QCD Beyond Leading Power

Gherardo Vita

Massachusetts Institute of Technology

Particle Theory Seminar

Zurich, 15 October 2019

Based on

Operators and Lagrangians: [Stewart, GV et al.] 1703.03408, 1712.04343, [Chang, Stewart, GV] to appear soon

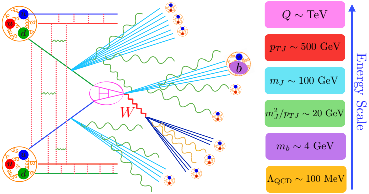
Factorization: [Moult, Stewart, GV] 1905.07411,

Fixed Order: [Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1804.04665, 1812.08189 [Bhattacharya, Moult, Stewart, GV] 1812.06950

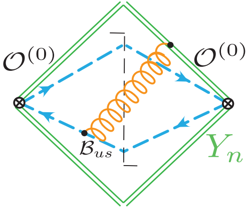
Resummation: [Moult, Stewart, GV, Zhu] 1804.04665 [Moult, Schunk, Stewart, Tackmann, GV, Zhu] to appear soon

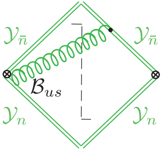
Outline

- Introduction:
  - Systematic Expansion of QCD using Soft and Collinear Effective Theory
  - Motivation for going beyond Leading Power



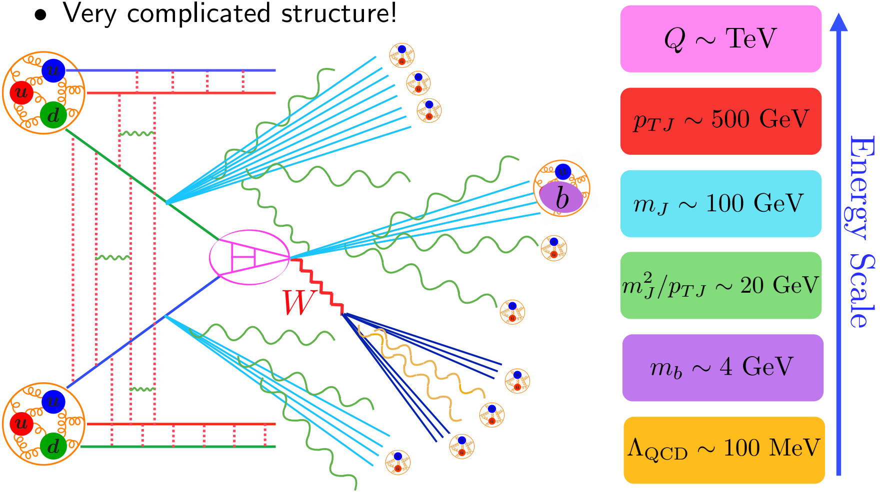
- SCET at Subleading Power:
  - Overview of recent developements in collider observables at Subleading Power
  - Computing Power Corrections at Fixed Order
  - Subleading Power Regularization and Renormalization
  - Leading Log Resummation at subleading power





An LHC Collision

- Very complicated structure!

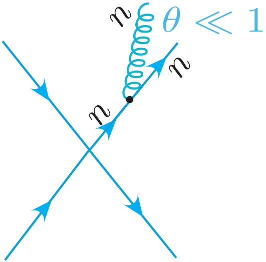


- Involves interactions at many hierarchical energy scales.
- It is very complicated to obtain precise theoretical predictions

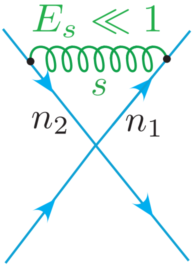
Limits of QCD

- Significant progress in understanding QCD made by considering limits where we have a power expansion in some small kinematic quantity.

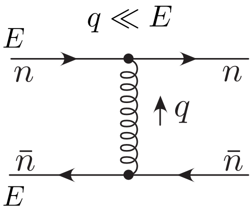
Collinear



Soft

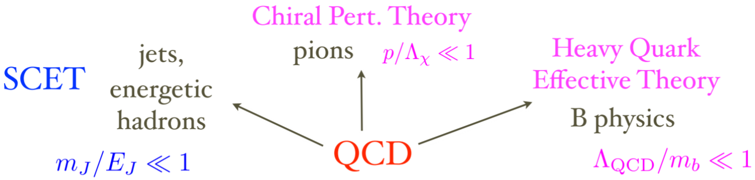


Regge



Soft and Collinear Effective Theory [Bauer, Fleming, Pirjol, Stewart]

Soft and Collinear Effective Theory (SCET) is limit of QCD



- Results derived with SCET must be equivalent to results derived directly from QCD.
- SCET systematizes the power expansion from the start
  $\to$ explicit power counting at any step
- Simplifies field theoretic derivation of factorization formulae
  $\to$ Scales separated in building the EFT once and for all, recycled among different processes
- Resummation of large logs from deriving anomalous dimensions of hard, collinear or soft operators $\to$ logs coming from IR poles in pQCD get related to UV divergences in SCET, hence we can define $\overline{\rm MS}$-like counterterms, anomalous dimensions, RGEs, etc..

Mode setup in SCET

- Light cone coordinates: $k^\mu = \frac{\bar{n}^\mu}{2} k^+ + \frac{n^\mu}{2} k^- + k_\perp^\mu \equiv (k^+,\, k^-,\, k_\perp)$

EFT expansion: $\lambda \ll 1$

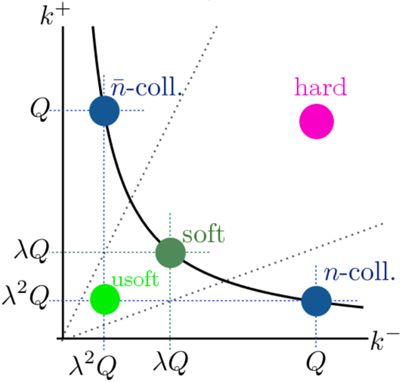
$(k^+, k^-, k_\perp)$

n-collinear: $k_n^\mu \sim Q(\lambda^2,\, 1,\, \lambda)$

$\bar{n}$-collinear: $k_{\bar{n}}^\mu \sim Q(1,\, \lambda^2,\, \lambda)$

$\text{SCET}_{\text{II}} \to$ soft: $k_s^\mu \sim Q(\lambda,\, \lambda,\, \lambda)$

$\text{SCET}_{\text{I}} \to$ usoft: $k_{us}^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$



hard scale: $k_{hard}^\mu \sim Q(1, 1, 1)$ (integrated out)

- Allows for a factorized description: Hard, Jet, Beam, Soft radiation

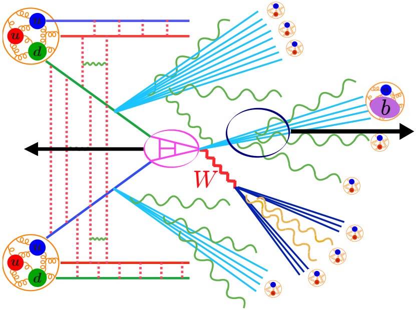
From Standard Model to SCET

$$\mathcal{L}_{SM} \to \mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_{\mathrm{hard}} + \mathcal{L}_{\mathrm{dyn}} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\mathrm{hard}} + \sum_{i\geq 0} \mathcal{L}^{(i)}_{\mathrm{dyn}} + \mathcal{L}^{(0)}_{\mathcal{G}}$$

$\mathcal{L}_{\mathrm{hard}}$ describes the hard scattering/the partonic interaction.

e.g. how to go from gg to $H + 2$ partons.

