NNLO+PS MATCHING USING MINLO

OVERVIEW AND RECENT DEVELOPMENTS

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Zurich (10/10/2017)

LARGE HADRON COLLIDER

- Higgs discovery: 04/07/2012
- After that no striking evidence of New Physics

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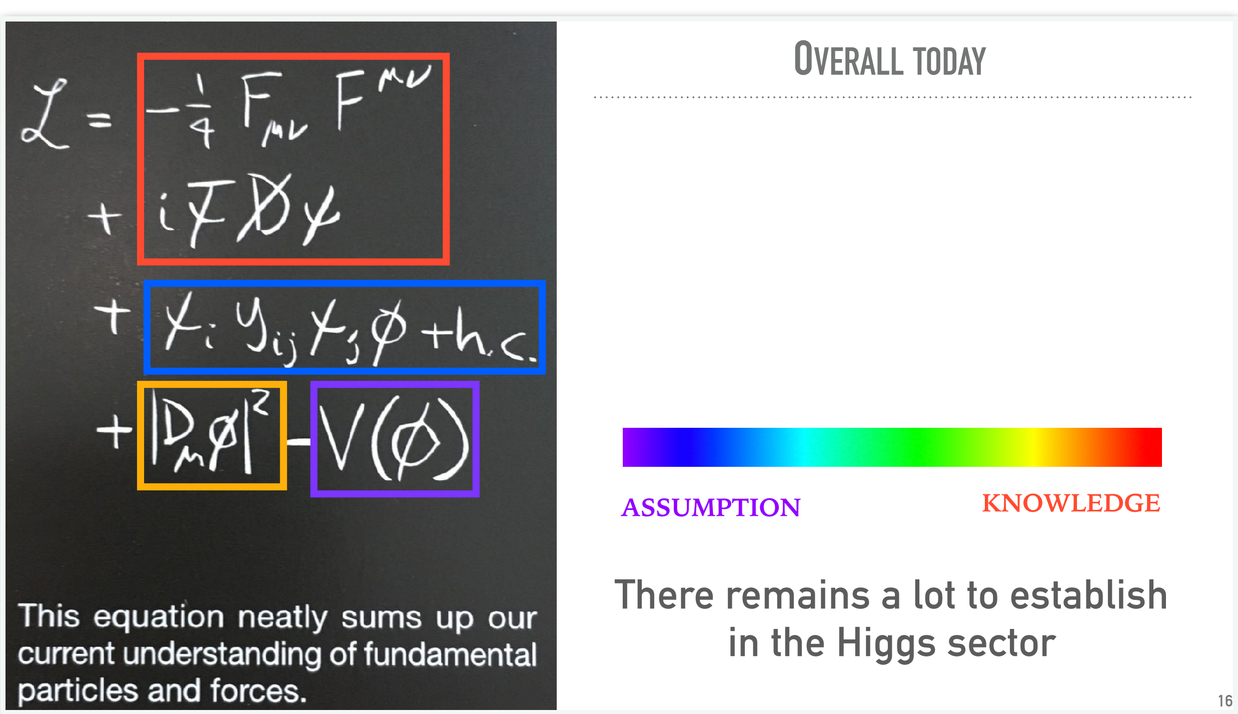

[Gavin Salam, Oxford Colloquium, Feb 2017]

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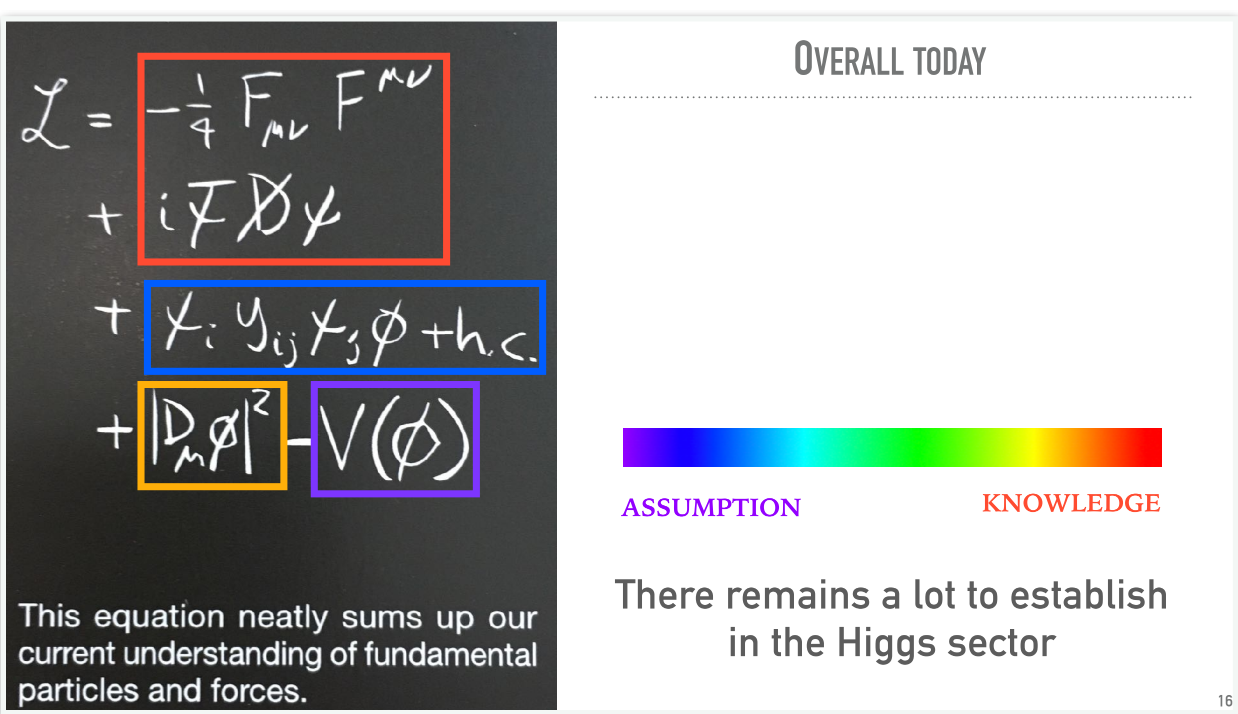

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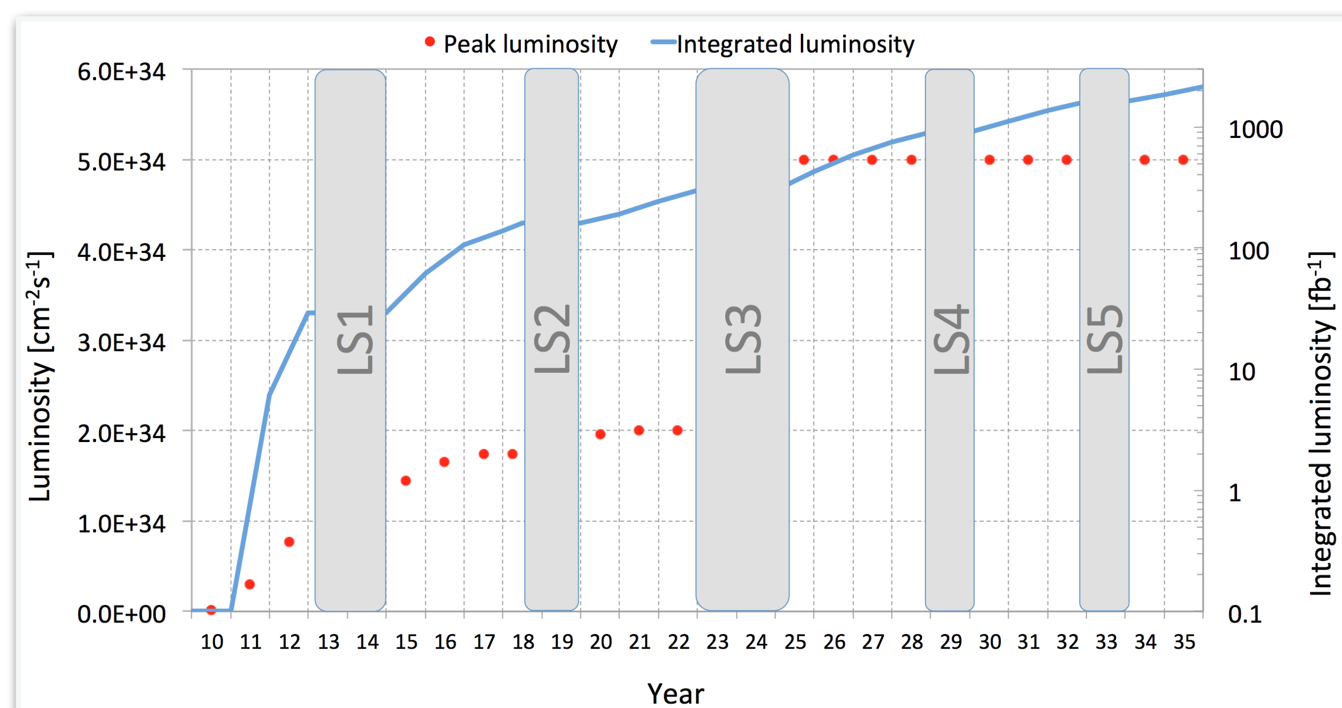
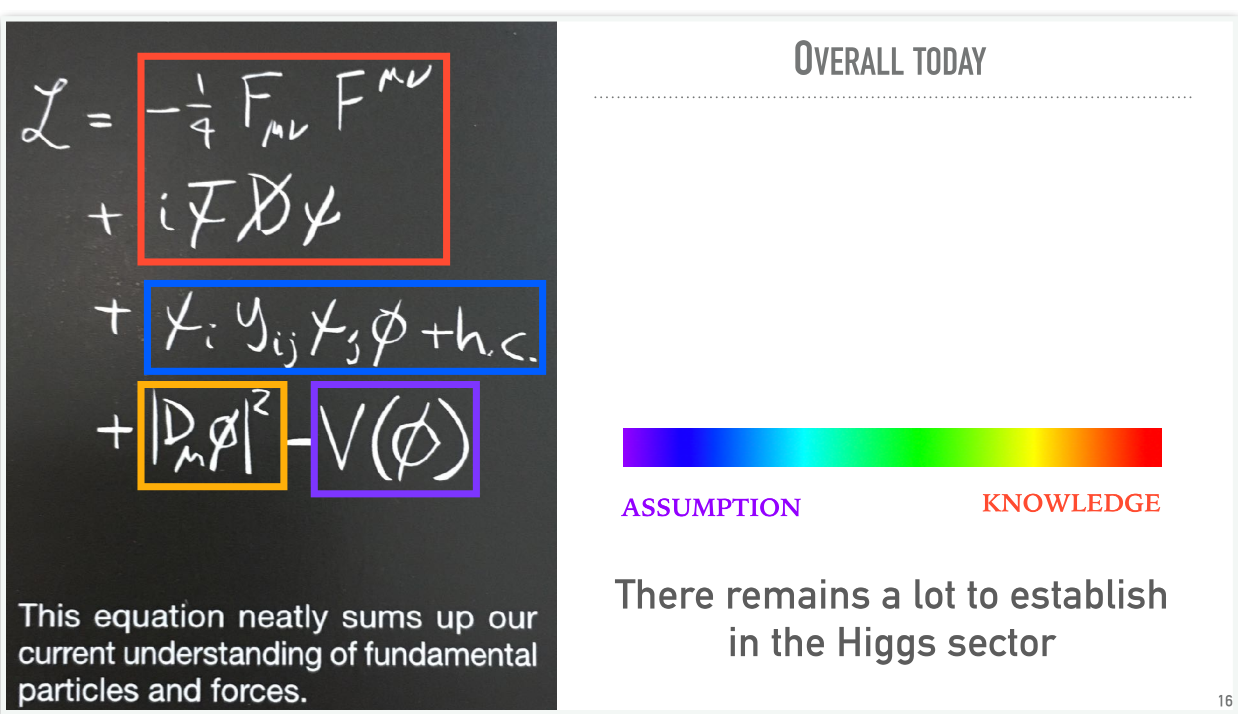


[LHC Schedule]

LARGE HADRON COLLIDER

- Higgs discovery: 04/07/2012
- After that no striking evidence of New Physics

- However:
  > there is still a lot of things to learn about the Standard Model
  > there is much more data coming…
- Despite being a hadron collider, will the LHC ultimately turn into a precision machine?
- For doing the precision physics we need the right tools!



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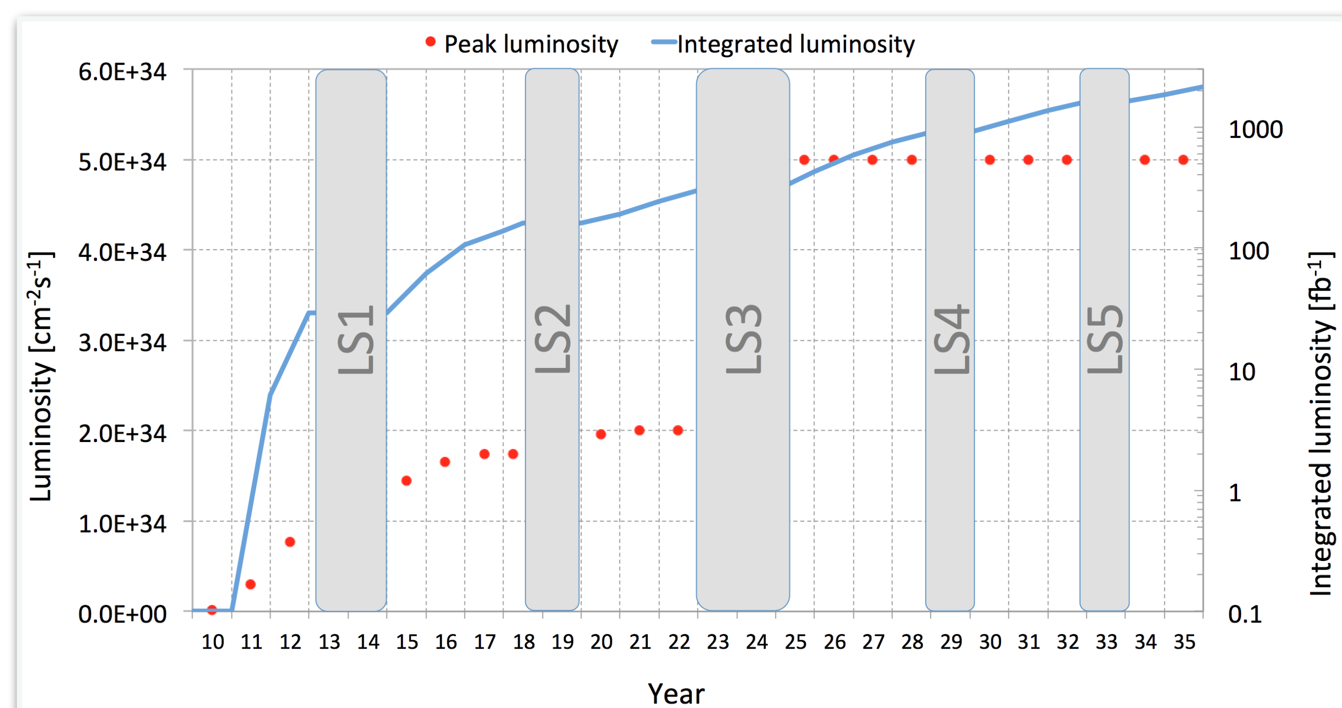


[LHC Schedule]

