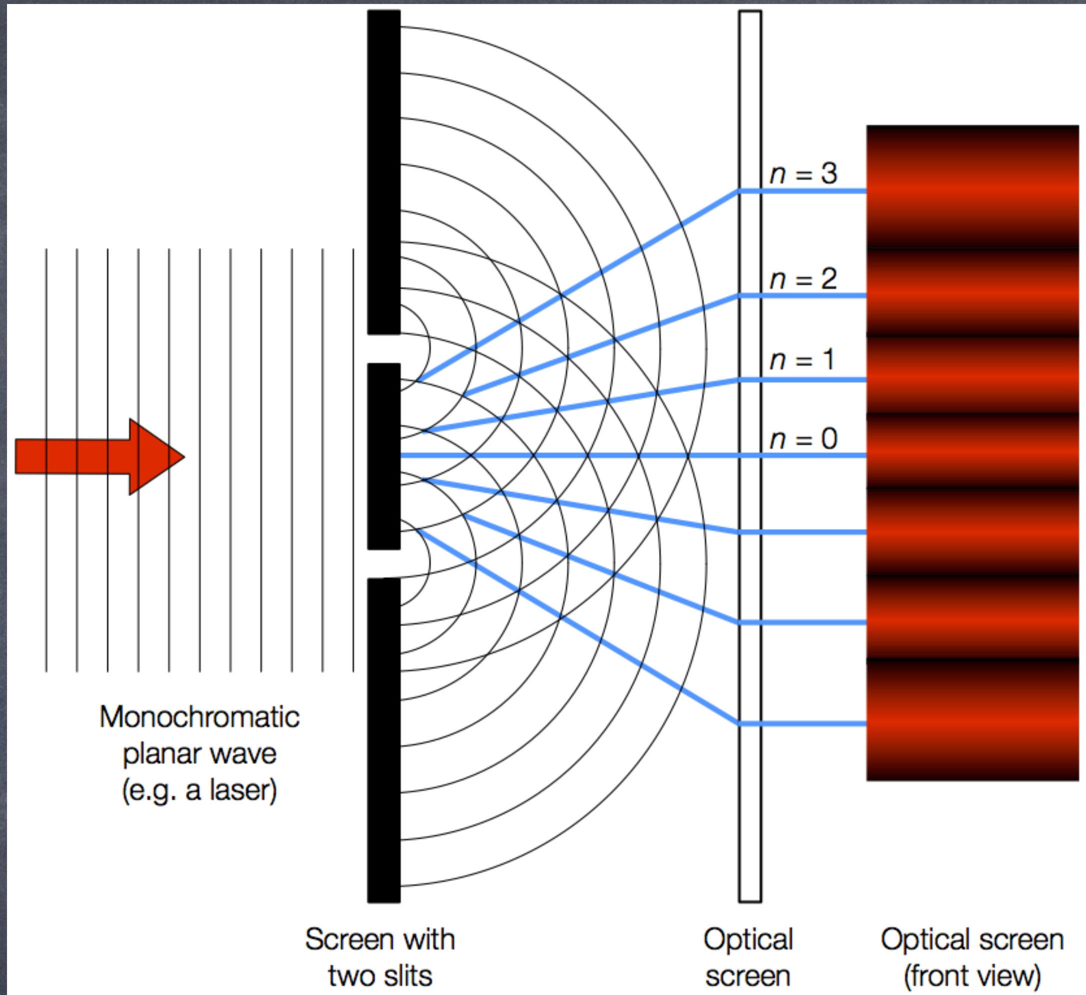
The ~~inten~~ instantaneous intensity I is

$$\frac{\text{power}}{\text{area}} = \text{energy density} * \text{velocity}$$

$$I = \eta c = c\varepsilon_0 E^2 = c\varepsilon_0 E_{y_0}^2 \sin^2(kx - \omega t)$$

The intensity of light is proportional to the square of the electric field.

Remember:



measure of intensity

zero intensity

maximum intensity

Intensity has a special meaning $I \propto E^2$

But this is also a measure of the probability of where we find the photons.

$P \propto E^2$

we have $I \propto \epsilon(x)^2$ $\quad$ ϵ is the electric field wave function

$$\downarrow$$

$$P(x) = \psi^2(x)$$

probability

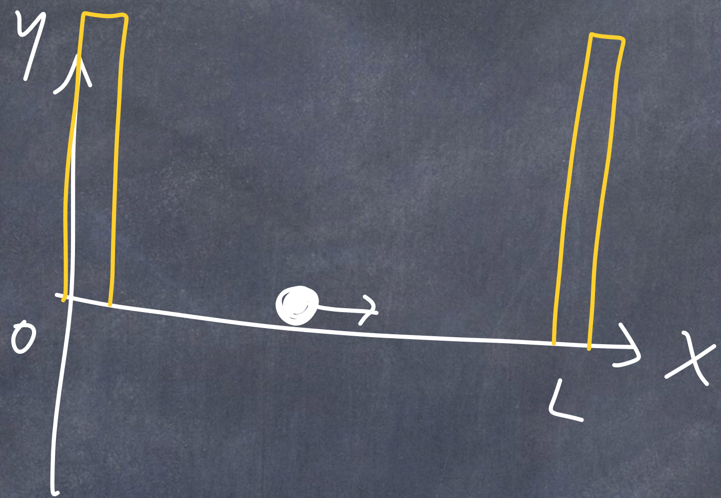
$P(x)$ = probability distribution function

$\psi(x)$ = wave function

(a measure of the amplitude of the wave)

Consider the classical world (that we know):

Consider a 1-D box with an electron inside.



box has length, L

classically, the electron moves back + forth crashing into walls.

If we know the starting position + velocity, we can predict its location at any time.

If we don't know its starting position, then we only know a probability of where it is.

equally likely to be anywhere in the box.

But it must be in the box.

$$\text{So} \quad \int_0^L P(x)\,dx = 1 \qquad\qquad \int_0^L P\,dx = 1$$

because is constant
with x.

so we can solve this: $\quad Px\Big]_0^L = 1$

$$P(L) - P(0) = PL = 1$$

$$P = \frac{1}{L}$$

So actually we want a probability for it to be in a finite space.

$$P_{x_1 \to x_2} = \int_{x_1}^{x_2} P(x)\,dx = \int_{x_1}^{x_2} \frac{1}{L}\,dx$$

$$= \frac{x}{L}\Big]_{x_1}^{x_2} = \frac{x_2 - x_1}{L}$$

For $t_1 = 0$, $t_2 = \frac{1}{2}L \Rightarrow \frac{\frac{1}{2}L - 0}{L} = \frac{1}{2}$

A wave is a particle,
and a particle is a wave.
This is true for any particle.

The wavelength of a particle is $\boxed{\lambda = \frac{h}{p}}$

p: momentum

This is the de Broglie wavelength.

An electron also behaves like a wave.

The energy of a particle : $E = K + U$

kinetic energy　　potential energy

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = mV$$

$$E = \frac{p^2}{2m} + U = \frac{h^2}{\lambda^2 2m} + U$$