Note: it can come from non-QCD interactions



$\mathcal{L}_{\mathrm{dyn}}$ describes the evolution of the strongly interacting final/initial states

e.g. how to go from 2 partons to 2 jets/ how the jets evolve

EFT of pure QCD

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim \sigma_0 H(Q,\mu) \otimes J(Q,\tau,s,\mu) \otimes S(s,\mu) + \dots$$

Power expansion for generic $\mathcal{O}$ observable

- A large class of observables $\mathcal{O}$ (q_T, event shapes, angularities, etc.) exhibit singularities in perturbation theory as $\mathcal{O} \to 0$.
- Standard factorization theorems describe only leading power term.
- To be concrete let's take $\mathcal{O} = p_T^2$.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \frac{\log^m \frac{p_T^2}{Q^2}}{p_T^2}$$

Leading Power (LP)

- Relate observable $\frac{p_T^2}{Q^2} \ll 1$ to the SCET power counting parameter $\lambda \ll 1$
- Use SCET to study this limit

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$$+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(1)} \log^m \frac{p_T^2}{Q^2}$$

Next to Leading Power (NLP)

- Relate observable $\frac{p_T^2}{Q^2} \ll 1$ to the SCET power counting parameter $\lambda \ll 1$
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Next to Leading Power (NLP)

$$+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)}\, p_T^2 \log^m \frac{p_T^2}{Q^2} + \cdots$$

$$= \frac{d\sigma^{(0)}}{\mathrm{d}p_T^2} + \frac{d\sigma^{(1)}}{\mathrm{d}p_T^2} + \frac{d\sigma^{(2)}}{\mathrm{d}p_T^2} + \cdots$$

- Relate observable $\frac{p_T^2}{Q^2} \ll 1$ to the SCET power counting parameter $\lambda \ll 1$
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Leading Power

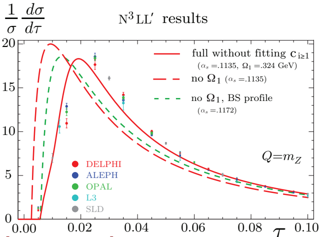
Leading power well understood for a wide variety of observables.

- We can prove factorization theorems

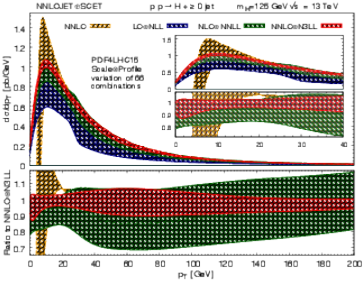
$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\tau} = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ =$$

$$= H^{(0)} J_\tau^{(0)} \otimes J_\tau^{(0)} \otimes S_\tau^{(0)} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\tau Q}\right)$$

- We can resum logs and get very accurate theory predictions



[Stewart, et al.]

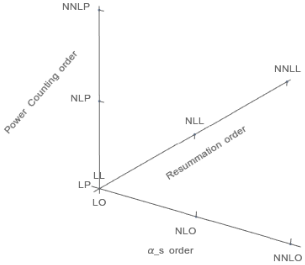


[Chen, Gehrmann, Glover, Huss, Li, Neill, Schulze, Stewart, Zhu]

So, why bother going beyond leading power?

NLP field theoretical motivations

- **Power counting** is a different **direction** in which amplitudes and cross sections can be **expanded**



- Various interesting field theoretical questions to answer at subleading power:
  - ◇ What is the **structure of factorization theorems** at each power?

$$\frac{d\sigma^{(n)}}{d\mathcal{O}} = \sum_j H_j^{(n_{Hj})} \otimes J_j^{(n_{Jj})} \otimes S_j^{(n_{Sj})}$$

  - ◇ What is the degree of **universality**?
  - ◇ Appearance of universal structures, e.g. $\Gamma_{\rm cusp}(\alpha_s)$?
  - ◇ Appearance of **new RGE structures**, functions, objects, etc

Application: Fixed Order Computations via Slicing

- IR divergences in fixed order calculations can be regulated using slicing parameter (e.g. q_T [Catani,Grazzini], N-jettiness [Gaunt et. al], [Boughezal et al.]).

$$\sigma(X) = \int_0 dq_T \frac{d\sigma(X)}{dq_T} = \int_0^{q_T^{\rm cut}} dq_T \frac{d\sigma(X)}{dq_T} + \int_{q_T^{\rm cut}} dq_T \frac{d\sigma(X)}{dq_T}$$

- q_T subtraction has been applied to many processes in pp at **NNLO**: $pp \to Z$, $pp \to W$, $pp \to H$, $pp \to \gamma\gamma$, $pp \to Z\gamma$, $pp \to W\gamma$, $pp \to ZZ$, $pp \to WW$, $pp \to WZ$ [Matrix collaboration]

- N-jettiness subtraction also applied to $W/Z/H + 1$ jet @NNLO

- Error, $\Delta\sigma(q_T^{\rm cut})$, (or computing time) can be exponentially improved by analytically computing power corrections.

$$\Delta\sigma(q_T^{\rm cut}) = \int_0^{q_T^{\rm cut}} dq_T \left(\frac{d\sigma(X)}{dq_T} - \frac{d\sigma(X)^{\rm LP}}{dq_T} \right) \equiv \sigma^{\text{non sing.}}(q_T^{\rm cut})$$

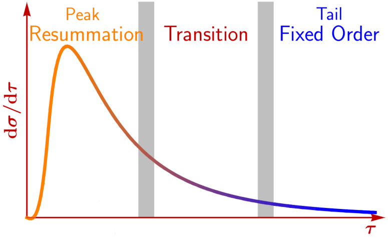
- Understanding of power corrections crucial for applications to more complicated processes (fully differential N^{3}LO calculations, H + jets, Z/W + jets)

Matching resummation with FO

If observable τ needs resummation:

- Use Leading Power EFT for resummed XS at small τ

$$\frac{d\sigma}{d\tau} \underset{\tau\to 0}{\sim} \alpha_s^n \frac{\log^m \tau}{\tau} \xrightarrow[\text{resummation}]{\text{EFT}} \frac{e^{-\alpha_s^k \log^{2k} \tau}}{\tau}$$

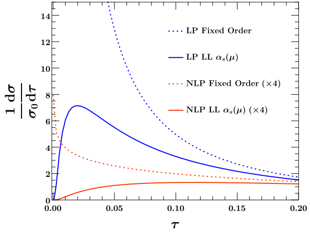


- For large τ use Fixed Order calculation to get full $\mathcal{O}(\alpha_s^n)$ contribution
- Need matching procedure in transition region between the two.
- Computing **Power Corrections** analytically **improves** convergence of the EFT at larger values of τ
  $\implies$ **smaller transition regions**
  $\implies$ smaller uncertainties from matching procedure

Taming log divergence of NLP

- Fixed order power correction (NLP) exhibits an integrable divergence for $\tau \to 0$
- If Leading Power (singular) is resummed and NLP is not, the NLP (integrable) divergence dominates.

$$\alpha_s^n \frac{\log^m \tau}{\tau} \longrightarrow \frac{e^{-\alpha_s^k \log^{2k} \tau}}{\tau} \quad \text{vs} \quad \alpha_s^n \log^m \tau$$



[Moult, Stewart, Vita, Zhu]

Bootstrap for observables

- Bootstrap approaches aim to completely reconstruct amplitudes or cross sections from limits.
- Intensively applied for amplitudes in $\mathcal{N} = 4$.
- Recenty, some success in QCD for soft matrix elements [Zhu et al.]

Remaining Parameters in Symbol of 6-Point MHV Remainder Function

Constraint	$L=2$	$L=3$	$L=4$
1. Integrability	75	643	5897
2. Total S_3 symmetry	20	151	1224
3. Parity invariance	18	120	874
4. Collinear vanishing (T^0)	4	59	622
5. OPE leading discontinuity	0	26	482
6. Final entry	0	2	113
7. Multi-Regge limit	0	2	80
8. Near-collinear OPE (T^1)	0	0	4
9. Near-collinear OPE (T^2)	0	0	0

LL All Powers → (row 5)

NLP, NNLP → (rows 8, 9)

[Dixon et al.], [Basso, Sever, Vieira]

- Can the bootstrap be extended from amplitudes to cross section? For example, can we bootstrap an event shape observable using the information from limits at leading and subleading power?