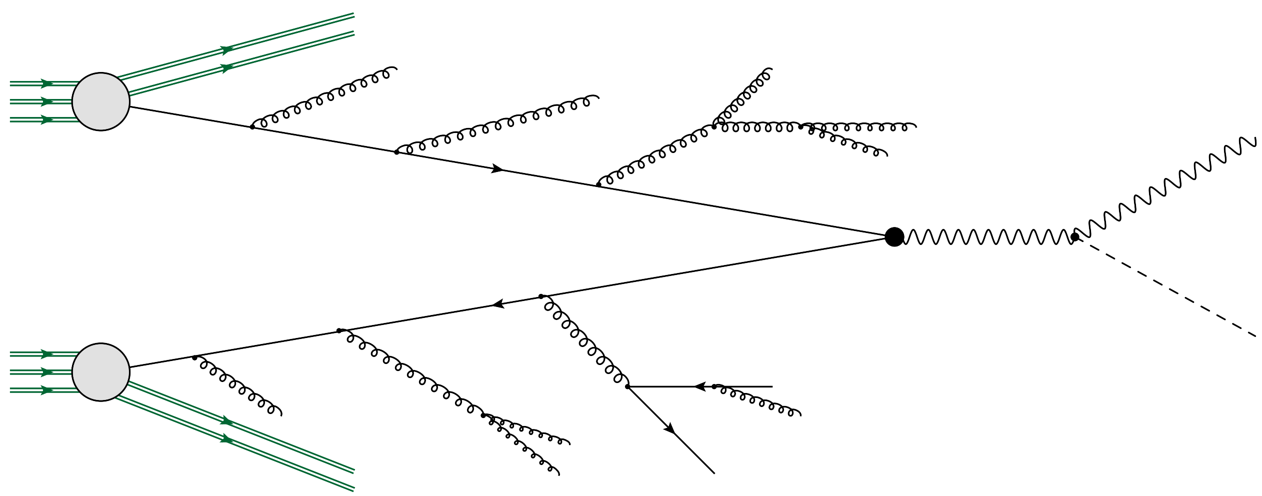
NNLO REVOLUTION

- The major challenge is posed by treatment of the strong interactions
- There has been a substantial progress in the next-to-next-to-leading order (NNLO) calculations
- Especially in the past two/three years, plenty of independent calculations…
  > 2-to-2, 2-to-3 processes with special kinematics
  > colour-singlet production, colour-singlet+jet, dijet production…
  [seminar two weeks ago: Raoul Rontsch]
- At NNLO:
  > scale uncertainty reduced
  > sometimes essential (large K-factors)
  > necessary step while moving towards precision physics…

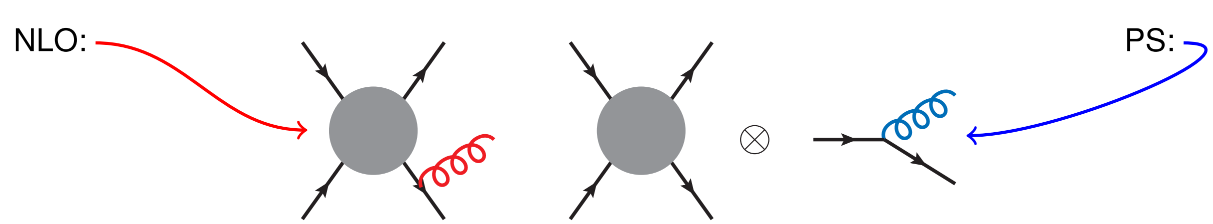
- Unfortunately: fixed-order perturbation theory:
  > handles only a limited number of particles in final-state (a few)
  > fails in regions dominated by enhanced soft and collinear radiation

PARTON SHOWERS

- **Goal:** transition from limited multiplicity to a realistic situation with 100–1000 particles in the final state
  Solution: parton shower (PS) algorithm based on knowledge of QCD in the soft/collinear region



- **Goal:** improve the accuracy of Monte Carlo event generators including as much information as possible from higher-order perturbative QCD (fixed-order, but also from resummation)



Consistent matching to fixed-order:

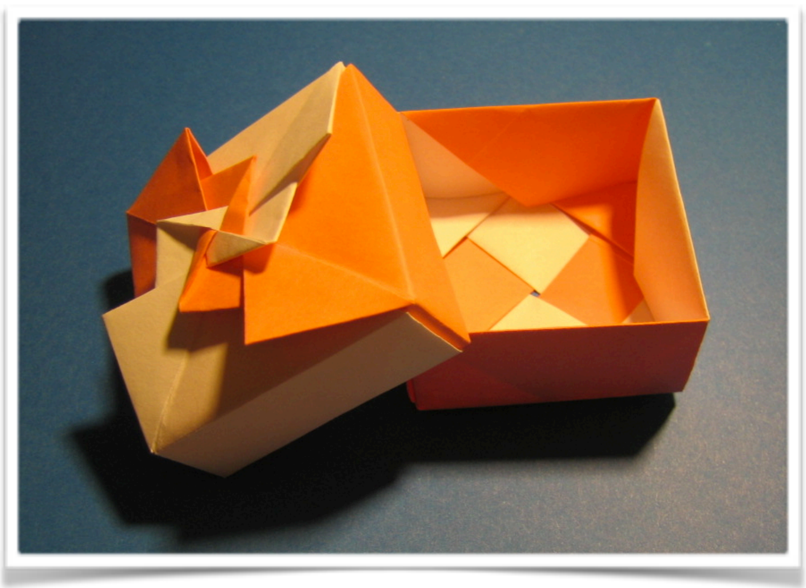
– at NLO+PS various methods constructed during last decades (automated, very advanced stage)

– at NNLO+PS first results about 4 years ago, the frontier is treatment of more complex processes – this talk

POWHEG

- POsitive Weight Hard Emission Generator – tool that enables a user to generate samples of hadronic collision events, which provide NLO QCD predictions for observables inclusive in radiation.
- In POWHEG, for each event, hardest radiation is generated. This provides upper scale for parton shower algorithms (so that NLO accuracy is not spoiled by parton shower).
- A large library of processes available.
- Continuous development of the software (POWHEG-BOX-RES).
- Frequently used by experimentalists

[Nason; hep-ph/0409146]
[Frixione, Nason, Oleari; 0709.2092]
[Alioli, Nason, Oleari, Re; 1002.2581]
[Campbell, Ellis, Nason, Re; 1412.1828]
[Jezo, Nason; 1509.09071]



MERGING VARIOUS JET-MULTIPLICITIES

- Consider generators for processes: X, X+j (X being a colour-singlet) producing an NLO accurate sample of events…
  Question: what happens when we want to investigate various observables?

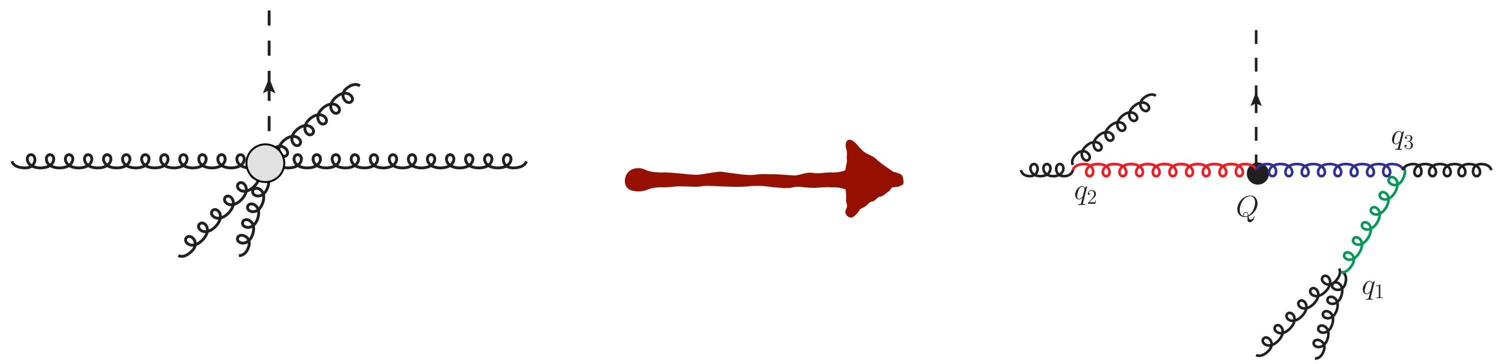
	0-jet	1-jet	2-jet	3-jet	…
X **@NLO**	NLO	LO	PS	PS	PS
X+j **@NLO**	infinite	NLO	LO	PS	PS

- Ideally we would like to have a single generator that would provide NLO accurate results for various jet multiplicities…
  Why?
  > high-pT tails more accurately described by NLO-matrix elements than by PS algorithm
- Often one uses a merging scale (Q_{MS}) and uses events from X/X+j based on a scale assigned to a jet (below/above Q_{MS})
- POWHEG uses a different approach – a prescription for assigning scales in multi-jet computation and correcting weights…

POWHEG + MINLO: RECIPE

[Hamilton, Nason, Zanderighi; 1206.3572]

➡ Multi-scale improved NLO (MiNLO): a recipe for assigning scales in NLO computation



Recipe:

(a) Find the most likely CKKW branching history of n-partons with clustering scales: q1 < q2 < … < q(n).

Q

(b) Evaluate strong coupling constant at each vertex according to scale q(i).

(c) Set the renormalisation scale to the geometric average of q1….q(n).

(d) Attach Sudakov form factors for each coloured line in Born, virtual and real.
[for the real after the first clustering, i.e. on the underlying Born event]

(e) Subtract the NLO bit present in the Sudakov of Born (avoid double-counting).

`Quick example >> next slide`

Sudakov FF = probability of not emitting a parton between scales Q[in] and Q[out]

POWHEG + MINLO: EXAMPLE

[Hamilton, Nason, Zanderighi; 1206.3572]

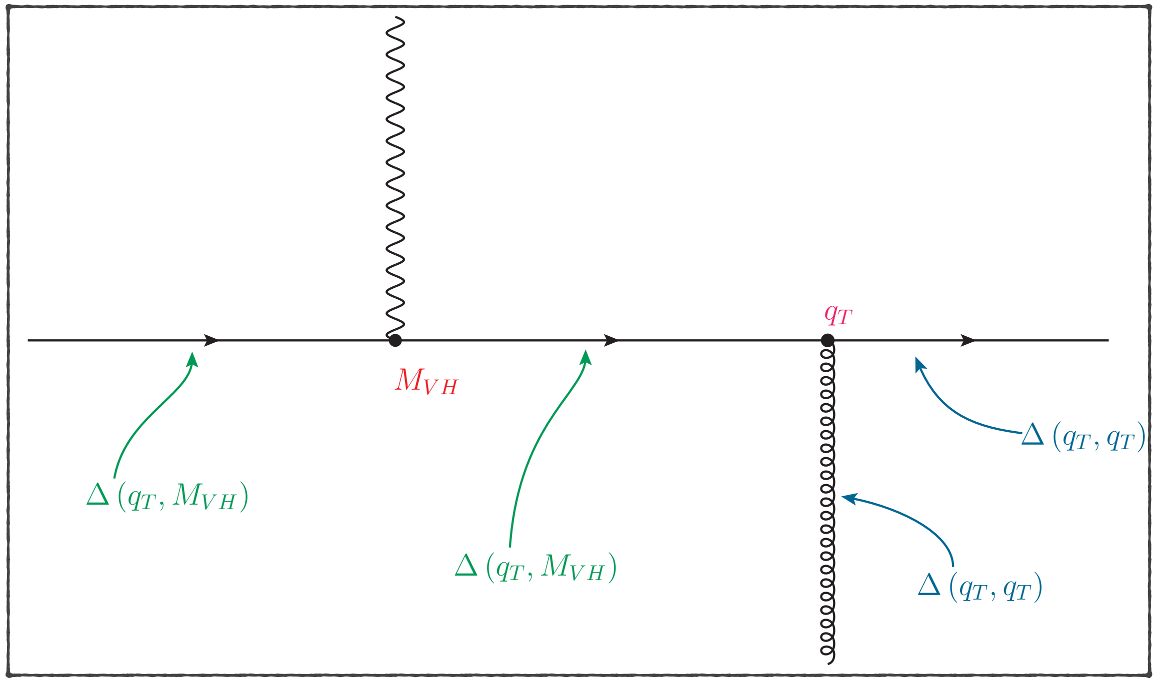
- POWHEG: X+j generator improved with MiNLO:

  Recipe:
  (a) Start with your old renormalisation scale (M_{VH}).
  (b) Change scale for each QCD vertex (CKKW-like clustering)
  (c) attach Sudakov form factors for each coloured line

- Resulting function to integrate:

$$\tilde{B}_{\mathtt{MiNLO}} = \alpha_s(q_T)\Delta^2(q_T,\bar{\mu}_R)\left[B\left(1-2\Delta^{(1)}(q_T,\bar{\mu}_R)\right)+\alpha_s(\bar{\mu}_R)\left(V(\bar{\mu}_R)+\int d\Phi_r R\right)\right]$$

  Result:
  (a) emissions at low qT are damped
  (b) finite result in the vanishing qT limit (unresolved jet)
  (c) no generation cut / Born suppression factor needed
  (d) possible to retain NLO accuracy >> next slides



POWHEG + MINLO: NLO ACCURACY PROOF

- Take a look at the resummed formula for colour-singlet production

$$\frac{d\sigma}{d\Phi_B dq_T^2} = \left(\frac{\hat{\sigma}_0}{d\Phi_B}\right)_{ij} \frac{d}{dq_T^2}\Big\{ \left[C_{ia} \otimes f_a\right](x_1, q_T) \times \left[C_{jb} \otimes f_b\right](x_2, q_T) \times \Delta_i(Q, q_T) \times \Delta_j(Q, q_T)\Big\} + R_f$$

- after integration over qT from 0 (strongly suppressed by Sudakov FF) up to the hard scale (Q):

$$\frac{d\sigma}{d\Phi_B} = \left(\frac{\hat{\sigma}_0}{d\Phi_B}\right)_{ij} \left[C_{ia} \otimes f_a\right](x_1, Q) \times \left[C_{jb} \otimes f_b\right](x_2, Q) + \int dq_T^2 R_f + \dots$$

Conclusion: the formula is NLO(X) accurate if:

> coefficient functions C are accurate up to first order, O(as)

> R_f (non-singular part of the cross-section) is accurate at O(as), meaning LO(Xj)

- NLO(X) accuracy is maintained by construction, independently of particular form of the Sudakov form factor, as long as we include the aforementioned terms…
- However, inside the POWHEG-BOX code we integrate the formula after taking the derivative… Next slide: which terms do we need to keep and which ones might be discarded?

POWHEG + MINLO: NLO ACCURACY PROOF

- Take the derivative of the resumed expression:

$$\frac{d\sigma}{d\Phi_B dq_T^2} = \left(\frac{\hat{\sigma}_0}{d\Phi_B}\right)_{ij} \frac{d}{dq_T^2}\Big\{ \left[C_{ia} \otimes f_a\right](x_1, q_T) \times \left[C_{jb} \otimes f_b\right](x_2, q_T) \times \Delta_i(Q, q_T) \times \Delta_j(Q, q_T)\Big\} + R_f$$

- obtain terms of the form ($L = \log\left(Q^2/q_T^2\right)$):

$$\left(\frac{\hat{\sigma}_0}{d\Phi_B}\right)_{ij} \frac{1}{q_T^2}\left[\alpha_\mathrm{s},\, \alpha_\mathrm{s}^2,\, \alpha_\mathrm{s}^3,\, \alpha_\mathrm{s}^4,\, \alpha_\mathrm{s} L,\, \alpha_\mathrm{s}^2 L,\, \alpha_\mathrm{s}^3 L,\, \alpha_\mathrm{s}^4 L,\, \right] \times \Delta_i(Q, q_T) \times \Delta_j(Q, q_T)$$

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$\mathcal{O}(\alpha_s^{1/2})$ $\mathcal{O}(\alpha_s^{5/2})$ $\mathcal{O}(\alpha_s^2)$ $\mathcal{O}(\alpha_s^3)$

$\mathcal{O}(\alpha_s^{3/2})$

$\mathcal{O}(\alpha_s)$

Counting:

$\alpha_s L^2 \sim 1$

$\Rightarrow L \sim 1/\sqrt{\alpha_s}$

$$\boxed{\int_{\Lambda^2}^{Q^2} \frac{dq_T^2}{q_T^2}\,\alpha_s^n(q_T^2)\log^m\left(\frac{Q^2}{q_T^2}\right)\exp\left\{\mathcal{S}(Q,q_T)\right\} \approx \left[\alpha_s(Q^2)\right]^{n-\frac{(m+1)}{2}}}$$

POWHEG + MINLO: NLO ACCURACY PROOF

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- obtain terms of the form ($L = \log\left(Q^2/q_T^2\right)$):

$$\left(\frac{\hat{\sigma}_0}{d\Phi_B}\right)_{ij} \frac{1}{q_T^2} \left[\boxed{\alpha_s}, \boxed{\alpha_s^2}, \boxed{\alpha_s^3}, \alpha_s^4, \boxed{\alpha_s L}, \boxed{\alpha_s^2 L}, \boxed{\alpha_s^3 L}, \alpha_s^4 L, \right] \times \Delta_i(Q, q_T) \times \Delta_j(Q, q_T)$$

$\mathcal{O}(\alpha_s^{1/2})$ $\boxed{\mathrm{B}_1}$

$\mathcal{O}(\alpha_s^{3/2})$ $\boxed{\mathrm{B}_2}$

$\cancel{\mathcal{O}(\alpha_s^{5/2})}$

$\mathcal{O}(\alpha_s)$ $\boxed{\mathrm{A}_1}$

$\mathcal{O}(\alpha_s^2)$ $\boxed{\mathrm{A}_2}$

$\cancel{\mathcal{O}(\alpha_s^3)}$

can be neglected…

have to be included…

<u>*Counting:*</u>

$\alpha_s L^2 \sim 1$

$\Rightarrow L \sim 1/\sqrt{\alpha_s}$

$$\boxed{\int_{\Lambda^2}^{Q^2} \frac{dq_T^2}{q_T^2}\, \alpha_s^n(q_T^2) \log^m\left(\frac{Q^2}{q_T^2}\right) \exp\left\{\mathcal{S}(Q, q_T)\right\} \approx \left[\alpha_s(Q^2)\right]^{n - \frac{(m+1)}{2}}}$$

POWHEG + MINLO: NLO ACCURACY PROOF

	0-jet	1-jet	2-jet	3-jet	…
X @NLO	NLO	LO	PS	PS	PS
X+j @MiNLO	NLO	NLO	LO	PS	PS

SUMMARY:

A single generator for the process (X+j), improved with MiNLO recipe can yield NLO accurate results both for (X+j) observables as well as inclusive X observables without any merging scale or a generation cut for a jet.