SCET beyond leading power

Subleading Power QCD for collider observables

Before 2016

	SCET$_{\text{I}}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	**SCET$_{\text{II}}$** soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	
Fixed Order (fully differential)		
Hard Scattering Operators		
Resummation		

Lagrangians

[Stewart et al.] [Beneke et al.] (2002-2004)

Fixed Order

Hard Scattering

Resummation

Subleading Power QCD for collider observables

2016

	SCET$_{\text{I}}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	**SCET$_{\text{II}}$** soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	
Fixed Order (fully differential)	LL	
Hard Scattering Operators		
Resummation		

Lagrangians	Fixed Order	Hard Scattering	Resummation
[Stewart et al.] [Beneke et al.] (2002-2004)	[Moult et al.] LL at $\mathcal{O}(\alpha_s^2)$ 1612.00450 ($q\bar{q}V$)		

2017

	SCET$_{\text{I}}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	**SCET$_{\text{II}}$** soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	
Fixed Order (fully differential)	LL	
Hard Scattering Operators	ggH, $Vq\bar{q}$, $Hq\bar{q}$, N-jet	
Resummation		

Lagrangians

[Stewart et al.] [Beneke et al.] (2002-2004)

Fixed Order

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Hard Scattering

[Moult, Stewart, GV] 1703.03408
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[Chang, Stewart, GV] $q\bar{q}H$ 1712.04343

[Beneke et al.] (N-jet operators) 1712.04416

Resummation

Subleading Power QCD for collider observables

2018

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Subleading Lagrangians	✓	
Fixed Order (fully differential)	LL, NLL	NLL
Hard Scattering Operators	ggH, $Vq\bar{q}$, $Hq\bar{q}$, N-jet	
Resummation	$H \to gg$	

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Subleading Power QCD for collider observables

(to appear in) **2019**

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Subleading Lagrangians	✓	✓
Fixed Order (fully differential)	LL, NLL	NLL
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Lagrangians

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Resummation

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[IM, LS, IS, FT, GV, HXZ] $pp \to H, pp \to V$

Subleading Power QCD for collider observables

	$\mathrm{SCET_I}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	$\mathrm{SCET_{II}}$ soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	✓
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Other works on at subleading powers in different contexts:

- *B*-**physics**: [Lee, Stewart],[Neubert, Becher, Paz, Hill] [Beneke, Feldmann] [Tackmann, Mannel] (and many others)
- **Threshold** (only soft radiation): [Bonocore, Laenen, Magnea, Vernazza, White] (next-to-eikonal), [Beneke, Broggio, Garny, Jaskiewicz, Szafron, Vernazza] (resummation) and many other references...
- **Inclusive fixed order**: [Boughezal, Liu, Petriello], [Boughezal, Isgrò, Petriello]
- **Subleading power in light quark mass expansion**: [Liu, Penin]
- ...

What's in this talk

	$\mathbf{SCET_I}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	$\mathbf{SCET_{II}}$ soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
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[Chang, Stewart, GV] Subleading Lagrangians in SCET_{II}

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Next: Computing power corrections at Fixed order

	$\mathbf{SCET_I}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	$\mathbf{SCET_{II}}$ soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	✓
Fixed Order (fully differential)	LL, NLL	NLL
Hard Scattering Operators	ggH, $Vq\bar{q}$, $Hq\bar{q}$, N-jet	✓
Resummation	$H \to gg$, $pp \to H/V$	

Lagrangians

[Stewart et al.] [Beneke et al.] (2002-2004)

[Chang, Stewart, GV] Subleading Lagrangians in $\mathrm{SCET_{II}}$

Fixed Order

[Moult et al.] LL at $\mathcal{O}(\alpha_s^2)$ 1612.00450 ($q\bar{q}V$), 1710.03227 (ggH)

[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1807.10764, $\mathcal{T}_0$ (beam thrust) at NLL NLP

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[Chang, Stewart, GV]

Resummation

[Moult, Stewart, GV, Zhu] 1804.04665, $H \to gg$

[IM, LS, IS, FT, GV, HXZ] $pp \to H, pp \to V$

Power corrections at FO: General Setup

- Take as example the **fully differential cross section** $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}}$ for color singlet production (0-jettiness) including $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\mathcal{T}/Q)$ corrections .
- Power corrections in $\mathcal{O}(\mathcal{T}/Q)$:
  - **Perturbative**
  - **NOT** higher twist PDFs/non-perturbative power corrections.

Beam Thrust (0-jettiness)

$$\mathcal{T}_0 = \sum_{k \in \text{event}} \min(p_k^+, p_k^-)$$

- Used as a slicing parameter for FO calculations
- Represents the "crossed" version of thrust

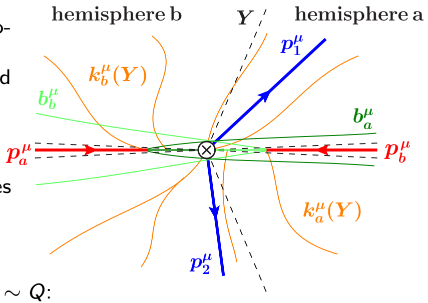
- $\mathcal{O}(\mathcal{T}/Q)$ corrections contained in:
  - **Phase space:** $\Phi = \Phi^{(0)} + \frac{\mathcal{T}}{Q}\Phi^{(2)} + \mathcal{O}(\frac{\mathcal{T}^2}{Q^2})$
  - **Matrix element squared:** $|\mathcal{M}|^2 = A^{(0)} + \frac{\mathcal{T}}{Q}A^{(2)} + \mathcal{O}(\frac{\mathcal{T}^2}{Q^2})$

Schematically:
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \int \frac{\mathrm{d}z}{z}\left[A^{(0)}\Phi^{(0)} + \frac{\mathcal{T}}{Q}A^{(0)}\Phi^{(2)} + \frac{\mathcal{T}}{Q}A^{(2)}\Phi^{(0)}\right] + \mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}, \alpha_s^2\right)$$

Power corrections at FO: PDF expansion

- Need to keep track of $\mathcal{O}(\mathcal{T})$ component of momenta: both for phase space expansion and mandelstams entering $|\mathcal{M}|^2$.

- Solving Q and Y measurements uniquely fixes how factors of $\mathcal{T}$ enter the PDFs.



Example n-collinear emission, $k^+ \sim \mathcal{T}$, $k^- \sim Q$:

$$p_a^\mu = Qe^Y\left[\left(1 + \frac{k^- e^{-Y}}{Q}\right) + \frac{\mathcal{T}}{Q}\frac{k^-}{2Q} + \mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}\right)\right]\frac{n^\mu}{2}$$

$$p_b^\mu = Qe^{-Y}\left[1 + \frac{\mathcal{T}}{Q}\left(e^Y + \frac{k^-}{2Q}\right) + \mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}\right)\right]\frac{\bar{n}^\mu}{2}$$

$$n^\mu = (1, 0, 0, 1)$$
$$\bar{n}^\mu = (1, 0, 0, -1)$$

- At subleading power **both** PDF momenta contain power corrections **regardless** of the direction of the emission $\Longrightarrow$ derivative of both PDFs

$\mathcal{T}$ power corrections from residual momenta in PDFs for an n-collinear emission:

$$f_a\left(\frac{p_a}{E_{cm}}\right) \sim f_a\left(\frac{x_a}{z_a} + \frac{\mathcal{T}}{Q}\Delta_a\right) = f_a\left(\frac{x_a}{z_a}\right) + \frac{\mathcal{T}}{Q}\Delta_a f_a'\left(\frac{x_a}{z_a}\right)$$

$$f_b\left(\frac{p_b}{E_{cm}}\right) \sim f_b\left(x_b + \frac{\mathcal{T}}{Q}\Delta_b\right) = f_b\left(x_b\right) + \frac{\mathcal{T}}{Q}\Delta_b f_b'\left(x_b\right)$$

Power corrections at FO: Master formulae

- Expansion of **phase space** and **matrix element squared** in **soft** and **collinear** limits has a general (universal) structure

n-Collinear Master Formula for 0-Jettiness power corrections

$$\frac{\mathrm{d}\sigma_n^{(2)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \int_{x_a}^{1} \frac{\mathrm{d}z_a}{z_a} \frac{z_a^\epsilon}{(1-z_a)^\epsilon} \left(\frac{Q\mathcal{T}e^Y}{\rho}\right)^{-\epsilon} \Bigg\{ f_a f_b\, A^{(2)}(Q,Y,z_a) + \frac{e^Y}{\rho} A^{(0)} \frac{\mathcal{T}}{Q} \left[f_a f_b \frac{(1-z_a)^2-2}{2z_a} + x_a \frac{1-z_a}{2z_a} f_a' f_b + x_b \frac{1+z_a}{2z_a} f_a f_b' \right] \Bigg\}$$

Soft Master Formula for 0-Jettiness power corrections

$$\frac{\mathrm{d}\sigma_s^{(2)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \frac{1}{\epsilon} \frac{\mathcal{T}^{-2\epsilon}}{Q} \Bigg\{ \bar{A}^{(0)}(Q,Y) \left[f_a f_b \left(-\frac{\rho}{e^Y} - \frac{e^Y}{\rho} \right) + x_a \frac{\rho}{e^Y} f_a' f_b + x_b \frac{e^Y}{\rho} f_a f_b' \right] + f_a f_b \left[\rho Q\, \bar{A}_+^{(2)}(Q,Y) + \frac{Q}{\rho}\, \bar{A}_-^{(2)}(Q,Y) \right] \Bigg\}$$

Power corrections at FO: Cross section results

- Combining **soft** and **collinear** kernels, $\frac{1}{\epsilon}$ poles cancel (consistency check) and the differential cross section takes the form:

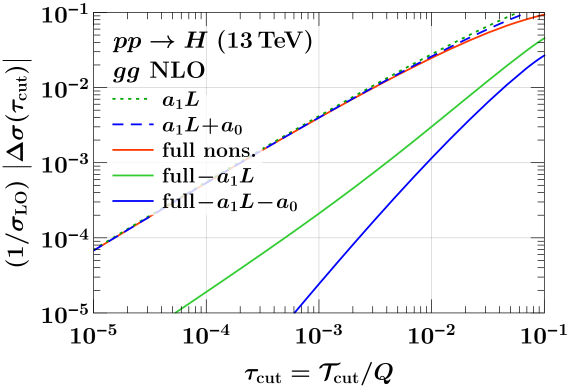
$$\frac{d\sigma^{(2,n)}}{dQ^2 dY d\mathcal{T}} = \hat{\sigma}^{\rm LO}\left(\frac{\alpha_s}{4\pi}\right)^n \int_{x_a}^1 \int_{x_b}^1 \frac{dz_a}{z_a}\frac{dz_b}{z_b}\left[f_i f_j C^{(2,n)}_{f_i f_j}(z_a, z_b, \mathcal{T}) + \frac{x_a}{z_a} f_i' f_j C^{(2,n)}_{f_i' f_j}(z_a, z_b, \mathcal{T}) + \frac{x_b}{z_b} f_i f_j' C^{(2,n)}_{f_i f_j'}(z_a, z_b, \mathcal{T})\right]$$

- By consistency, the kernel must have trivial z_a, z_b dependence (soft kinematic) at Leading Log.
- We can compute the full NLO kernels with master formulae.
  Non-trivial z_a, z_b dependence at NLL.
- Example for gg channel in H production at NLL:

$$C^{(2,1)}_{f_g' f_g}(z_a, z_b, \mathcal{T}) = 4C_A \frac{\rho}{Qe^Y}\delta(1-z_a)\left[\left(-\ln\frac{\mathcal{T}e^Y}{Q\rho} - 1\right)\delta(1-z_b) + \frac{(1+z_b)(1-z_b+z_b^2)^2}{2z_b^2}\mathcal{L}_0(1-z_b)\right]$$
$$+ 4C_A \frac{e^Y}{Q\rho}\frac{(1-z_a+z_a^2)^2}{2z_a}\delta(1-z_b)$$

Power corrections at FO: full NLO results for $pp \to H$

[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1807.10764



$F_{\rm NLO}(\tau) = \frac{\mathrm{d}}{\mathrm{d}\ln\tau}\Big\{\tau\big[a_1 \ln\tau + a_0 + \mathcal{O}(\tau)\big]\Big\}$

Numerical fit at percent level matches analytic calculation within 1 σ

NLO $\mathcal{T}_0^{\rm lep}$ $gg \to Hg$	a_1	a_0
earlier fit	$+0.6090 \pm 0.0060$	$+0.1824 \pm 0.0043$
analytic	$+0.6040$	$+0.1863$

Next: Regularization and Renormalization at NLP

	SCET_{I} ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	**SCET_{II}** soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	✓
Fixed Order (fully differential)	LL, NLL	NLL
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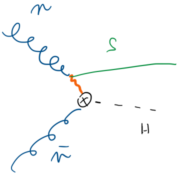
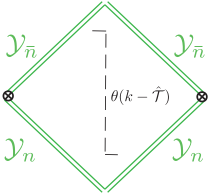
New features of Regularization and Renormalization at Subleading Power





Regularization of subleading power Rapidity divergences

(Ebert, Moult, Stewart, Tackmann, GV, Zhu)

[1812.08189]

Renormalization with θ functions

(Moult, Stewart, GV, Zhu)

[1804.04665]

Rapidity Divergences

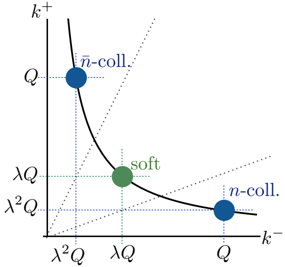
- Large class of observables e.g. $\vec{q}_T$, broadening, EEC, p_T^{veto}, ... belong to the class of $\mathrm{SCET_{II}}$ observables
- $\mathrm{SCET_{II}}$ calculations are affected by **Rapidity Divergences**
- Measurement fixes $\perp$ component of momentum, i.e. $k^+k^- \sim k_\perp^2$ hyperbola

Light cone coordinates: $k^\mu = (k^+,\, k^-,\, \vec{k_\perp})$

n-collinear: $p_n \sim Q(\lambda^2,\, 1,\, \lambda)$

$\bar{n}$-collinear: $p_{\bar{n}} \sim Q(1,\, \lambda^2,\, \lambda)$

soft: $p_s \sim Q(\lambda,\, \lambda,\, \lambda)$



- Example of massless soft real emission with $\mathrm{SCET_{II}}$ measurement:

$$\int \mathrm{d}^d k\, \delta_+(k^2)\delta^{(d-2)}(\vec{q}_\perp - \vec{k}_\perp) f(k^+, k^-, \vec{k}_\perp) = q_T^{-2\epsilon} \int_0^\infty \frac{\mathrm{d}k^-}{k^-} f(k^-, \vec{q}_\perp)$$

- Divergence when modes overlap

$$k^\pm \to 0\,, \qquad y = 1/2 \log(k^+/k^-) \to \pm\infty\,,$$

not regulated by dimensional regularization $\Longrightarrow$ need a rapidity regulator

Rapidity Divergences beyond leading power

- **Leading Power** (in $q_T^2 \ll Q^2$) representative rapidity divergent integral:

$$\frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}q_T^2} \sim \frac{1}{q_T^{2+2\epsilon}} \int_0^Q \frac{\mathrm{d}k^-}{k^-}$$

  - ◇ **Log divergent**, from eikonal propagators from Wilson Lines. (typically...)
  - ◇ It can be regulated in many ways: [Collins] , [Beneke, Feldmann, Chiu, Manohar, . . .], [Becher, Bell] [Bell, Rahn, Talbert], [Chiu, Jain, Neill, Rothstein] [Rothstein,Stewart], [Chiu, Fuhrer, Hoang, Kelley, Manohar], [Echevarria, Idilbi, Scimemi], [Li, Neill, Zhu], . . .