H/Z/W >> [Hamilton, Nason, Oleari, Zanderighi; 1212.4504]
HW/HZ >> [Luisoni, Nason, Oleari, Tramontano; 1306.2542]
and others….

POWHEG + MINLO: NLO ACCURACY PROOF

	0-jet	1-jet	2-jet	3-jet	…
X @NLO	NLO	LO	PS	PS	PS
X+j @MiNLO	NLO	NLO	LO	PS	PS
X+2j @MiNLO’	NLO	NLO	NLO	LO	PS

[Hamilton,Frederix; 1512.02663]
// numerical estimation of B2 coefficient, as a function of the Born phase-space //

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ROAD TO NNLO+PS

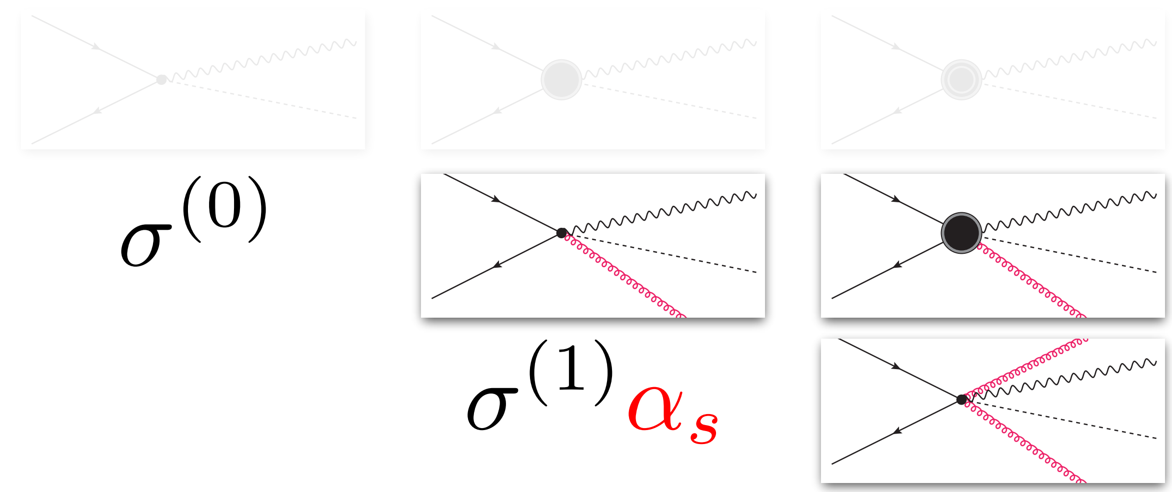
	0-jet	1-jet	2-jet	3-jet	…
X @NLO	NLO	LO	PS	PS	PS
X+j @MiNLO	NNLO (?)	NLO	LO	PS	PS

➡ What is missing??

- NLO (VH+J) computation:

$$\sigma_{\mathtt{PWHG}}(VH+j) = \tilde{\sigma}^{(1)}\alpha_s + \tilde{\sigma}^{(2)}{\alpha_s}^2$$

$\mathcal{O}(1)$ $\quad$ $\mathcal{O}(\alpha_s)$ $\quad$ $\mathcal{O}({\alpha_s}^2)$



$\sigma^{(0)}$

$\sigma^{(1)}\alpha_s$

$\sigma^{(2)}{\alpha_s}^2$

ROAD TO NNLO+PS

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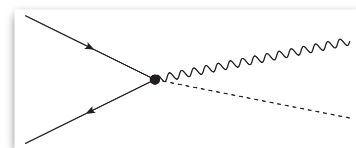
- NLO (VH+J) computation:

$$\sigma_{\texttt{PWHG}}(VH+j) = \tilde{\sigma}^{(1)}\alpha_s + \tilde{\sigma}^{(2)}\alpha_s{}^2$$

- after integrating out real radiation (`MiNLO`):

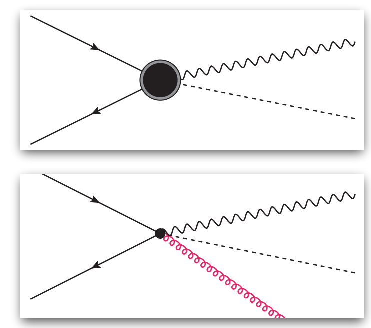
$$\sigma_{\texttt{PWHG}}(VH) = \sigma^{(0)} + \sigma^{(1)}\alpha_s + \tilde{\sigma}^{(2)}\alpha_s{}^2$$

$\mathcal{O}(1)$



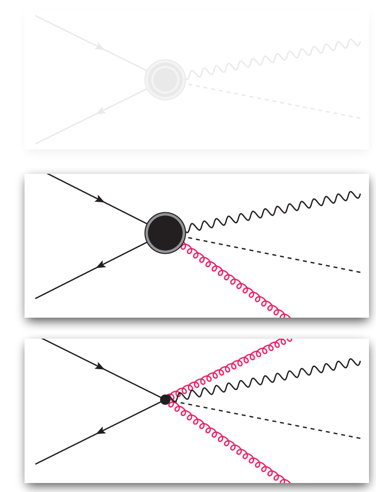
$\sigma^{(0)}$

$\mathcal{O}(\alpha_s)$



$\sigma^{(1)}\alpha_s$

$\mathcal{O}(\alpha_s{}^2)$



$\sigma^{(2)}\alpha_s{}^2$

ROAD TO NNLO+PS

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- NLO (VH+J) computation:

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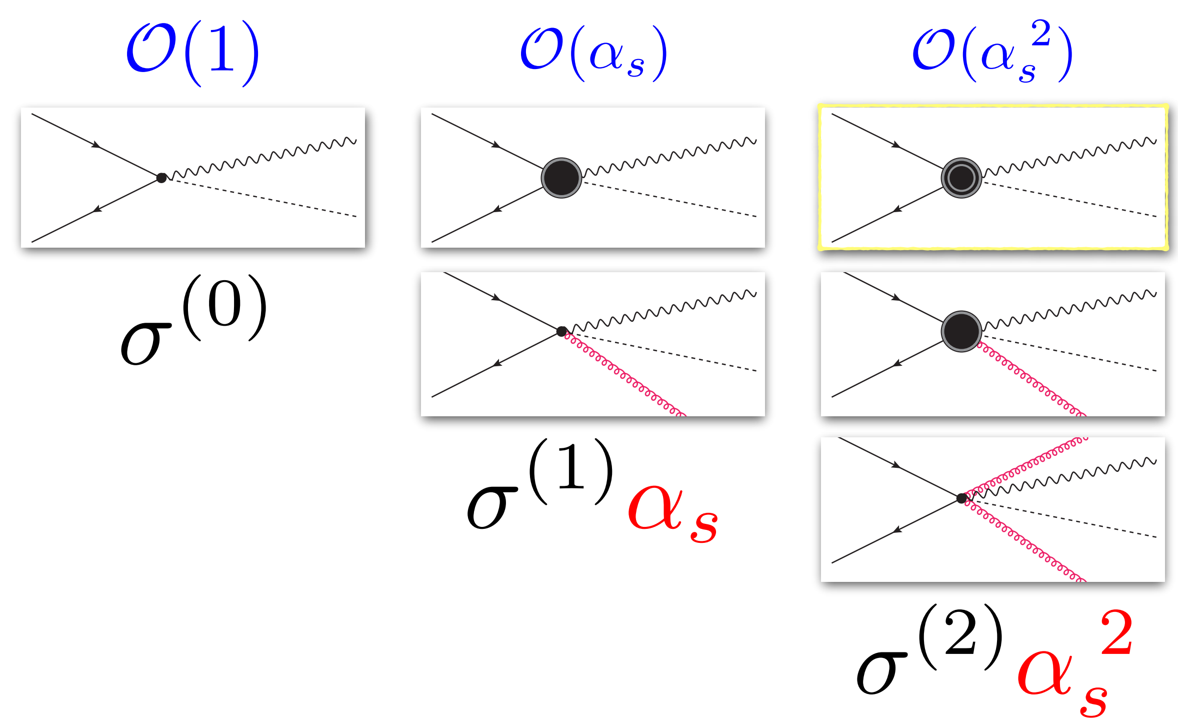
- after integrating out real radiation (`MiNLO`):

$$\sigma_{\mathtt{PWHG}}(VH) = \sigma^{(0)} + \sigma^{(1)}\alpha_s + \tilde{\sigma}^{(2)}\alpha_s^{\,2}$$

- whereas full NNLO:

$$\sigma_{\mathtt{NNLO}}(VH) = \sigma^{(0)} + \sigma^{(1)}\alpha_s + \sigma^{(2)}\alpha_s^{\,2}$$



ROAD TO NNLO+PS

- NLO accurate predictions from set of events produced by MiNLO generator:

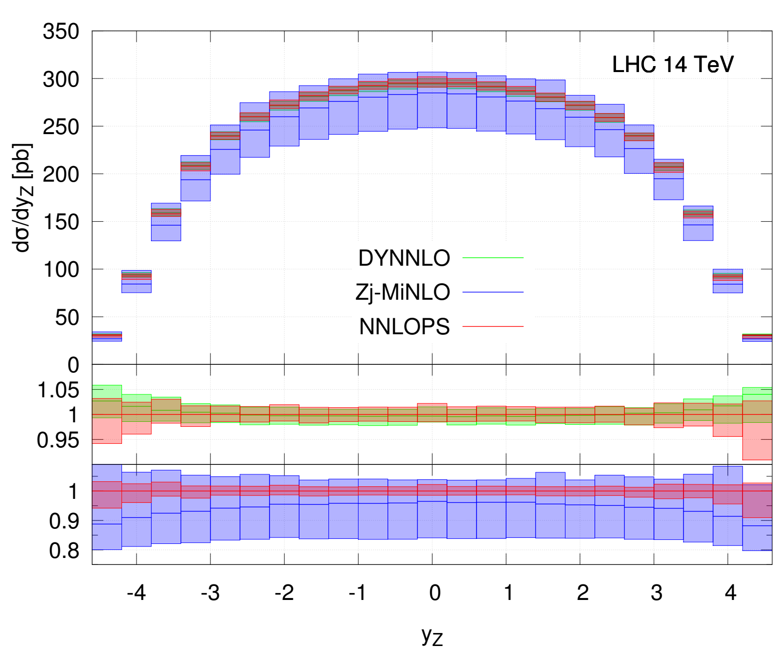
$$\texttt{MiNLO-events:} \quad \sum_i w_i \longrightarrow \sigma_{\texttt{MiNLO}} = \sigma^{(0)} + \sigma^{(1)}\alpha_s + \tilde{\sigma}^{(2)}{\alpha_s}^2$$

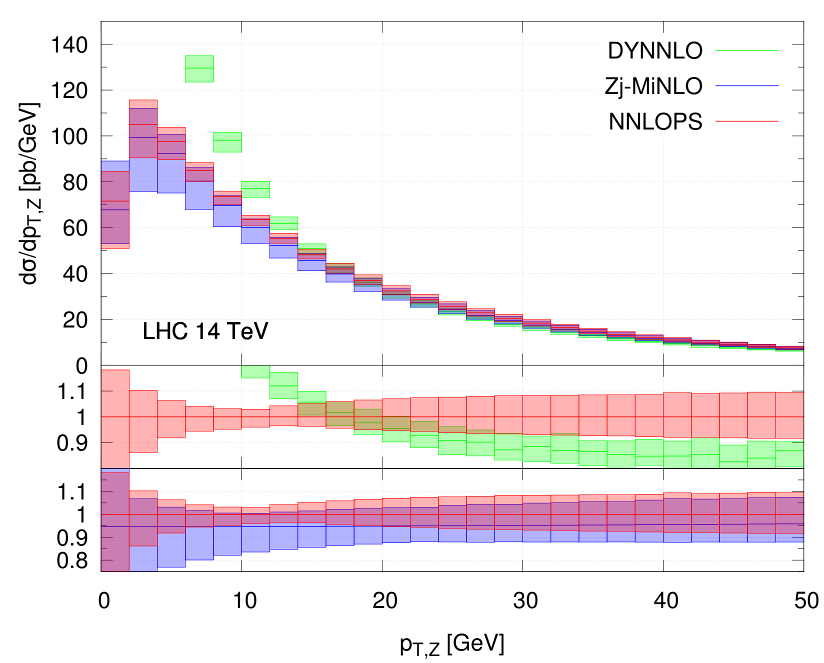
- Rescale all weights by a factor W which is differential in Born kinematics:

$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\texttt{NNLO}}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\texttt{MiNLO}}} = \frac{d\sigma^{(0)} + d\sigma^{(1)}\alpha_s + d\sigma^{(2)}{\alpha_s}^2}{d\sigma^{(0)} + d\sigma^{(1)}\alpha_s + d\tilde{\sigma}^{(2)}{\alpha_s}^2} = 1 + \frac{d\sigma^{(2)} - d\tilde{\sigma}^{(2)}}{d\sigma^{(0)}}{\alpha_s}^2 + \mathcal{O}({\alpha_s}^3)$$

- Such rescaling gives NNLO accurate set of events (by construction):

$$\texttt{NNLO-events:} \quad \sum_i w_i \times W(\Phi_B) \longrightarrow \sigma_{\texttt{NNLO}}$$





Conclusions:

1. Significant reduction of scale uncertainty for inclusive observables.
2. No singular behaviour at small Z-transverse momentum (Sudakov peak).

[Karlberg, Re, Zanderighi; 1407.2940]

ROAD TO NNLO+PS

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$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\texttt{NNLO}}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\texttt{MiNLO}}} = \frac{d\sigma^{(0)} + d\sigma^{(1)}\alpha_s + d\sigma^{(2)}\alpha_s{}^2}{d\sigma^{(0)} + d\sigma^{(1)}\alpha_s + d\tilde{\sigma}^{(2)}\alpha_s{}^2} = 1 + \frac{d\sigma^{(2)} - d\tilde{\sigma}^{(2)}}{d\sigma^{(0)}}\alpha_s{}^2 + \mathcal{O}(\alpha_s{}^3)$$

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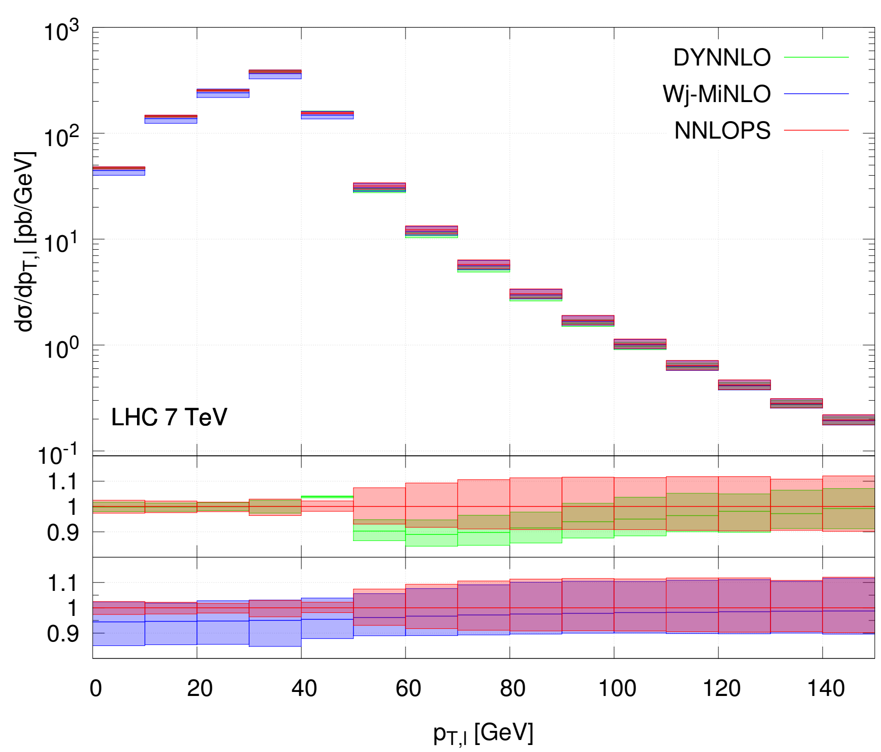


Interesting observable:

Interesting observable: lepton pT in W-production. Transition from small to large scale-uncertainty (above Jacobi peak).