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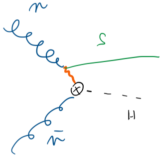
- **Subleading Power**: much broader class of rapidity divergent integrals appearing
  - ◇ Prototypical integrals take the form: $\frac{\mathrm{d}\sigma^{\mathrm{NLP}}}{\mathrm{d}q_T^2} \sim \frac{1}{q_T^{2\epsilon}} \int_0^Q \frac{\mathrm{d}k^-}{(k^-)^{\alpha}}$
  - ◇ α can be negative, hence not only log divergences

$$\int \frac{\mathrm{d}k^-}{(k^-)^2}, \quad \int \frac{\mathrm{d}k^-}{(k^-)^3} \quad \Longrightarrow$$ **Power Law Rapidity Divergences**

  - ◇ **Regulating only Wilson lines is not sufficient**.
    Note that this is also true at LP for Glaubers, see [Rothstein, Stewart]
  - ◇ Divergences also from soft-quark emissions, hard-collinear propagators, phase space expansion.



Rapidity Regularization at Subleading Power

Hence, at Subleading Power:

- Regulating only Wilson lines **is not sufficient**.
- Regularization should conveniently treat power law rapidity divergent integrals
- Common simplifications always used at Leading Power **no longer true**

Example: non-homogeneous regulators (as k^0 or η regulator with $|k_z|$) generate power corrections!

$$\text{n collinear regulator:}\quad \left(\frac{k_n^- + k_n^+}{\nu}\right)^{-\eta} = \nu^\eta \left(k_n^- + \frac{k_T^2}{k_n^-}\right)^{-\eta} = \left(\frac{k_n^-}{\nu}\right)^{-\eta}\left[1 - \eta\,\frac{k_T^2}{(k_n^-)^2} + \mathcal{O}(\lambda^4)\right]$$

$$\mathcal{I}_n^{(0)} = \underbrace{\nu^\eta \int_0^Q \mathrm{d}k^- \frac{g_n(k^-/Q)}{(k^-)^{1+\eta}}}_{\text{LP collinear integral}} - \underbrace{k_T^2 \nu^\eta \int_0^Q \mathrm{d}k^- \,\eta\, \frac{g_n(k^-/Q)}{(k^-)^{3+\eta}}}_{\text{NLP integral induced by non homogeneous reg.}} + \mathcal{O}(\lambda^4)$$

The NLP integral induced by the regulator is $\frac{1}{\eta}$ divergent $\Longrightarrow$ the η prefactor cancels out and the term does NOT vanish for $\eta \to 0$

Introduce the *pure rapidity regulator*

$$\int \mathrm{d}^d k \to \int \mathrm{d}^d k\, \omega^2 \upsilon^\eta \left|\frac{\bar{n}\cdot k}{n\cdot k}\right|^{-\eta/2} = \int \mathrm{d}^d k\, \omega^2 \upsilon^\eta e^{-y_k \eta}$$

- It doesn't introduce power corrections
- It breaks boost symmetry in the most minimal way.
- Includes **dimensionless** (pure) rapidity scale υ (**upsilon**)

Leading-Logarthmic power corrections

- Compute power corrections in q_T^2/Q^2 in the n-collinear, $\bar{n}$-collinear and soft limits (soft is scaless for homogeneous regulators)
- Sum together results
- Rapidity divergences cancel between sectors, finite terms add up.
  (In rapidity regularization this is trivial since $g_n(\eta) = g_{\bar{n}}(-\eta)$)

At Leading Log the result is quite simple. Here a couple of examples:

- Drell Yan production ($q\bar{q} \to Vg$)

$$\frac{\mathrm{d}\sigma^{(2),\mathrm{LL}}_{q\bar{q}\to Vg}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} = \hat{\sigma}^{\mathrm{LO}}_{q\bar{q}\to V}(Q) \times \frac{\alpha_s C_F}{4\pi}\frac{2}{Q^2}\ln\frac{Q^2}{q_T^2}\left[f^{q\bar{q}}_{\mathrm{uni}}(x_a, x_b)\right],$$

- Gluon fusion Higgs production ($gg \to Hg$)

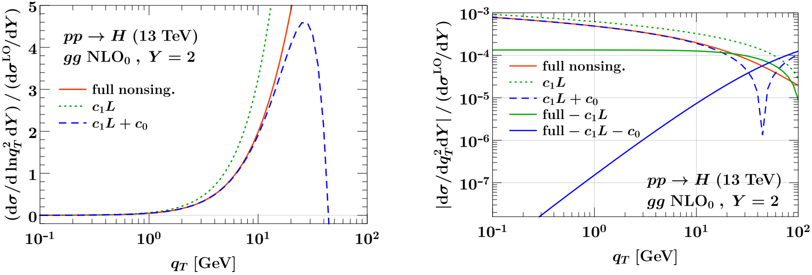
$$\frac{\mathrm{d}\sigma^{(2),\mathrm{LL}}_{gg\to Hg}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} = \hat{\sigma}^{\mathrm{LO}}_{gg\to H}(Q) \times \frac{\alpha_s C_A}{4\pi}\frac{2}{Q^2}\ln\frac{Q^2}{q_T^2}\left[8 f_g(x_a) f_g(x_b) + f^{gg}_{\mathrm{uni}}(x_a, x_b)\right],$$

- Common factor

$$f^{ij}_{\mathrm{uni}}(x_a, x_b) = -x_a f_i'(x_a) f_j(x_b) - f_i(x_a)\, x_b f_j'(x_b) + 2 x_a f_i'(x_a)\, x_b f_j'(x_b)$$

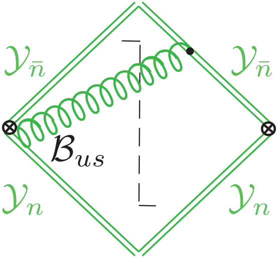
Next Leading-Logarithmic power corrections

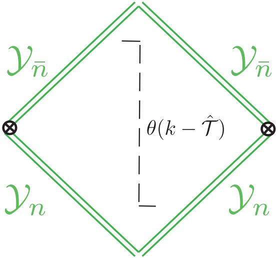
- We computed also the NLL kernels at $\mathcal{O}(\alpha_s)$ for all channels both in DY and ggH.
- z_a, z_b kernels pretty complicated. They involve $\mathcal{L}_0^{++}(1 - z_a)$, etc.
- Remainder is q_T^2/Q^2 suppressed
- Describes q_T distribution up to 10 GeV



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2} - \frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}Y\mathrm{d}q_T^2} = c_1(Y)\ln\frac{Q^2}{q_T^2} + c_0(Y) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

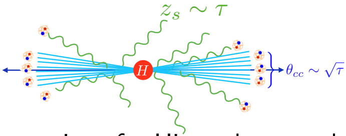
Renormalization at subleading powers



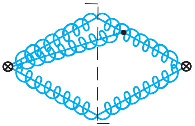


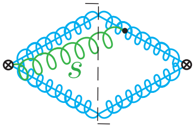
(Moult, Stewart, GV, Zhu) [1804.04665]

Fixed Order Calculation of Thrust



- Compute power corrections for Higgs thrust at lowest order





$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\tau} = 8C_A\left(\frac{\alpha_s}{4\pi}\right)\left[\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{Q^2\tau}\right) - \left(\frac{1}{\epsilon} + \log\frac{\mu^2}{Q^2\tau^2}\right)\right]\theta(\tau) + \mathcal{O}(\alpha_s^2)$$
$$= 8C_A\left(\frac{\alpha_s}{4\pi}\right)\log\tau\ \theta(\tau) + \mathcal{O}(\alpha_s^2)$$

- No virtual corrections at lowest order ($\delta(\tau) \sim 1/\tau$).
- Divergences cancel between soft and collinear.
- Log appears at first non-vanishing order:
  - At LP, $\log(\tau)/\tau$ arises from RG evolution of $\delta(\tau)$
  - At NLP $\log(\tau)$ arises from RG evolution of "nothing"?

Elements of Subleading Power Factorization [Moult, Stewart, GV, Zhu]

- Analogously to what we have seen at FO power corrections at the operator level arise from two distinct sources:
    - Power corrections to scattering amplitudes.
    - Power corrections to kinematics.

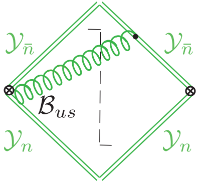
Note: Fields/Lagrangians have a definite power counting in λ!

Operator	$\mathcal{B}^{\mu}_{n\perp}$	χ_n	$\mathcal{P}^{\mu}_{\perp}$	$\mathcal{B}^{\mu}_{us}$	ψ_{us}	∂^{μ}_{us}
Power C.	λ	λ	λ	λ^2	λ^3	λ^2

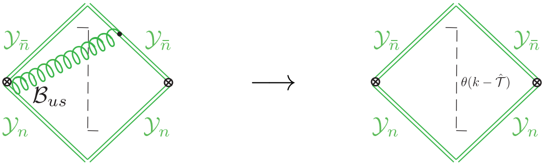
$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_{\text{hard}} + \mathcal{L}_{\text{dyn}} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\text{hard}} + \sum_{i\geq 0} \mathcal{L}^{(i)} + \mathcal{L}^{(0)}_{\mathcal{G}}$$

- Power corrections to scattering amplitudes can be computed from subleading SCET operators [Moult, Stewart, GV]

- They give rise to new jet and soft functions, whose renormalization was not previously known



Renormalization of Subleading Soft Functions



- The subleading soft function satisfies a 2×2 mixing RG

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix} = \begin{pmatrix} \gamma_{11}(y,\mu) & \gamma_{12} \\ 0 & \gamma_{22}(y,\mu) \end{pmatrix} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix}$$

- It mixes with "θ-soft" functions

$$S^{(2)}_{g,\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\theta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

  - It is power suppressed due to $\theta(\tau) \sim 1$ instead of $\delta(\tau) \sim 1/\tau$.
  - In collinear sector, analogous subleading Jet functions and θ-jet functions appear
  - We find this type of mixing is a generic behavior at subleading power.

Resummed Soft Function

- Solve RGE mixing equation to renormalize the operators, and resum subleading power logarithms.
- We find the final result for the renormalized subleading power soft function:

$$S^{(2)}_{g,\mathcal{B}_{us}}(Q\tau,\mu) = \theta(\tau)\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right)e^{\frac{1}{2}\gamma_{11}\log^2\left(\frac{\mu}{Q\tau}\right)}$$

- Expanded perturbatively, we see a simple series:

$$S^{(2)}_{g,\mathcal{B}_{us}}(Q\tau,\mu) = \theta(\tau)\left[\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right) + \frac{1}{2}\gamma_{12}\gamma_{11}\log^3\left(\frac{\mu}{Q\tau}\right) + \cdots\right]$$

- In particular, we find:
  - First log generated by mixing with the θ function operators.
  - The single log is then dressed by Sudakov double logs from the diagonal anomalous dimensions.

- Example also useful for understanding power suppressed RG consistency.

Next: Resummation of beam thrust at NLP

	SCET$_{\text{I}}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	SCET$_{\text{II}}$ soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
Subleading Lagrangians	✓	✓
Fixed Order (fully differential)	LL, NLL	NLL
Hard Scattering Operators	ggH, $Vq\bar{q}$, $Hq\bar{q}$, N-jet	✓
Resummation	$H \to gg$, $pp \to H/V$	

Lagrangians

[Stewart et al.] [Beneke et al.] (2002-2004)

[Chang, Stewart, GV] Subleading Lagrangians in SCET$_{\text{II}}$

Fixed Order

[Moult et al.] LL at $\mathcal{O}(\alpha_s^2)$ 1612.00450 ($q\bar{q}V$), 1710.03227 (ggH)

[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1807.10764, $\mathcal{T}_0$ (beam thrust) at NLL NLP

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Hard Scattering

[Moult, Stewart, GV] 1703.03408
[Moult et al.] 1703.03411

[Chang, Stewart, GV] 1712.04343

[Beneke et al.] (N-jet operators) 1712.04416, 1808.04742

[Chang, Stewart, GV]

Resummation

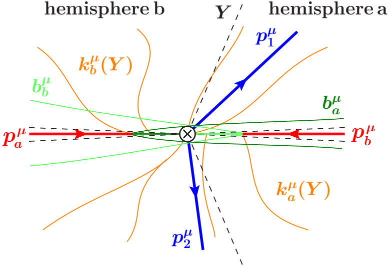
[Moult, Stewart, GV, Zhu] 1804.04665, $H \to gg$

[IM, LS, IS, FT, GV, HXZ] $pp \to H, pp \to V$

Resummation of Beam thrust at NLP

- Starting point for studying resummation at NLP for fully differential observables at the LHC

$$\mathcal{T}_0 = \sum_{k \in \text{event}} \min(p_k^+, p_k^-)$$



- Used as a slicing parameter for FO calculations
- Represents the "crossed" version of thrust
- Contains the additional complication of treating the proton initial states (Jet → Beam functions) at subleading power
- At LL NLP, the fixed order calculation gives (see Fixed Order section):

$$\frac{d\sigma^{(2)}}{d\mathcal{T}_0 dY dQ} = \frac{\sigma_0}{Q}\frac{\alpha_s C_A}{\pi} \ln\frac{\mathcal{T}_0}{Q}\Big[2f_g(x_a)f_g(x_b) \underbrace{-x_a f_g'(x_a)f_g(x_b) - f_g(x_a)x_b f_g'(x_b)}_{\text{PDF derivative term, no analog at LP}}\Big]$$

Kinematic corrections for beam thrust

- LP factorization theorem for $\mathcal{T}_0$ contains LP beam functions

$$B_g\left(t = b^+\omega, \frac{\omega}{P^-}, \mu_c\right) = \langle p_n(P^-)|\mathcal{B}^c_{n\perp\mu}(0)\delta(b^+ - \hat{p}^+)\delta(\omega - \overline{P}_n)\mathcal{B}^{c\mu}_{n\perp}(0)|p_n(P^-)\rangle$$

- As we've seen at Fixed Order in the previous sections, at NLP we need to keep track of small components of momenta routed in beam functions:

$$\omega \to \omega + \Delta\omega \quad \text{where} \quad \Delta\omega \sim \mathcal{T}_0 \sim \mathcal{O}(\lambda^2)$$

- After expansion we get a new object, a *Derivative Beam Function*

$$B_g{}'\left(t, \frac{\omega}{P^-}, \mu_c\right) = \langle p_n(P^-)|\mathcal{B}^c_{n\perp\mu}(0)\delta(b^+ - \hat{p}^+)\delta'(\omega - \overline{P}_n)\mathcal{B}^{c\mu}_{n\perp}(0)|p_n(P^-)\rangle$$