[Karlberg, Re, Zanderighi; 1407.2940]

ROAD TO NNLO+PS

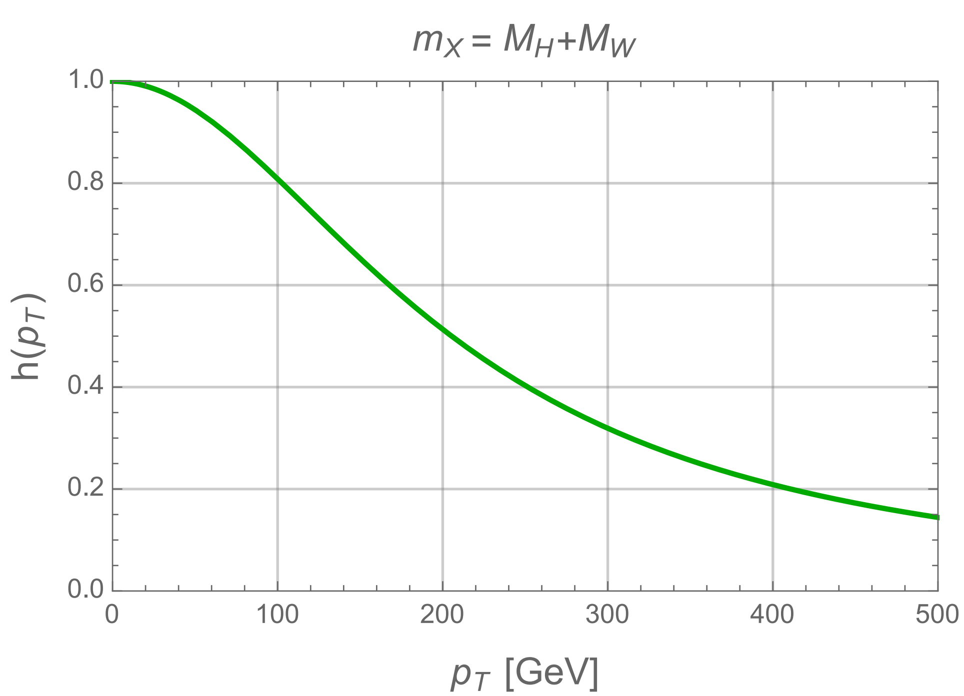
- Reminder: we are starting from X+j@MiNLO generator: all real corrections of the X@NNLO calculations are already included…
- We can use the variant of the reweighting procedure, splitting the cross-section

pT = transverse momentum of the hardest jet

$$h(p_T) = \frac{(m_X)^2}{(m_X)^2 + (p_T)^2}$$

$$d\sigma_A = d\sigma \cdot h(p_T)$$
$$d\sigma_B = d\sigma \cdot (1 - h(p_T))$$

$$d\sigma = d\sigma_A + d\sigma_B$$



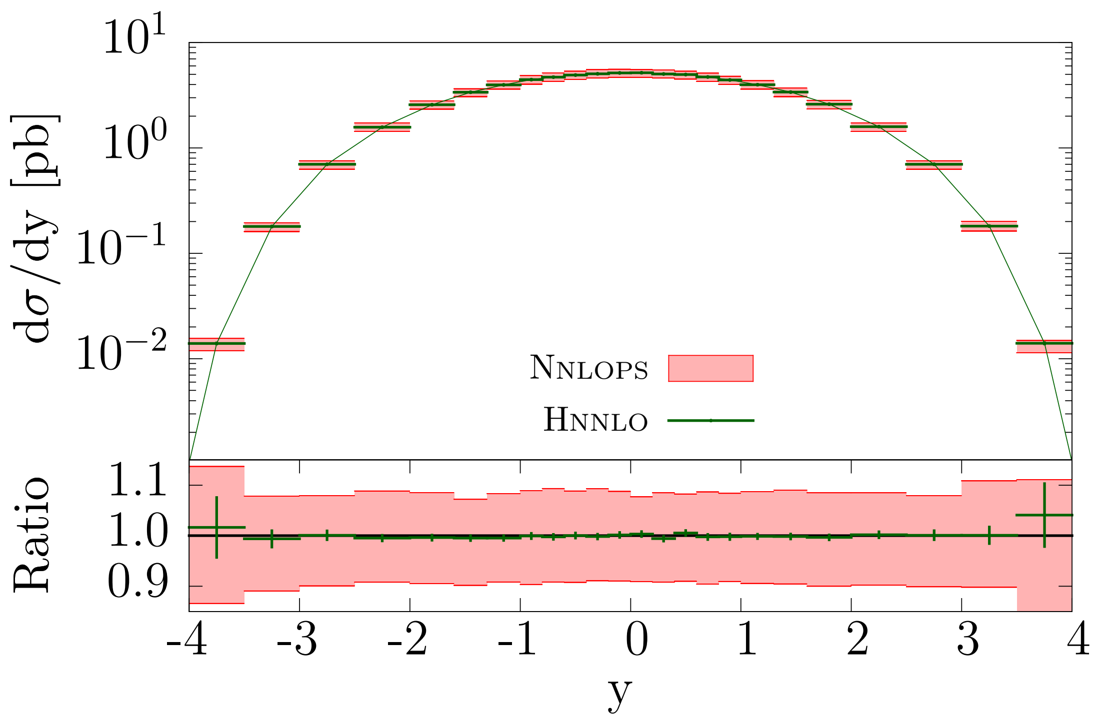
$$W(\Phi_B) = h(p_T) \cdot \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathtt{NNLO}}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathtt{MiNLO}}} + (1 - h(p_T))$$

Result:
NNLO corrections are concentrated around region with small-pT. Effect on high-pT tail is minimised, as it was already described with the same nominal accuracy in X+j@MiNLO generator.

ROAD TO NNLO+PS: GROWING COMPLEXITY

- NNLO reweighting factor W is a function of fully-differential kinematics.
- With more complicated phase-space, procedure (though formally simple) becomes computationally involving…

(a) Higgs production: (1-dimension) $\mapsto$ 1 variable (1D histogram, e.g. **25 bins**)



[Hamilton, Nason, Re, Zanderighi; 1309.0017]
[Karlberg, Re, Zanderighi; 1407.2940]
[Astill, Bizon, Re, Zanderighi; 1603.01620]

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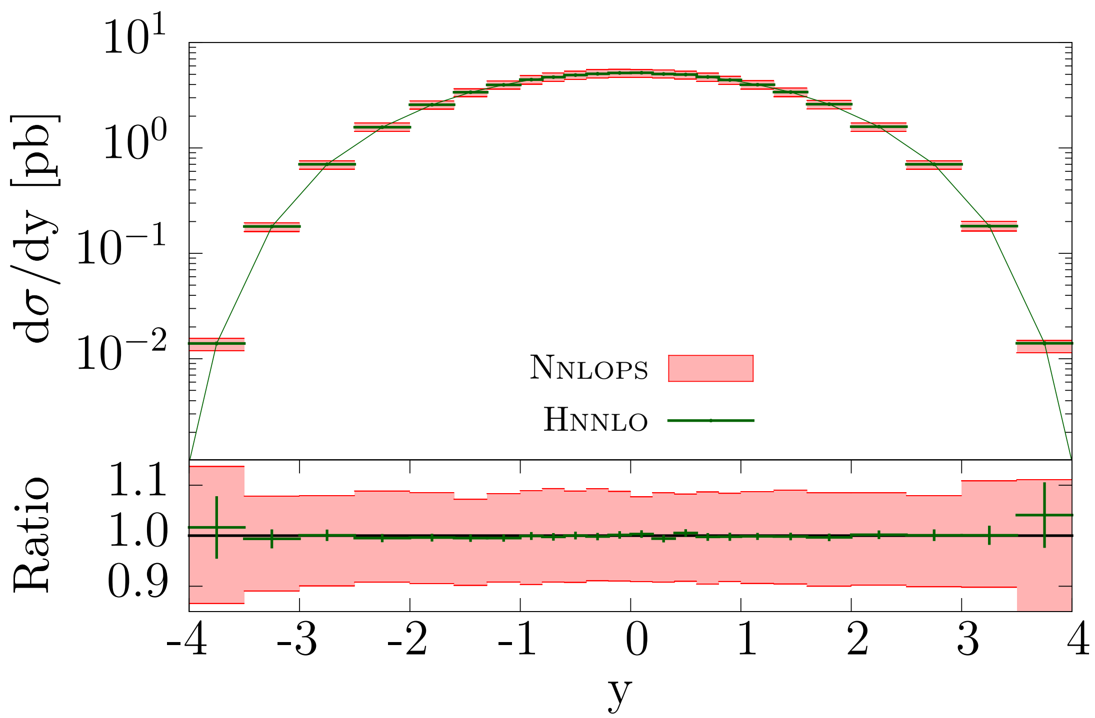
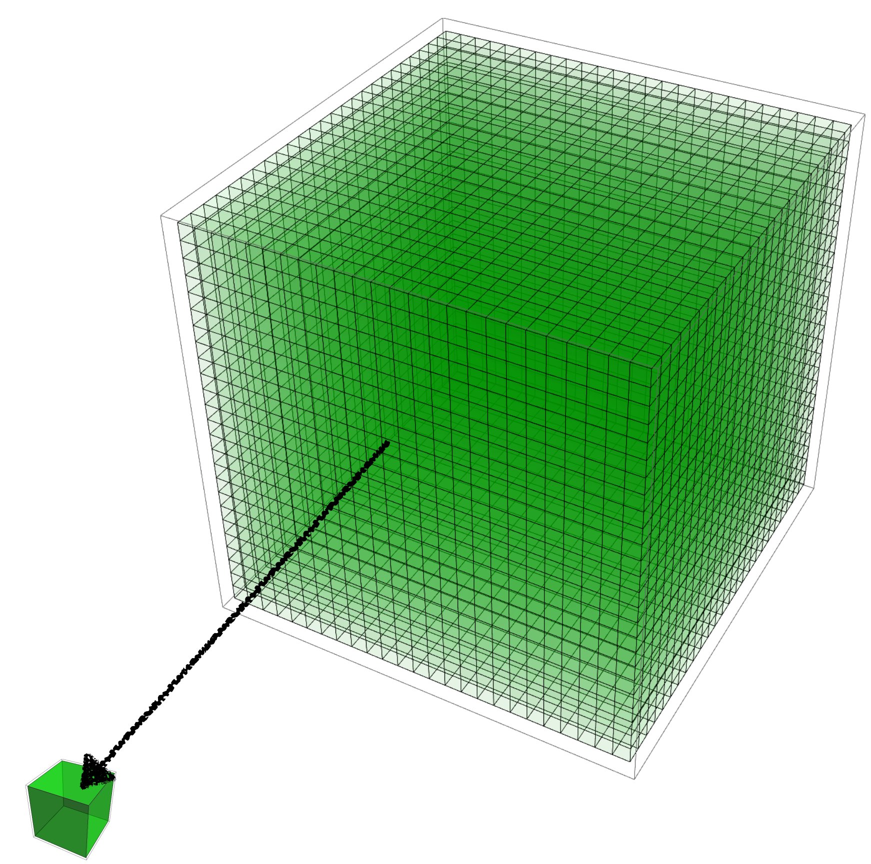
(b) Drell-Yan production: (3-dimentions) $\longmapsto$ 3 variables (3D histogram, 25^3 = **15 625 bins**)





$$\frac{d^3\sigma}{dy_V d\theta_l dm_V} = \texttt{SINGLE-NUMBER}$$

[Hamilton, Nason, Re, Zanderighi; 1309.0017]
[Karlberg, Re, Zanderighi; 1407.2940]
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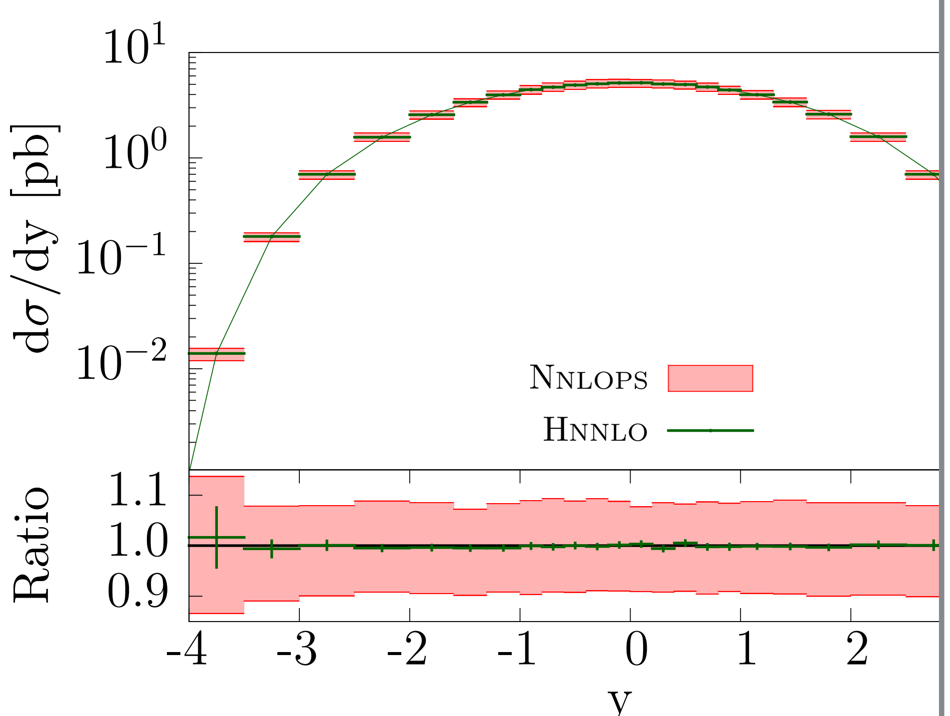
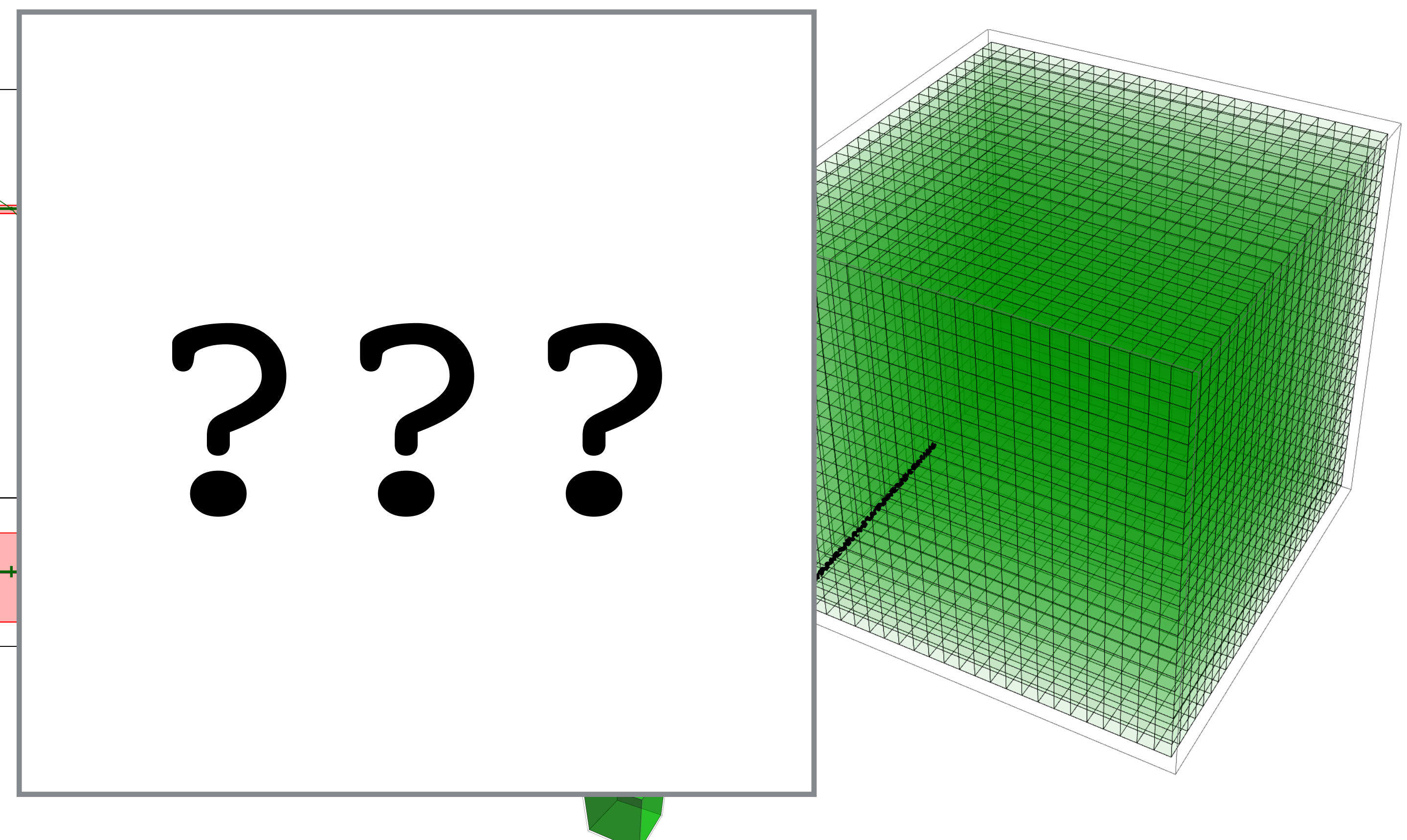
ROAD TO NNLO+PS: GROWING COMPLEXITY