Note that the derivative beam function is LP $(\omega \sim \lambda^0)$

Derivative beam functions and OPE

- The derivative beam function B' is the object entering the factorization theorem
- We can OPE it onto PDFs operators using a matching kernel $\tilde{\mathcal{I}}$

$$B'_g\left(t, x = \frac{\omega}{P^-}, \mu_c\right) = \sum_j \int \frac{d\xi}{\xi} \tilde{\mathcal{I}}_{gj}\left(t, \frac{x}{\xi}, \mu\right) f_j(\xi, \mu) \left[1 + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{t}\right)\right]$$

- The matching kernel is shown to be related to the LP kernel via

$$\tilde{\mathcal{I}}_{gj}\left(t, \frac{x}{\xi}, \mu\right) = \frac{\mathrm{d}}{\mathrm{d}x}\mathcal{I}_{gj}\left(t, \frac{x}{\xi}, \mu\right)$$

- Hence, **to all orders in** α_s, the derivative beam function OPE is

$$B'_g(t, x, \mu_c) = \sum_j \int \frac{d\xi}{\xi} \frac{\mathrm{d}}{\mathrm{d}x}\mathcal{I}_{gj}\left(t, \frac{x}{\xi}, \mu\right) f_j(\xi, \mu) \left[1 + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{t}\right)\right]$$

- The $\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{t}\right)$ power corrections include *non-perturbative* corrections which include higher twist PDFs. We neglect these by considering values of $\mathcal{T}_0$ s.t. $t \sim Q\mathcal{T}_0 \gg \Lambda^2_{QCD}$

Resummed cross section for beam thust at NLP

- At tree level $$\mathcal{I}_{gj}^{tree}\left(t, \frac{x}{\xi}, \mu\right) = \delta_{gj}\delta(t)\delta\left(1 - \frac{x}{\xi}\right)$$
- Hence $$B_g^{\prime\, tree}(t, x, \mu_c) = \delta(t) f_g'(x, \mu)$$

we got the PDF derivatives we expected from FO!

For the cross section:

- Other pieces work similar to thrust case (theta beam functions $B_\theta^{(2)}$, subleading Beam functions $B_{\tau\delta}^{(2)} \sim tB^{(0)}$, subleading operators, RGEs)
- Final result

$$\frac{\mathrm{d}\sigma_{ggH}^{\mathrm{LL}}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\tau_0} = \hat{\sigma}^{\mathrm{LO}}(Q) \underbrace{\left(\frac{\alpha_s}{4\pi}\right) 4C_A\theta(\tau_0)\log(\tau_0)}_{\text{single log from mixing with } \theta} \underbrace{e^{-\frac{\alpha_s}{4\pi}4C_A\log^2(\tau_0)}}_{\text{Sudakov}}$$
$$\times \Big[\underbrace{2f_g(x_a)f_g(x_b) - x_a f_g'(x_a)f_g(x_b) - f_g(x_a)x_b f_g'(x_b)}_{\text{non-perturbative functions}}\Big]$$

Future Directions

- Fixed order calculation of LL power corrections at N3LO
- Power corrections for diboson production
- Resummation beyond Sudakov for collider observables
- Systematic application of fixed order techniques (IBPs, DE, etc.) to calculate EFT objects at high loop order
- Regge/Small-x/Forward/high-energy limit beyond leading power
- Factorization beyond leading power and Factorization breaking effects
- Subleading power observables, spin asymmetries, p_T distributions in quarkonia production

Conclusions

- Described the recent developements for collider observables at subleading power

	SCET$_{\text{I}}$ ultra-soft modes $k_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ (thrust, N-jettiness, jet mass, ...)	SCET$_{\text{II}}$ soft modes $k_s^\mu \sim Q(\lambda, \lambda, \lambda)$ (q_T, broadening, EEC, Glaubers, ...)
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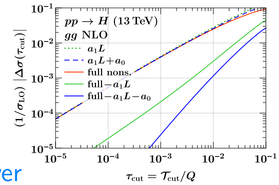
- Studied how to implement rapidity regularization at subleading powers and proposed a new regulator *purely* based on rapidity

$$\int \mathrm{d}^d k \to \int \mathrm{d}^d k \, \omega^2 \upsilon^\eta \left| \frac{\bar{n} \cdot k}{n \cdot k} \right|^{-\eta/2}$$

- Computed full $\mathcal{O}(\alpha_s)$ power correction of differential distribution for color singlet production



- Cross section level renormalization at subleading power involves a new class of universal jet and soft functions involving θ-functions.

- Achieved all orders resummation for fully differential color singlet production in *pp* at subleading power

$$\frac{\mathrm{d}\sigma^{\text{LL}}_{ggH}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\tau_0} = \hat{\sigma}^{\text{LO}}(Q) \underbrace{\left(\frac{\alpha_s}{4\pi}\right) 4 C_A \theta(\tau_0) \log(\tau_0)}_{\text{single log from mixing with } \theta} \underbrace{e^{-\frac{\alpha_s}{4\pi} 4 C_A \log^2(\tau_0)}}_{\text{Sudakov}}$$
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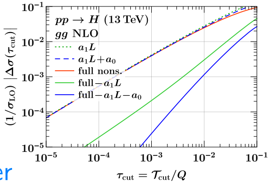

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THANK YOU!

Backup slides

How to treat power law divergences

- Consider rapidity divergent integral $\int_x^1 \mathrm{d}z \, \frac{g(z)}{(1-z)^{a+\eta}}$.
- When $g(z)$ is not known analytically (eg. when it involves PDFs), need to extract **pole** as $\eta \to 0$ without computing the integral.
- For $a = 1$, use standard distributional identity

$$\frac{1}{(1-z)^{1+\eta}} = -\frac{\delta(1-z)}{\eta} + \mathcal{L}_0(1-z) + \mathcal{O}(\eta) \,, \qquad \mathcal{L}_0(y) = [\theta(y)/y]_+ \,,$$

- For $a > 1$, these distributions need to be generalized to higher-order plus distributions subtracting higher derivatives as well. For example, for $a = 2$ one obtains

$$\boxed{\frac{1}{(1-z)^{2+\eta}} = \frac{\delta'(1-z)}{\eta} - \delta(1-z) + \mathcal{L}_0^{++}(1-z) + \mathcal{O}(\eta)} \,,$$

where the second-order plus function $\mathcal{L}_0^{++}(1-z)$ acts on a test function $g(z)$ as a double subtraction.

- Power law divergences generate new PDF derivatives

$$\int_{x_a}^1 \mathrm{d}z_a \, \frac{f(x_a/z_a) f(x_b/z_b)}{(1-z_a)^{2+\eta}} = \frac{f'(x_a) f(x_b/z_b)}{\eta} + \mathcal{O}(\eta^0)$$

Soft-Collinear Factorization at Subleading Power

- BPS field redefinition decouples LP soft and collinear interactions.
- Working in an expansion in EFT parameter λ (not α_s), subleading power Lagrangians enter as T-products:

$$\langle 0|T\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\, \mathcal{L}_{\rm dyn}]\}|X\rangle$$

$$= \langle 0|T\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\, (\mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots)]\}|X\rangle$$

$$= \langle 0|T\left\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\mathcal{L}^{(0)}]\left(1+i\int d^4y\mathcal{L}^{(1)}+\frac{1}{2}(i\int d^4y\mathcal{L}^{(1)})(i\int d^4z\mathcal{L}^{(1)})+i\int d^4z\mathcal{L}^{(2)}+\cdots\right)\right\}|X\rangle$$

$$= \langle 0|T\left\{\tilde{O}_j^{(k)}(0)\left(1+i\int d^4y\mathcal{L}^{(1)}+\frac{1}{2}(i\int d^4y\mathcal{L}^{(1)})(i\int d^4z\mathcal{L}^{(1)})+i\int d^4z\mathcal{L}^{(2)}\right)\right\}|X\rangle_{\mathcal{L}^{(0)}} + \cdots .$$

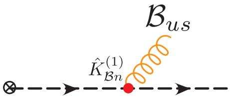
- Only need to consider a finite number of insertions.
- Decoupling of leading power dynamics $\implies$ states still factorize.

$$|X\rangle = |X_n\rangle|X_s\rangle$$

- Call resulting subleading Jet and Soft functions "Radiative" in analogy to *Next-to-eikonal soft gluon radiation* [Bonocore, Laenen, Magnea, Vernazza, White]

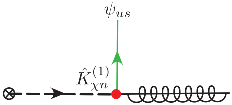
Radiative Functions: Examples

[Larkoski, Neill and Stewart], [Moult, Stewart and GV]



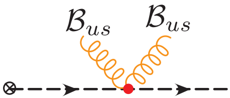
$$\mathcal{L}^{(1)}_{\chi_n} = g\bar{\chi}_n \mathcal{B}^{\perp}_{us(n)} \cdot \mathcal{P}_\perp \frac{\bar{n}\!\!\!/}{\bar{\mathcal{P}}} \chi_n = \hat{K}^{(1)\mu}_{\mathcal{B}_n} \mathcal{B}^{\perp}_{us\mu}$$

non-eikonal single soft emission from a collinear quark



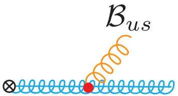
$$\mathcal{L}^{(1)\mathrm{BPS}}_{\chi_n \psi_{us}} = \bar{\chi}_n g \mathcal{B}\!\!\!/_{n\perp} \psi^{(n)}_{us} = \hat{K}^{(1)\bar{\alpha}}_{\chi_n} \psi^{\alpha}_{us(n)}$$

ultrasoft quark emission from a collinear field



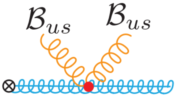
$$\mathcal{L}^{(2)}_{\chi\mathcal{B}\mathcal{B}} = \bar{\chi}_n \left[T^a \gamma^{\mu}_{\perp} \frac{1}{\bar{\mathcal{P}}} T^b \gamma^{\nu}_{\perp} \right] \frac{\bar{n}\!\!\!/}{2} \chi_n g \mathcal{B}^{a\mu}_{us(n)} g \mathcal{B}^{b\nu}_{us(n)}$$

λ^2 non-eikonal double soft emission from a collinear quark



$$\mathcal{L}^{(2)}_{g} \supset ig \left[\partial^{[\mu}_{\perp} \mathcal{B}^{\nu]}_{us} \right] [\mathcal{B}^{\perp}_{n\mu}, \mathcal{B}^{\perp}_{n\nu}]$$

λ^2 non-eikonal single soft emission from a collinear gluon



$$\mathcal{L}^{(2)}_{g} \supset g^2 \mathrm{Tr} \left([\mathcal{B}^{\mu\perp}_{us}, \mathcal{B}^{\nu\perp}_{us}][\mathcal{B}^{\perp}_{n\mu}, \mathcal{B}^{\perp}_{n\nu}] \right)$$

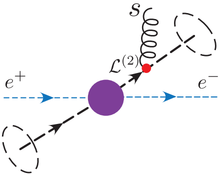
λ^2 non-eikonal double soft emission from a collinear gluon

Radiative Jet Function contribution to Power Corrections in Thrust

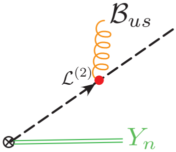
Leading Power $\qquad\qquad$ $\mathcal{O}(\lambda)$ typically vanishes

$$\mathrm{d}\sigma \sim \langle O^{(0)} O^{(0)} \rangle + \langle O^{(1)} O^{(0)} \rangle + \langle O^{(0)} \mathcal{L}^{(1)} O^{(0)} \rangle + \ldots$$

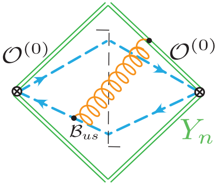
$$+ \langle O^{(0)} O^{(2)} \rangle + \langle O^{(1)} O^{(1)} \rangle + \langle O^{(0)} \mathcal{L}^{(2)} O^{(0)} \rangle + \langle O^{(0)} \mathcal{L}^{(1)} \mathcal{L}^{(1)} O^{(0)} \rangle + \ldots$$

Example: 

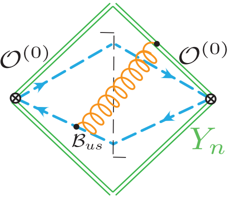
- LP hard scattering operator: $\mathcal{O}^{(0)} = \bar{\chi}_{\bar{n}} \gamma_\perp^\mu \chi_n$
- Subleading Lagrangian insertion on χ_n dynamics:



- Cross section:



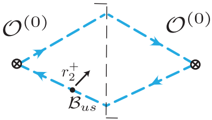
RJF contribution to Power Corrections in Thrust



$$\sim \int \mathrm{d}^4 y \langle 0| \overbrace{\left[\bar{\chi}_n(x) Y_n \gamma_\perp^\mu Y_{\bar{n}} \chi_{\bar{n}}(x)\right]}^{\mathcal{O}^{(0)}_{\mathrm{BPS}}(x)} \overbrace{\left[\bar{\chi}_n T^a g \not{\mathcal{B}}_{us\perp} \frac{1}{\bar{\mathcal{P}}} i\not{\partial}_{us\perp} \frac{\not{\bar{n}}}{2} \chi_n + \dots\right]}^{\mathcal{L}^{(2)}(y)\ \text{insertion}} \underbrace{\left[\bar{\chi}_{\bar{n}}(0) Y_{\bar{n}}^\dagger \gamma_\perp^\mu Y_n \chi_n(0)\right]}_{\mathcal{O}^{(0)\dagger}_{\mathrm{BPS}}(0)} |0\rangle$$

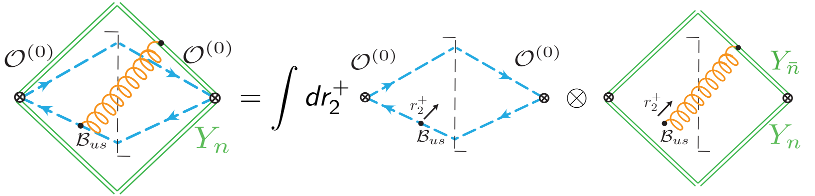
After fierzing, color algebra, reducing the allowed form of the convolutions, using simmetry to reduce the number of allowed object that appear we get a factorized expression in terms of matrix elements of soft and collinear fields.
Define **Radiative Jet Function**: $J_{\mathcal{B}}^{(2)}$. In picture, combine it with the LP jet function on $\bar{n}$ to give

$$J(\tau_{\bar{n}}) J_{\mathcal{B}}^{(2)}(\tau_n, r_2^+) =$$

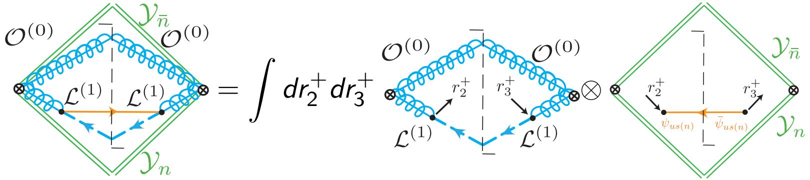


Factorization in Pictures

- Allows all orders factorization for Lagrangian insertions.
- Integral over soft and collinear matrix elements:



Other example: double insertion of soft quark emission



- Can separately compute radiative corrections to each matrix element
- Valid to all orders in α_s, but you need to address convergence and closure issues.