- NNLO reweighting factor W is a function of fully-differential kinematics.
- With more complicated phase-space, procedure (though formally simple) becomes computationally involving…

(a) Higgs production: (1-dimension) $\longmapsto$ 1 variable (1D histogram, e.g. **25 bins**)
(b) Drell-Yan production: (3-dimentions) $\longmapsto$ 3 variables (3D histogram, 25^3 = **15 625 bins**)
(c) VH production: (6-dimentions) $\longmapsto$ 6 variables (6D histogram, 25^6 = **??? (244M bins)**)





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ASSOCIATED HIGGS PRODUCTION AT NNLO+PS

➡ VH production: 6-dimensional Born phase-space

1	2	3	4	5	6

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➡ VH production: 6-dimensional Born phase-space

1	2	3	4	5	6
y_{VH}	$p_{t,H}$	Δy			

(a) 3D histograms

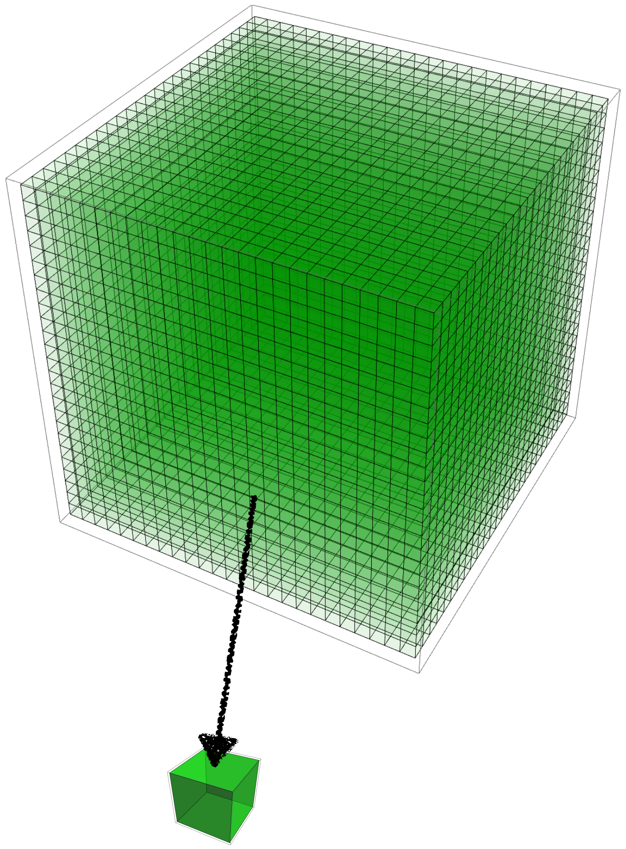
1. Use approach similar to the one from previous projects (3D histograms with 25 x 25 x 25 bins):

Dimensions being:

(1) X= y_{VH}

(2) Y= $p_{t,H}$

(3) Z= Δy



$$\frac{d^3\sigma}{dX\,dY\,dZ}$$

ASSOCIATED HIGGS PRODUCTION AT NNLO+PS

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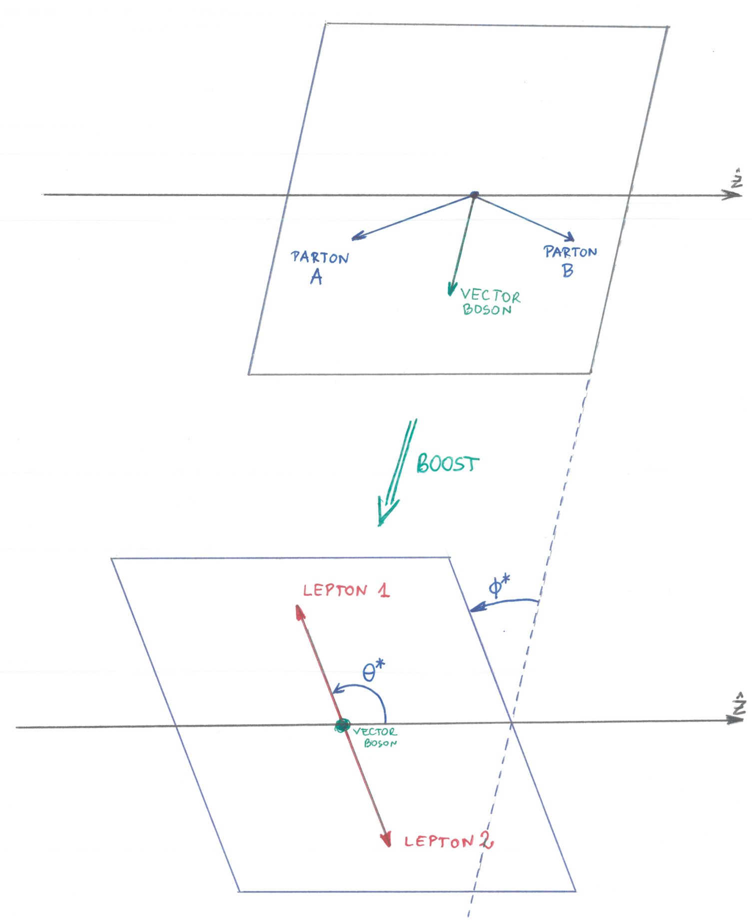
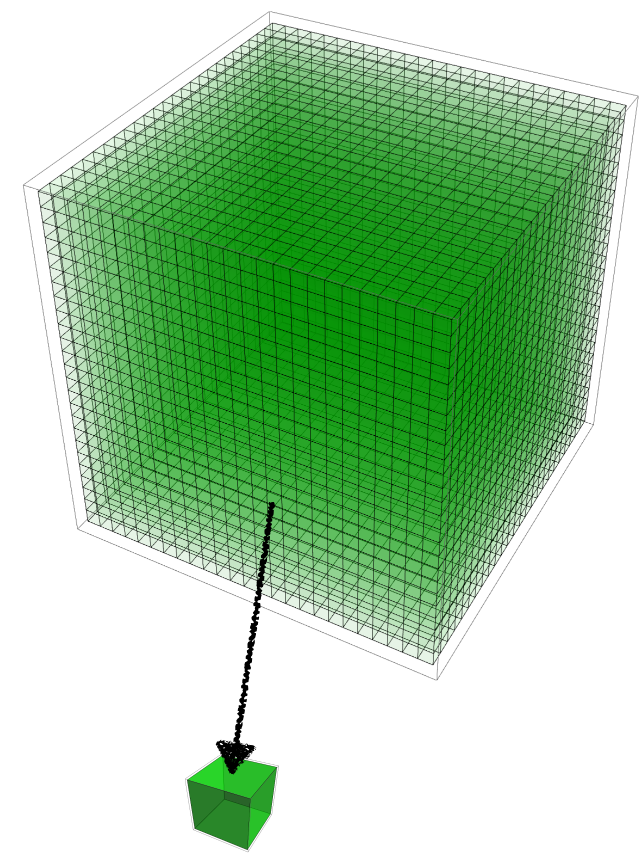
1	2	3	4	5	6
y_{VH}	$p_{t,H}$	Δy	θ^*	ϕ^*	

(b) Collins-Soper parametrisation

Definition:

- vector boson at rest
- **z-axis:** bisects angle between [PARTON A] and –[PARTON B]
- **x-axis:** –([PARTON A] + [PARTON B])





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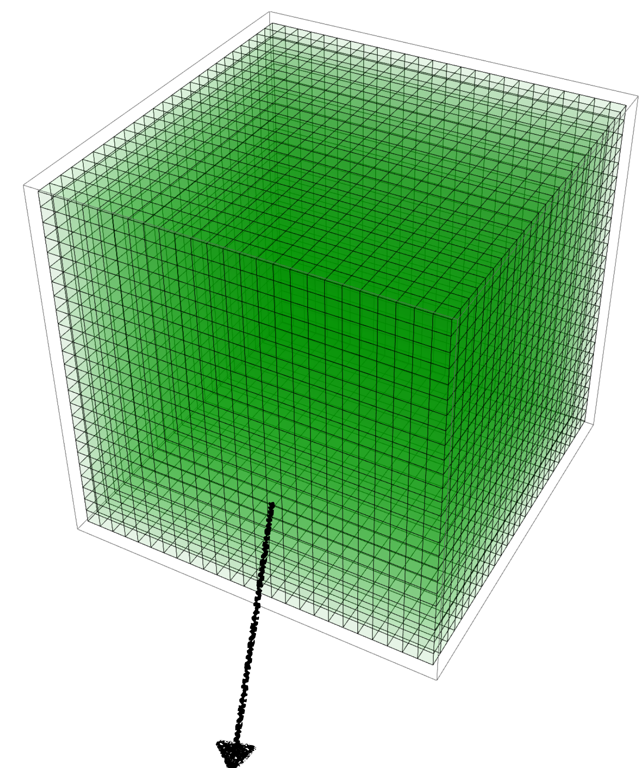
Why use this frame?

- cross-section in terms of (8+1) coefficients:
  Instead of 25^2=625 bins we have 9 numbers.

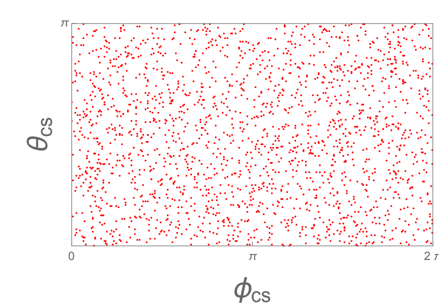
$$\frac{d\sigma}{d(\cos\theta^*)d\phi^*} = \frac{3}{16\pi}\frac{\sigma}{}\Big[(1+\cos^2\theta^*) + A_0\frac{1}{2}(1-3\cos^2\theta^*) + A_1 \sin 2\theta^* \cos\phi^* + A_2\frac{1}{2}\sin^2\theta^*\cos 2\phi^* + A_3\sin\theta^*\cos\phi^* + A_4\cos\theta^* + A_5\sin\theta^*\sin\phi^* + A_6\sin 2\theta^*\sin\phi^* + A_7\sin^2\theta^*\sin 2\phi^*\Big]$$

- analytical expressions are usually better than numbers
- frame often used in experiment



x 9

$$< A_i > = \int d\phi^* d\theta^* \ldots$$



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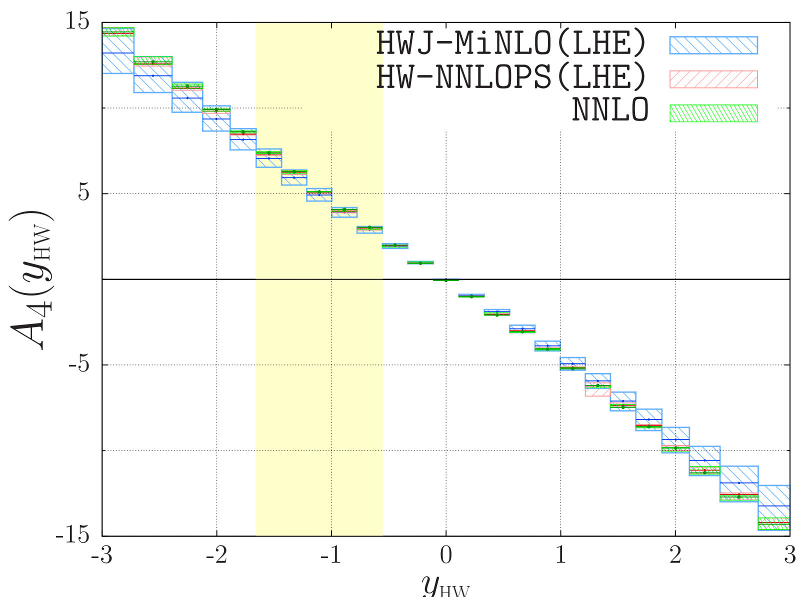
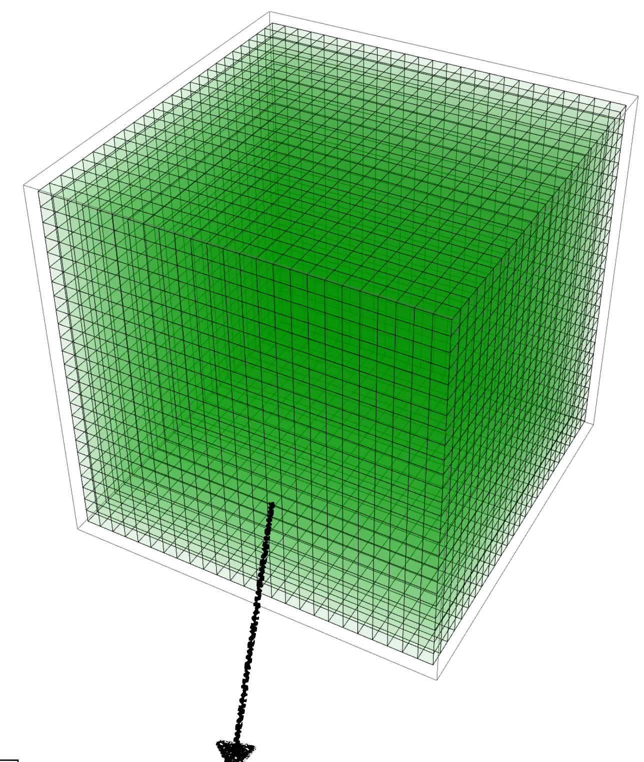
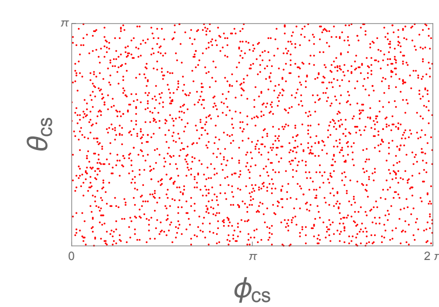


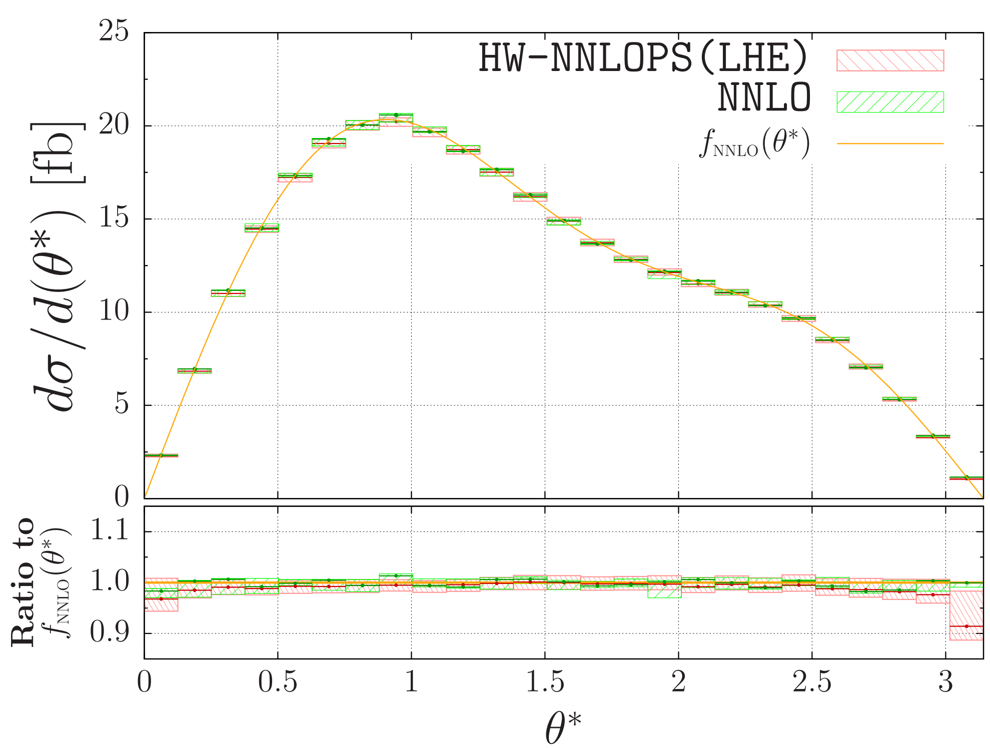
+ all other coefficients



x 9

$$< A_i >= \int d\phi^* d\theta^* \ldots$$





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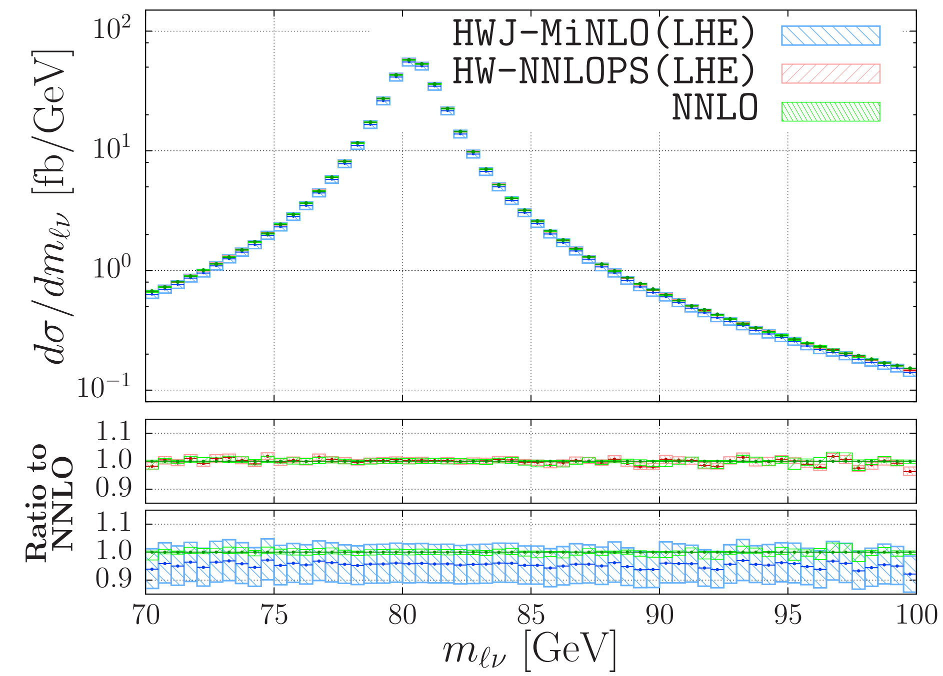
(c) Breit-Wigner shape of vector boson

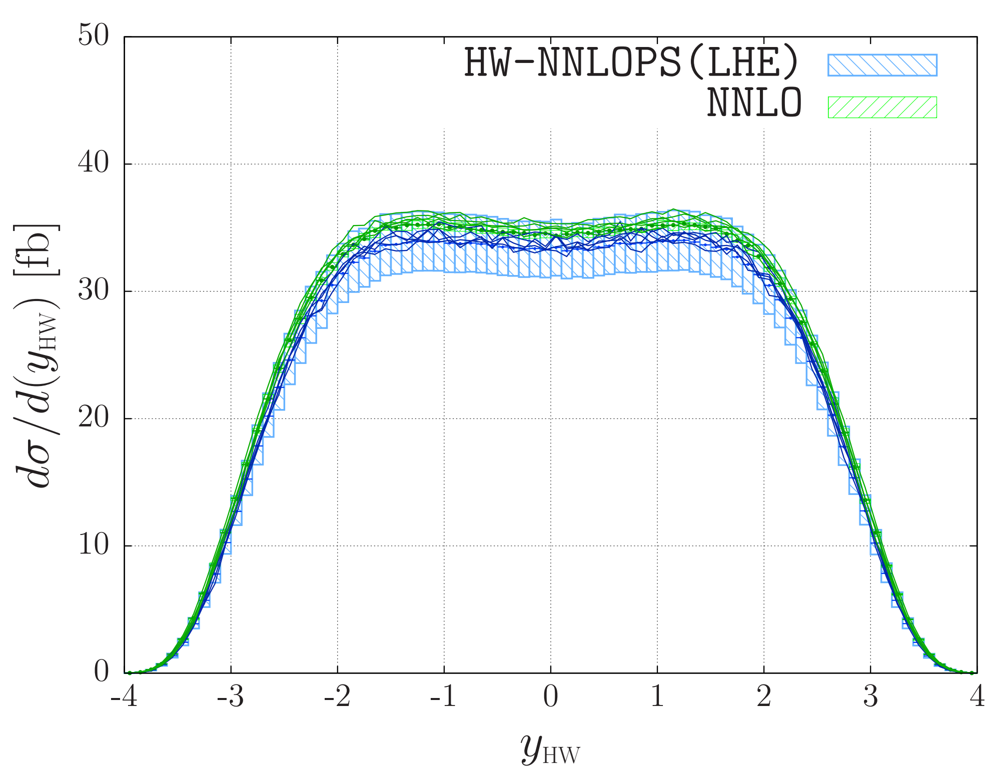
1. Distribution of lepton pair (from V-decay) invariant mass should take a form of Breit-Wigner shape.

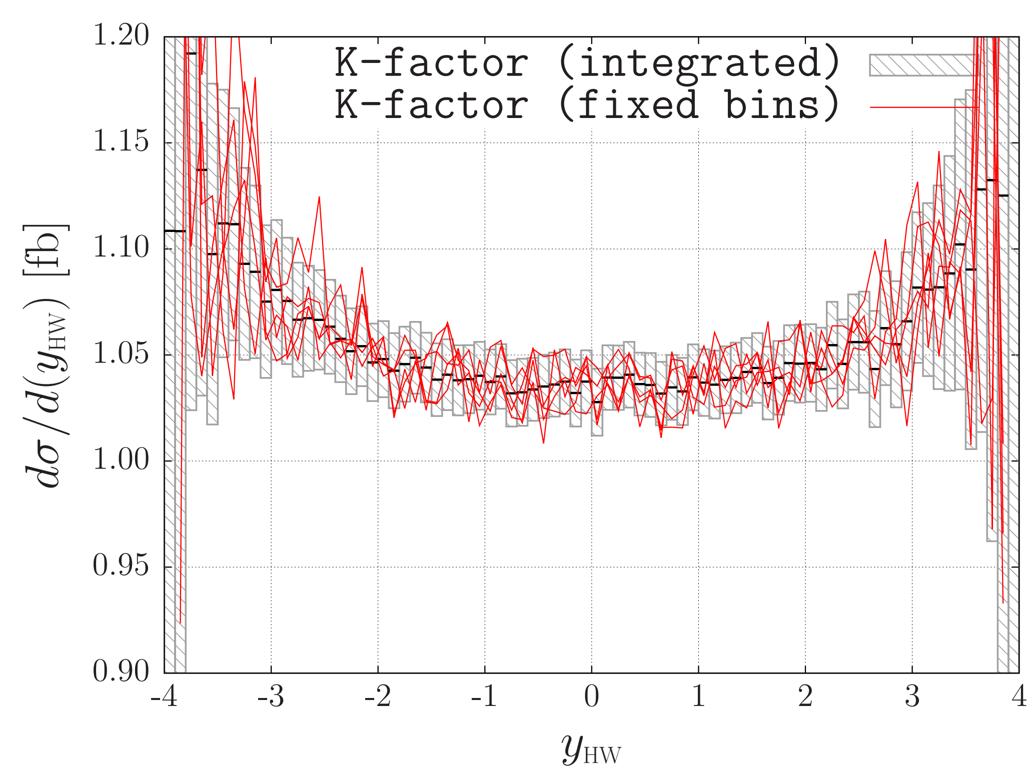
2. Expected that reweighting factor should be independent of lepton pair invariant mass.

Conclusion:

Neglect flat dimension of the phase space ($m_{\ell\bar{\ell}'}$) while doing reweighting.







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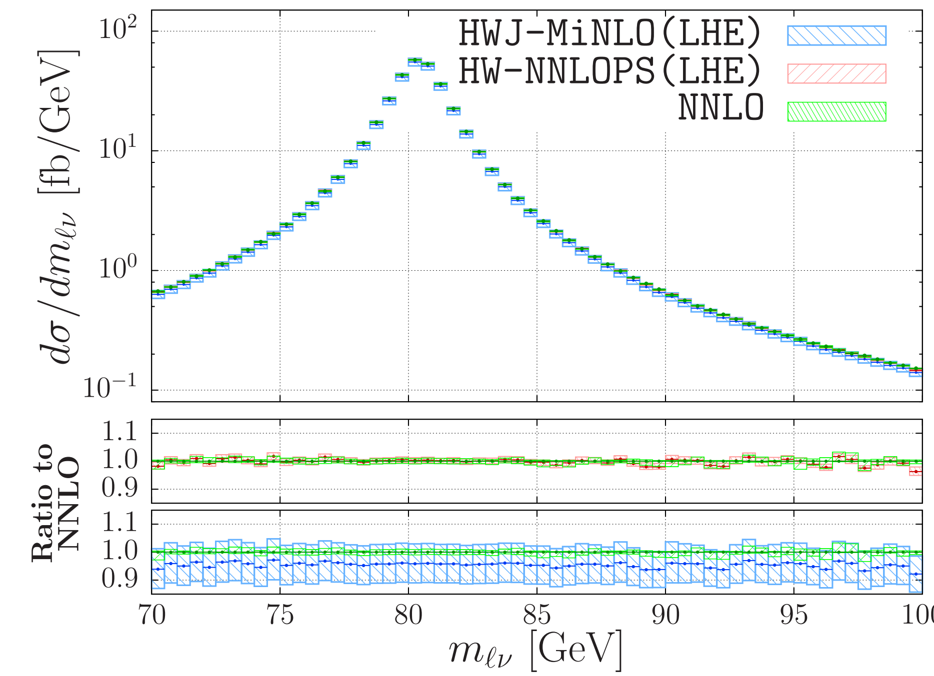
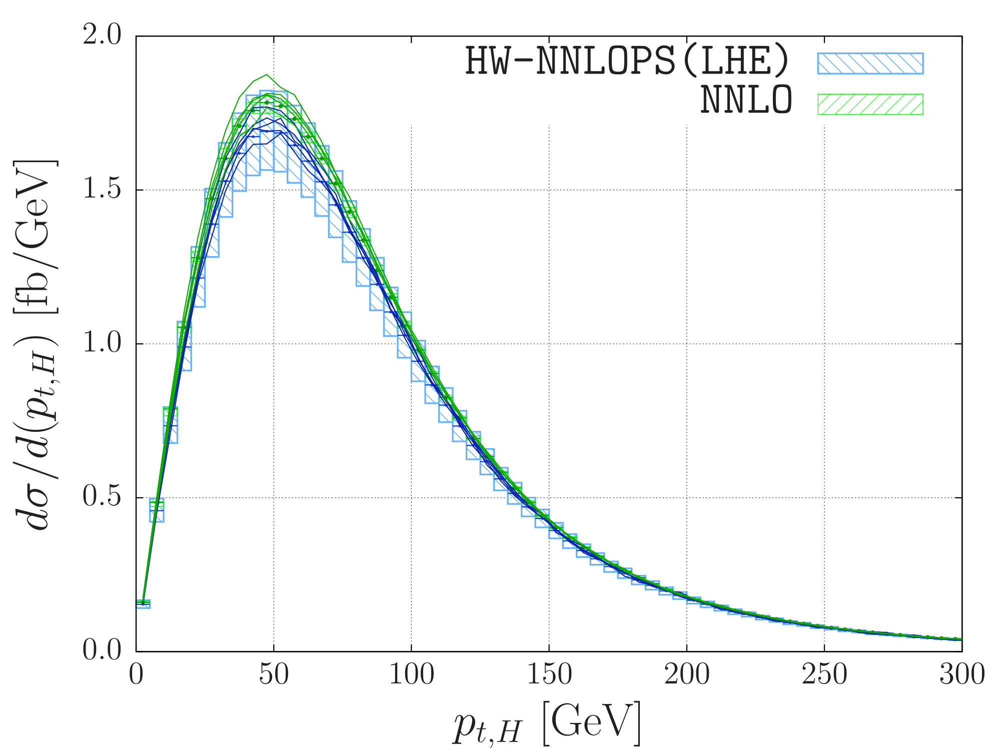
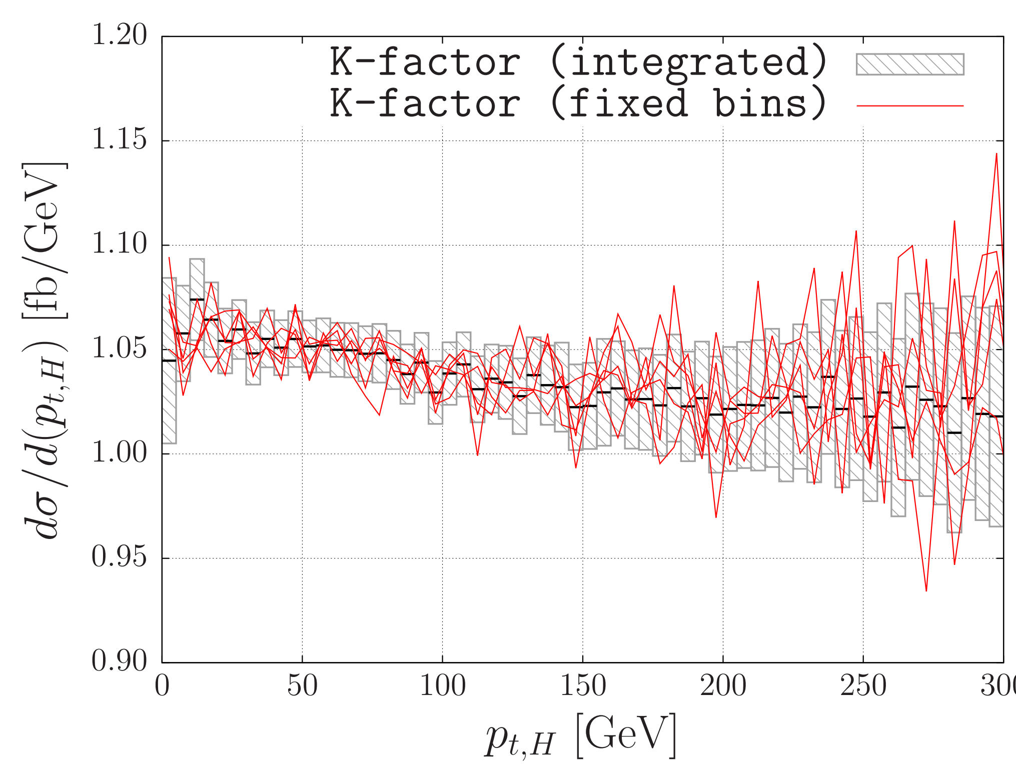
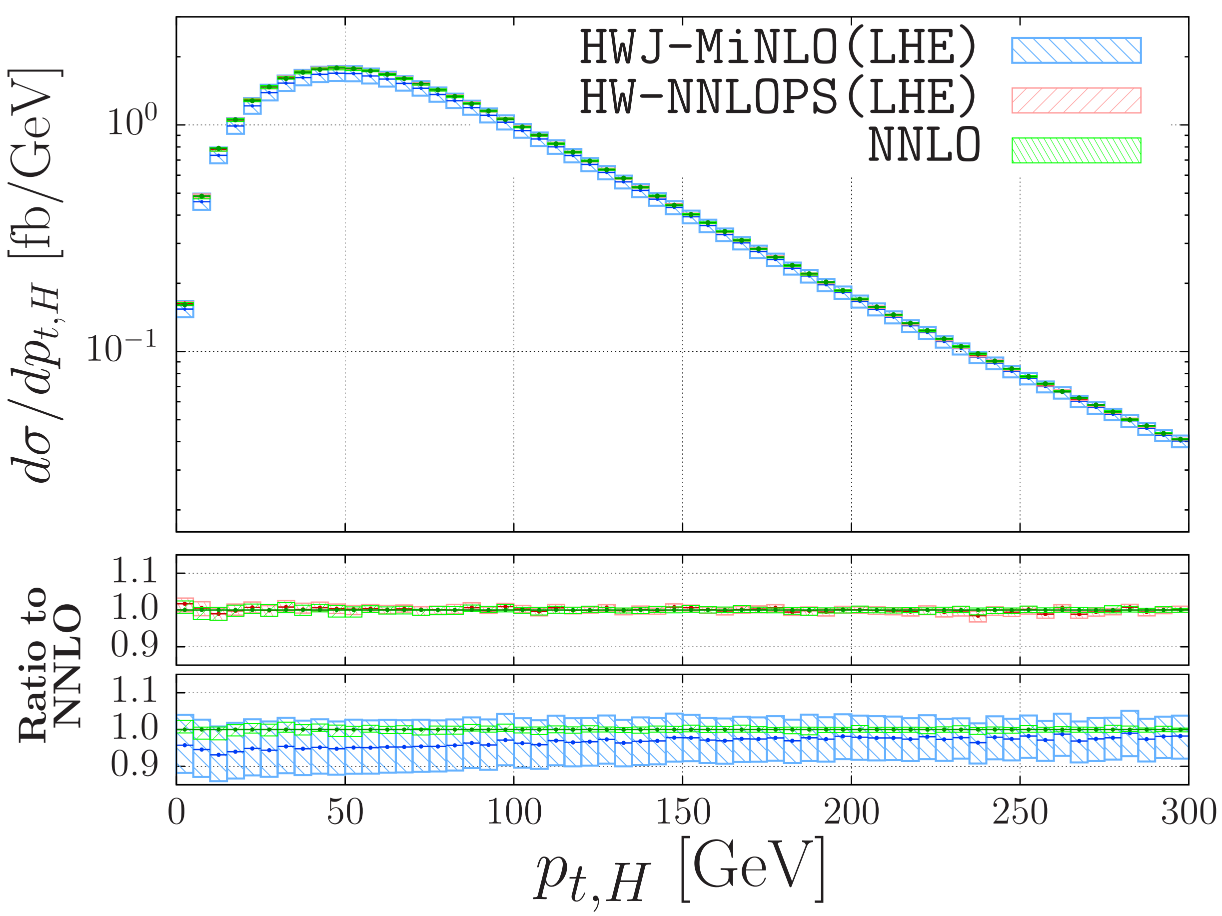
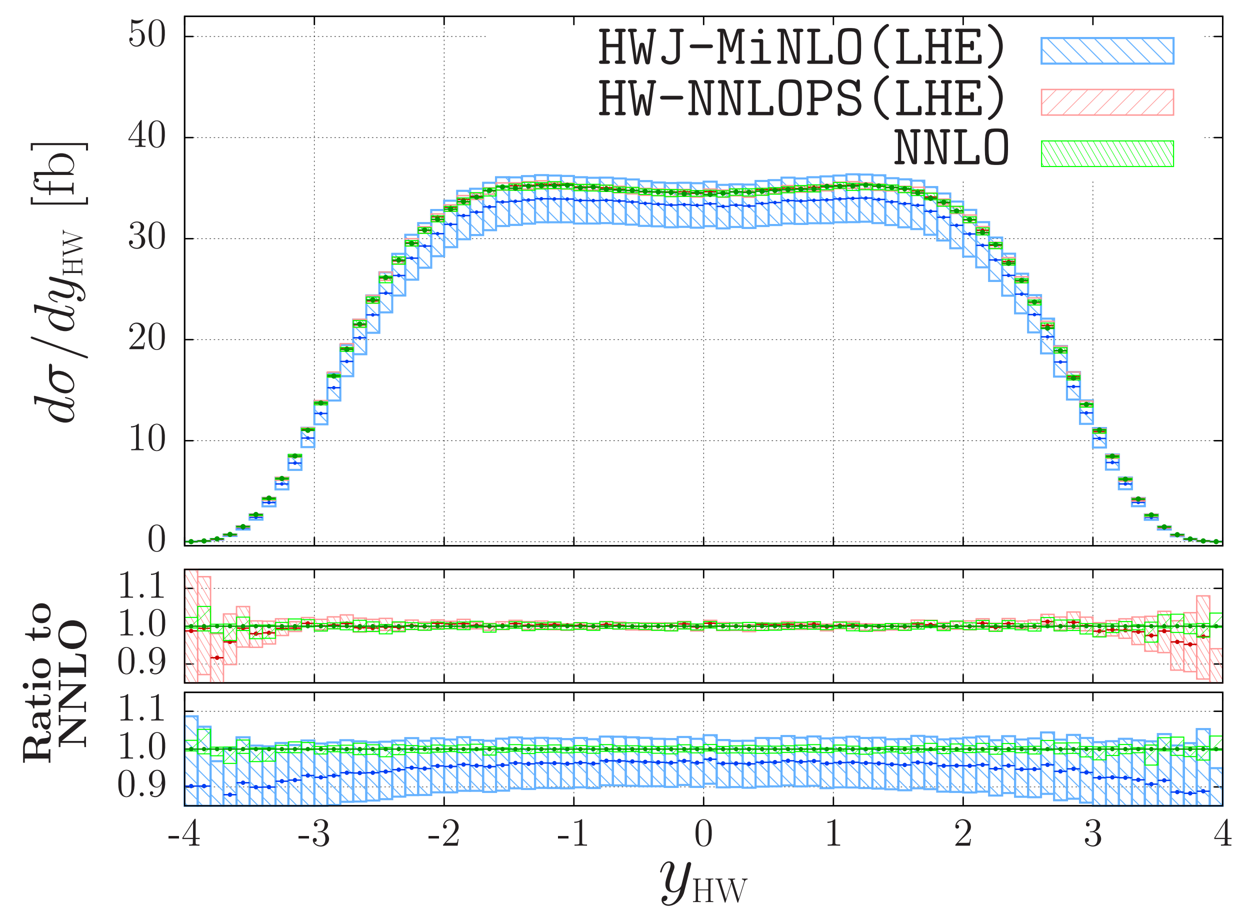






RESULTS (HW)





1. Reweighting for HW-inclusive observables:
   (a) reproduces fixed-order NNLO
   (b) reduction of scale uncertainty (as expected at NNLO)

[Astill, Bizon, Re, Zanderighi; 1603.01620]

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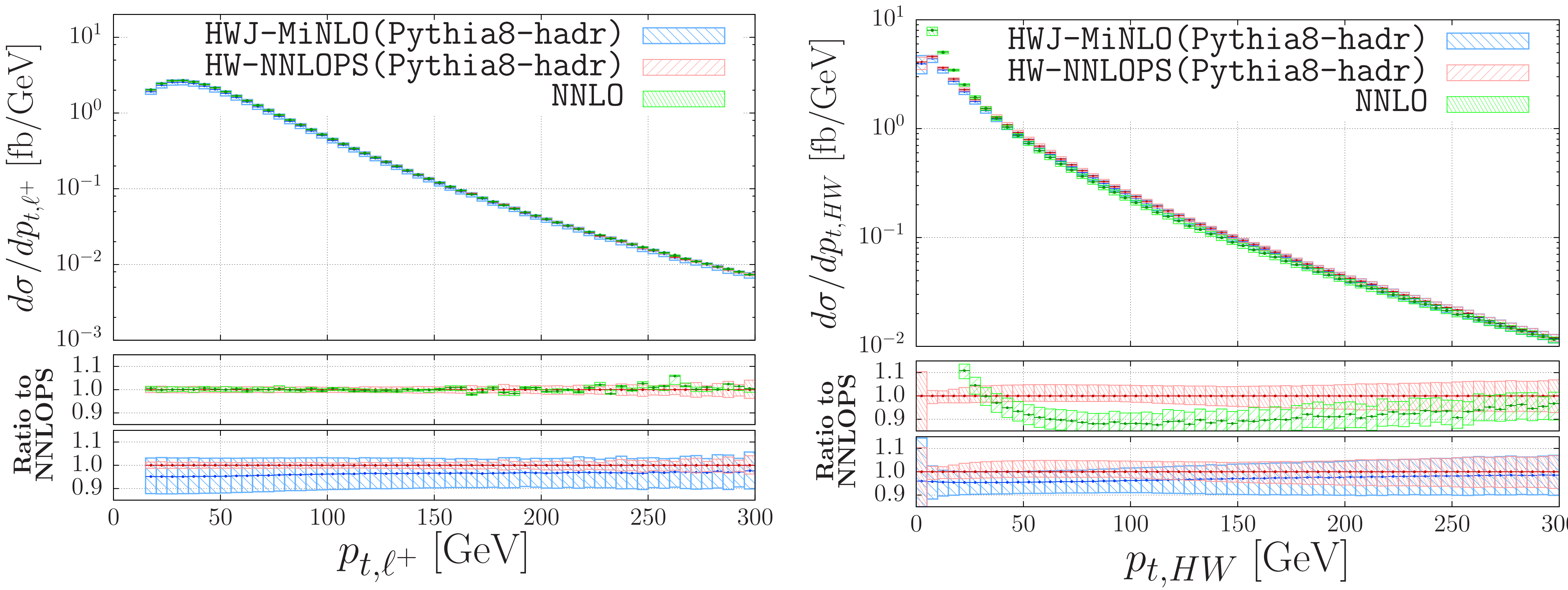


1. Reweighting for HW-inclusive observables:
   (a) reproduces fixed-order NNLO
   (b) reduction of scale uncertainty (as expected at NNLO)
   (c) observables that were not used for reweighting also reproduced

2. For observables which are singular at Born level (HW+j inclusive):
   (a) scale uncertainty is not affected (NLO accuracy)
   (b) differences in NNLO and NNLOPS due to different scale choice

[Astill, Bizon, Re, Zanderighi; 1603.01620]

RESULTS (HW)

Cross-section binned in 6 categories: according to presence of jets and transverse momentum of Higgs boson (YR4 recommendation):

(1) $0 < p_{t,H} < 150$ GeV

(2) $150 \text{ GeV} < p_{t,H} < 250$ GeV

(3) $250 \text{ GeV} < p_{t,H}$

<u>Large differences between NNLO and NNLOPS!</u>

(a) during parton shower evolution, some of QCD radiation ends up outside the jet hence jets are softened (jet-veto cross sections are larger)

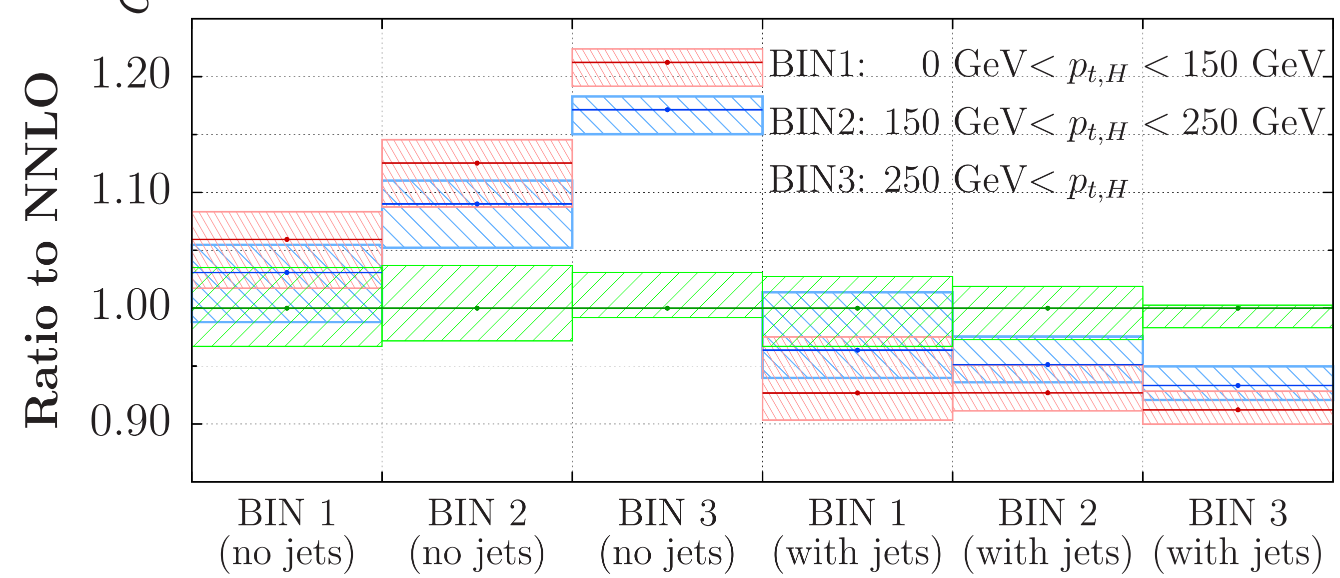
(b) pt-jet cut was set to 20 GeV which is close to the point where NNLO diverges

(c) further corrections due to hadronization

HW-NNLOPS(Pythia8-part)
HW-NNLOPS(Pythia8-hadr)
NNLO

σ_{TOTAL} [fb]

NNLO	75.95	7.51	2.02	58.65	10.65	3.97
HW-NNLOPS(Pythia8-part)	78.29	8.19	2.37	56.52	10.13	3.71
HW-NNLOPS(Pythia8-hadr)	80.46	8.45	2.45	54.35	9.87	3.62



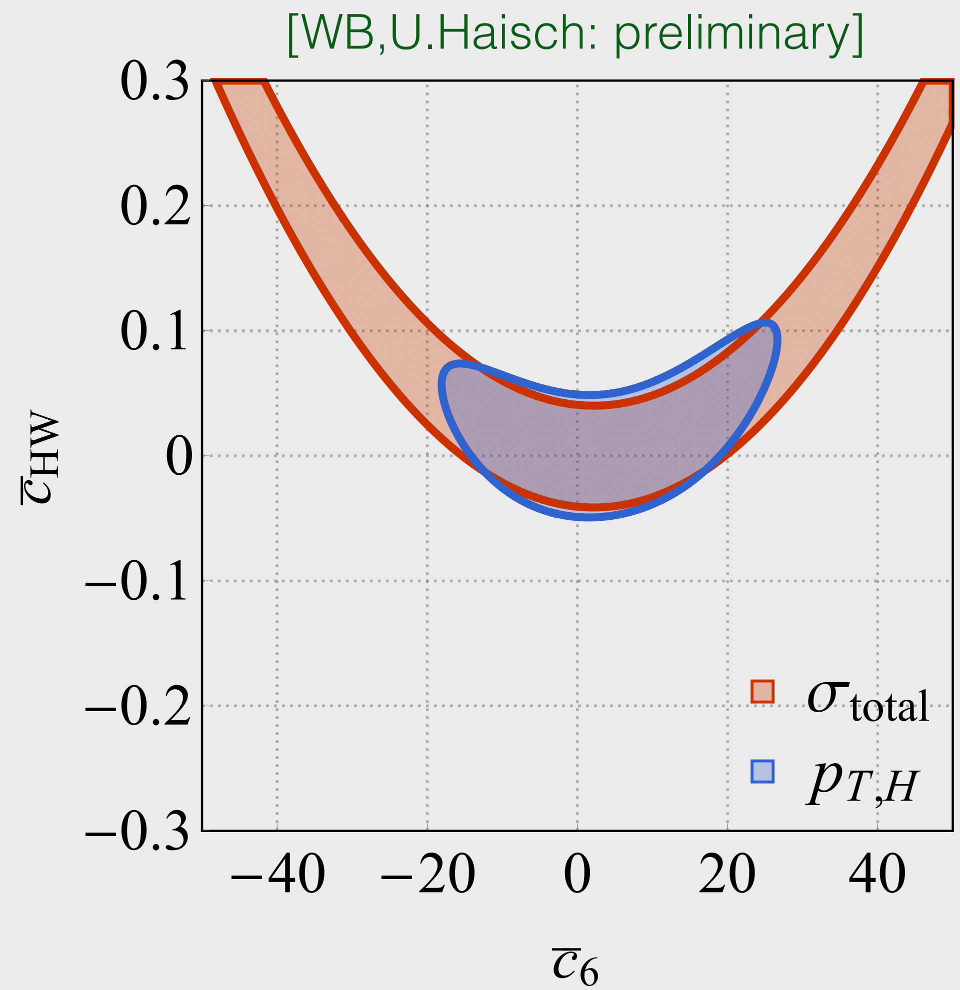
Jet definition:

→ anti-k_t algorithm

→ $R = 0.4$

→ $p_{t,j} > 20$ GeV

RESULTS (HW)



Example:

Putting tighter constraints on some of the SM EFT operators requires precise differential distributions.

Plot:

VH channel constraints on trilinear Higgs coupling ($\bar{c}_6$) and modifications of VVH coupling ($\bar{c}_{HW}$) with and without access to differential distributions.

HW-NNLOPS(Pythia8-part)
HW-NNLOPS(Pythia8-hadr)
NNLO

σ_{TOTAL} [fb]

	BIN 1 (no jets)	BIN 2 (no jets)	BIN 3 (no jets)	BIN 1 (with jets)	BIN 2 (with jets)	BIN 3 (with jets)
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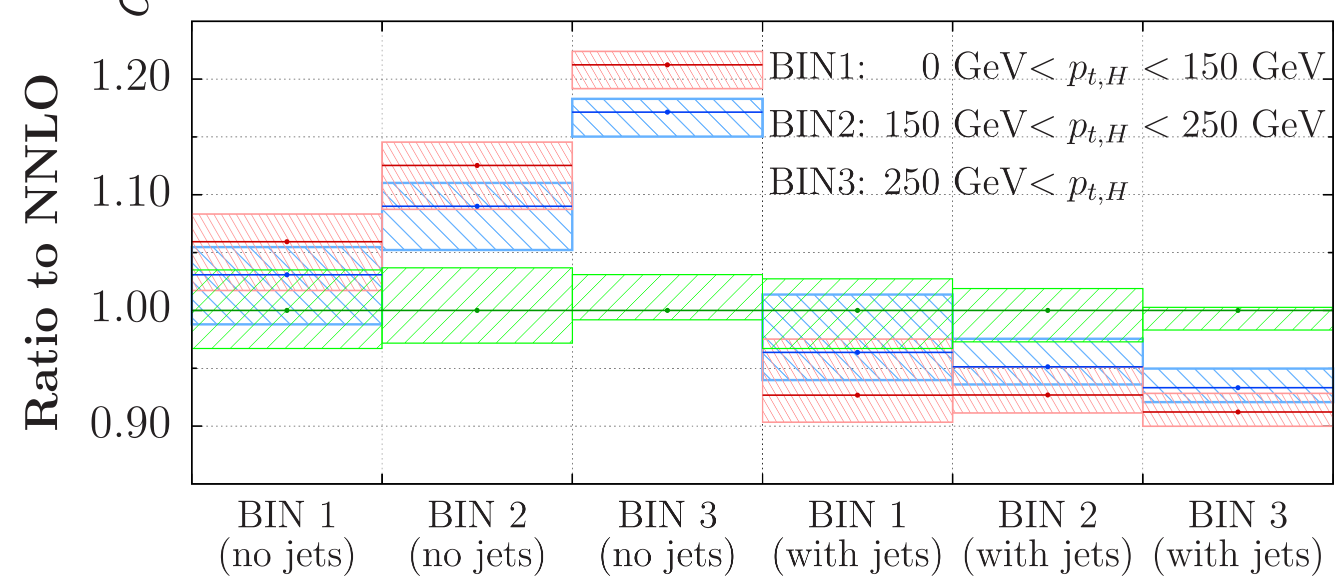

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ASSOCIATED HIGGS PRODUCTION AT NNLO+PS: UPDATES

- Problems with current approach:
  – even only a 3D histogram makes 25^3 $\sim$ 15k bins
  – some of the variables are non-trivially connected (it is hard to populate the bins with high $p_{t,H}$ and large rapidity y_{VH})

- **Question:** Can we do better with a semi-analytical approach?
  Answer: Play with variables: $(p_{T,H},\, y_{VH},\, \Delta y_{VH}) \longrightarrow (M_{VH},\, y_{VH},\, \cos\alpha)$

  – invariant mass and rapidity of VH resonance: easier to control from the point of view of the phase-space generation
  – $\cos\alpha$ is a polar angle in the VH rest-frame: $\cos\alpha = \dfrac{\vec{p}\,'_V \cdot \hat{z}'}{|\vec{p}\,'_V|\,|\hat{z}'|}$

  – analyse the Hadronic Tensor (like in derivation of Collins-Soper angles in DY process) contracted with tensor describing VH decay into the Higgs boson and leptons…

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  and la

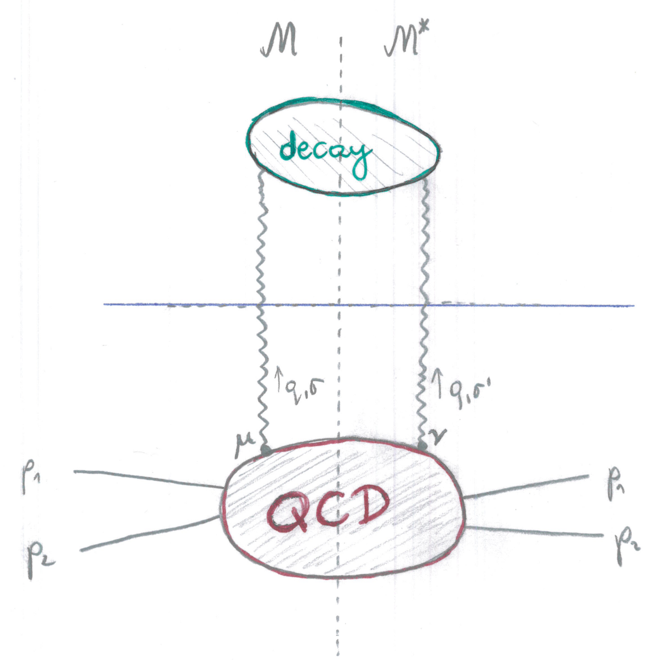
- **Ques**
  Answ

  – inva
  phase
  – cos

  – anal ... cted
  with t

Matrix Element Squared

$$
\begin{aligned}
\left|\mathcal{M}(\{p_1,p_2\} \to \{k_h,k_v\} \to \{k_h,k_{\ell 1},k_{\ell 2}\}\right|^2 &= \\
&= H_{\sigma\sigma'}B^{\sigma\sigma'} \\
&= H_{\sigma\sigma'}\left((\mathrm{VVH})^{\sigma\lambda}L_{\lambda\lambda'}(\mathrm{VVH})^{\lambda'\sigma'}\right)
\end{aligned}
$$



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&= H_{\sigma\sigma'} \left((\mathrm{VVH})^{\sigma\lambda} L_{\lambda\lambda'} (\mathrm{VVH})^{\lambda'\sigma'}\right) \\
&= \left[\epsilon^{\tilde{\mu}}(\sigma',q)\, H_{\tilde{\mu}\mu}\, \epsilon^{*,\mu}(\sigma,q)\right]\left[\epsilon^{\nu}(\sigma,q)\, g_{\nu\tilde{\nu}}\, \epsilon^{*,\tilde{\nu}}(\lambda,k)\right] \\
&\quad \times \left[\epsilon^{\tilde{\rho}}(\lambda,k)\, \mathcal{L}_{\tilde{\rho}\rho}\, \epsilon^{*,\rho}(\lambda',k)\right]\left[\epsilon^{\kappa}(\lambda',k)\, g_{\rho\tilde{\rho}}\, \epsilon^{*,\tilde{\kappa}}(\sigma',q)\right] \\
&= H_{\tilde{\mu}\mu} \left\{\epsilon^{*,\mu}(\sigma,q)\epsilon^{\nu}(\sigma,q)\right\} g_{\nu\tilde{\nu}} \left\{\epsilon^{*,\tilde{\nu}}(\lambda,k)\epsilon^{\tilde{\rho}}(\lambda,k)\right\} \\
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&= H_{\tilde{\mu}\mu} \left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{m_z^2}\right) g_{\nu\tilde{\nu}} \left(-g^{\tilde{\nu}\tilde{\rho}} + \frac{k^{\tilde{\nu}}k^{\tilde{\rho}}}{m_z^2}\right) \\
&\quad \times \mathcal{L}_{\tilde{\rho}\rho} \left(-g^{\rho\kappa} + \frac{k^{\rho}k^{\kappa}}{m_z^2}\right) g_{\rho\tilde{\rho}} \left(-g^{\tilde{\kappa}\tilde{\mu}} + \frac{q^{\tilde{\kappa}}q^{\tilde{\mu}}}{m_z^2}\right)
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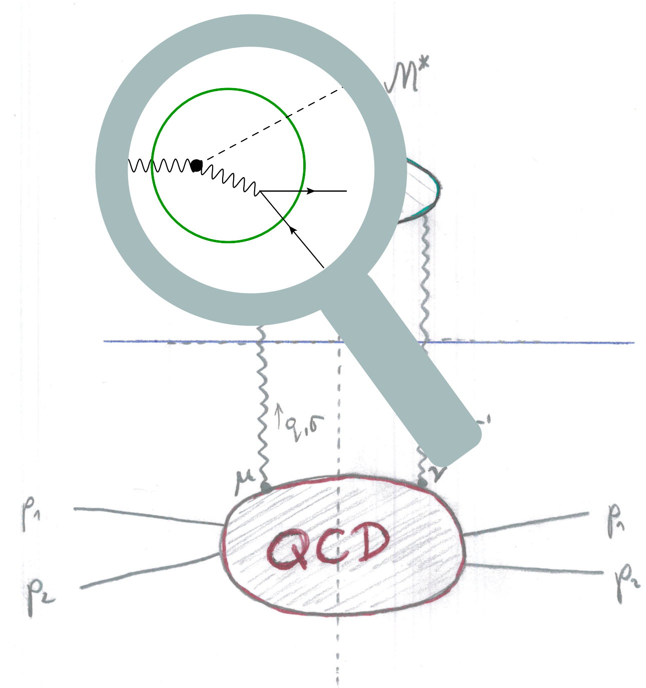
$$



Maximum 5 powers of k_v-momentum!

Parametrisation

Particle	Energy component	x-component	y-component	z-component
k_v	$\omega_v = \sqrt{m_v^2+\kappa^2}$	0	$+\kappa\sin\alpha$	$+\kappa\cos\alpha$
k_h	$\omega_h = \sqrt{m_h^2+\kappa^2}$	0	$-\kappa\sin\alpha$	$-\kappa\cos\alpha$

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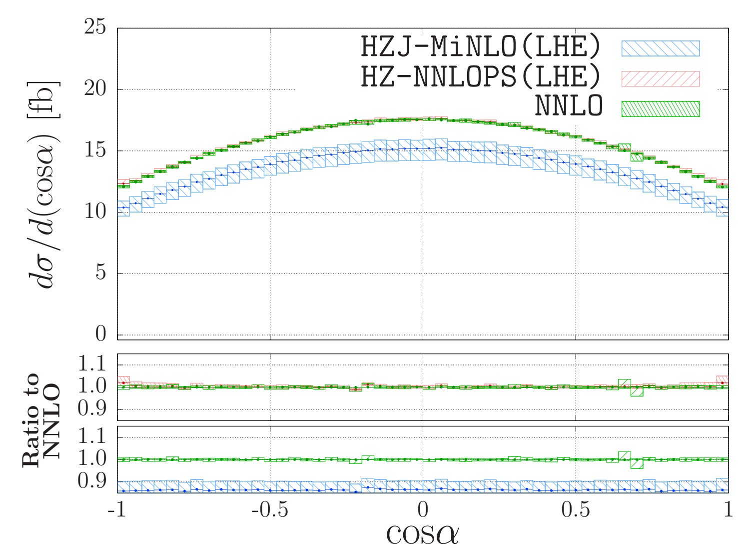
– analyse the Hadronic Tensor (like in derivation of Collins-Soper angles in DY process) contracted with tensor describing VH decay into the Higgs boson and leptons…
(C - coefficients, g - orthonormal basis of functions):

$$\frac{d^3\sigma}{dM\,dy\,d(\cos\alpha)} = \sum_j C_j(M,y)\,g_j(\cos\alpha)$$

> only a finite number of functions required (11)
> hierarchical structure of these spectral modes (stability!)

➡ improvement (factor of 6-8 in CPU time wrt. first implementation)



[preliminary]

ASSOCIATED HIGGS PRODUCTION AT NNLO+PS: WHY??

➡ VH channel:
> is not the most important one
> presence of two additional leptons is an advantage for tagging!
Result: a very good channel for probing **Hbb** decay channel

– largest decay channel: Br(Hbb)=0.58
– test the proportionality of mass/Yukawa coupling to fermions
– the direct coupling of the Higgs boson to quarks (down type!)

Channel	Importance
ggH	87%
VBF	7%
VH	5%
ttH	1%

➡ Add the NLO Hbb decay to the POWHEG generator:
[a flexible and precise tool for studying the signatures of decay to b-quarks]:
> more reliable M(bb) spectrum in the presence of relatively hard radiation
> possible differences PS/NLO when events categorised according to jet-cuts,…
> more reliable description of jet-distributions shapes (jet substructure)

[Butterworth, Davison, Rubin, Salam; 0802.2470]

Evidence for the Higgs boson decay to a bottom quark-antiquark pair → [ATLAS; 1708.03299] [CMS; 1709.07497]

LIMITATIONS OF THE METHOD

- More complex processes (with larger phase-space) are significantly harder to deal with.
  Main difficulties include:
  > obtaining smooth multi-differential distributions
  > non-trivial correlations between various observables (may have considerable impact when only finite precision available)

- There are still some tricks to exploit:
  > multi-differential grid adaptation and rebinning
  > semi-analytical analysis of matrix-elements

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POSSIBLE FURTHER DIRECTIONS

- WW@NNLOPS / ZZ@NNLOPS:
  > one could apply similar semi-analytical approach to prepare many-dimensional distributions
- Xj@NNLOPS:
  > in Z+j case one could possibly use very similar setup as in VH-implementation
  > in this case we also lack other inputs (i.e. B2 coefficient for MiNLO)

SUMMARY AND CONCLUSIONS

- Monte Carlo tools play a major role in many LHC searches
- NNLO+PS successfully implemented for a few processes:
  – colour-singlet production
  – 2-to-2 processes with decay of massive objects (like VH)
- A limiting factor for reweighting: large Born phase-space
- There are still some tricks to exploit (like the ones presented…)
- Interesting directions:
  – including decay of Higgs boson (@NLOPS, @NNLOPS?)
  – VV@NNLOPS
  – moving towards Xj@NNLOPS

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THANK YOU!

BACKUP: HIGGS DECAY

- Reweighting also works if we add Higgs decay:

In our case, the output of the fixed-order code and the POWHEG gives:

$$d\sigma_{\texttt{NNLO}}(\mathrm{HZ}) = \mathrm{Br}(\mathrm{H}\to \mathrm{b\bar{b}}) \cdot \left[\mathrm{d}\sigma^{(0)} \cdot \frac{\mathrm{d}\Gamma^{(0)}+\mathrm{d}\Gamma^{(1)}}{\Gamma^{(0)}+\Gamma^{(1)}} + (\mathrm{d}\sigma^{(1)} + \mathrm{d}\sigma^{(2)}) \cdot \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}}\right] \tag{2.6}$$

$$d\sigma_{\texttt{MiNLO}}(\mathrm{HZ}) = \mathrm{Br}(\mathrm{H}\to \mathrm{b\bar{b}}) \cdot \left[\left(\mathrm{d}\sigma^{(0)} + \mathrm{d}\sigma^{(1)}\right) \cdot \frac{\mathrm{d}\Gamma^{(0)}+\mathrm{d}\Gamma^{(1)}}{\Gamma^{(0)}+\Gamma^{(1)}} + \mathrm{d}\tilde{\sigma}^{(2)} \cdot \frac{\mathrm{d}\Gamma^{(0)}}{\Gamma^{(0)}+\Gamma^{(1)}}\right], \tag{2.7}$$

where $\mathrm{Br}(\mathrm{H}\to \mathrm{b\bar{b}})$ is the best prediction for Standard Model $\mathrm{H}\to \mathrm{b\bar{b}}$ branching ratio. The $d\tilde{\sigma}$ denotes NLO part of the HZj computation in POWHEG, which corresponds to double-real and real-virtual parts of HZ production at NNLO.

It is easy to check that after integrating out the decay of the Higgs boson in equation (2.6) one recovers (2.3), up to the overall branching ratio. One can also verify that

$$\frac{d\sigma_{\texttt{NNLO}}(\mathrm{HZ})}{d\sigma_{\texttt{MiNLO}}(\mathrm{HZ})} = 1 + \frac{\left(\sigma^{(2)} - \tilde{\sigma}^{(2)}\right)}{\sigma^{(0)}} + \mathcal{O}\left(\alpha_s^3\right) , \tag{2.8}$$

which means that reweighting does not spoil the NLO accuracy of the event sample (rescaling is equal to one up to $\mathcal{O}(\alpha_s^2)$ terms).