
On the Top Quark Mass

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Particles and Interactions

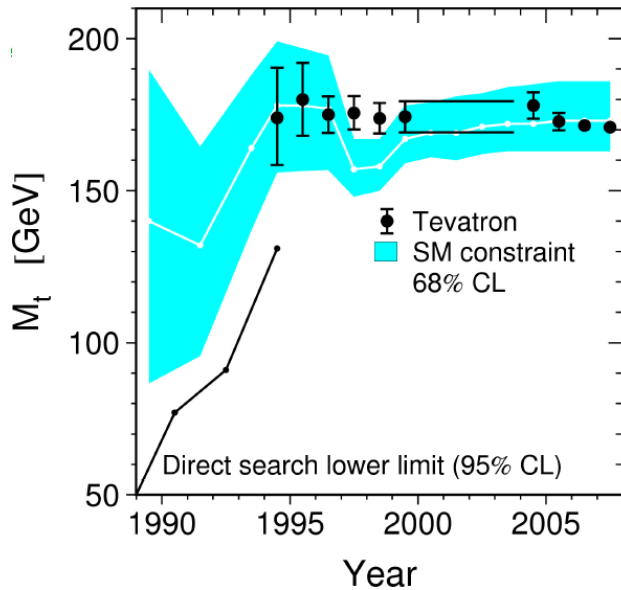


FWF
Der Wissenschaftsfonds.

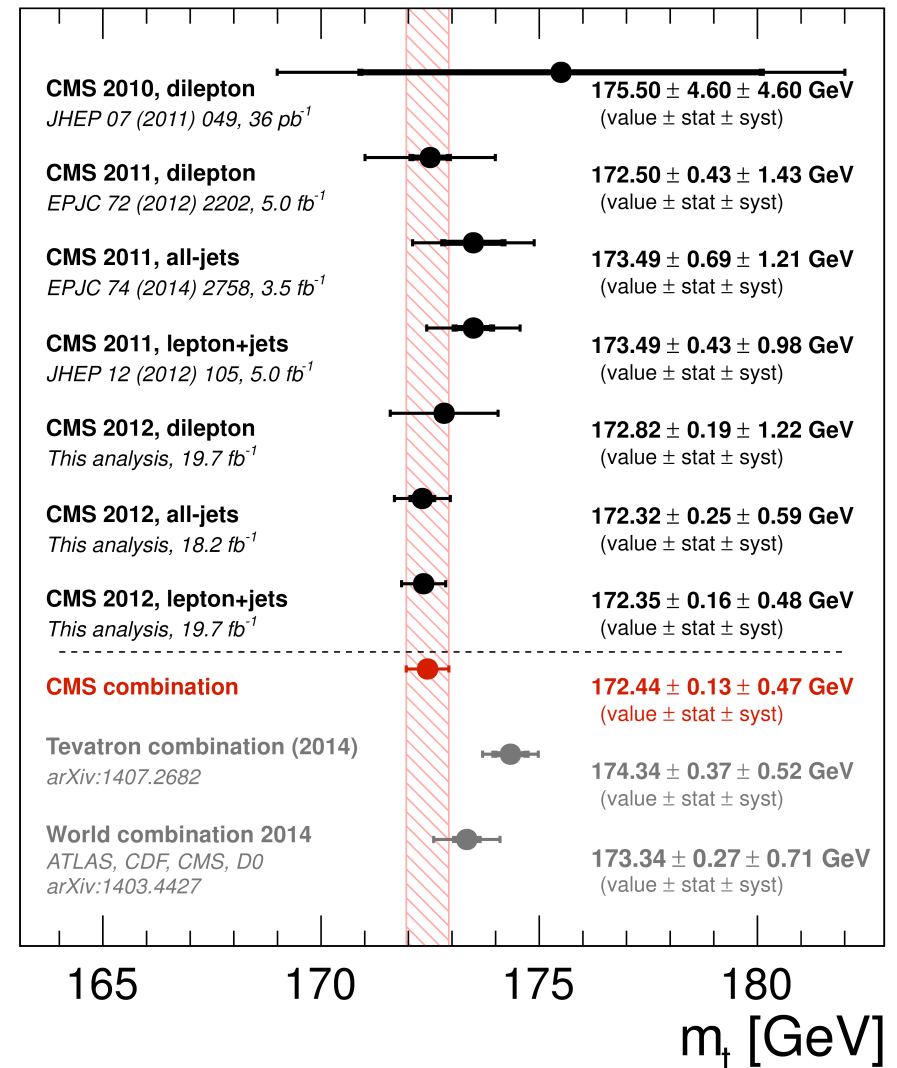
Outline

- Introduction
- Top quark mass schemes
- Calibration of the Monte Carlo top mass parameter: $e^+e^- \rightarrow t \bar{t}$ (2-jettiness)
Butenschön, Dehnadi, Mateu, Preisser, Stewart,AH; PRL 117 (2016) 153
- MSR mass and pole mass “ultimate precision”
Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart, AHH; arXiv:1704.01580
Lepenik, Preisser, AHH, JHEP 1709 (2017) 099
- Relation of $M_t^{\text{Pythia 8.2}}$ and m_t^{pole}
- Factorization for $pp \rightarrow t \bar{t}$ with and w/o jet grooming
- Studies for LHC top mass measurements with SoftDrop
Mantry, Pathak, Stewart, AHH; arXiv:1708.02586
- Summary, future plans

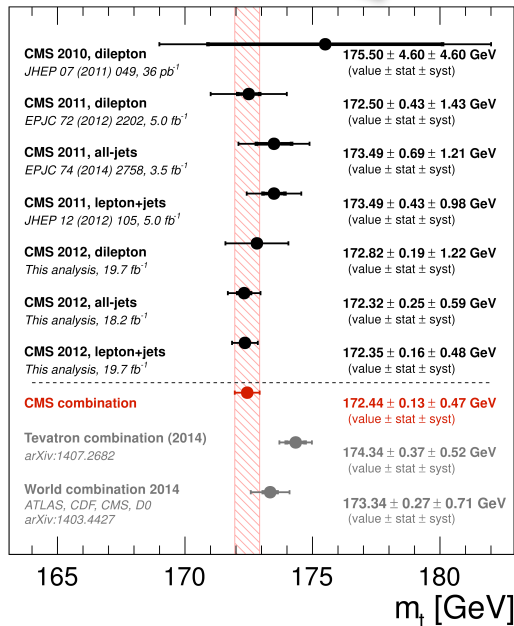
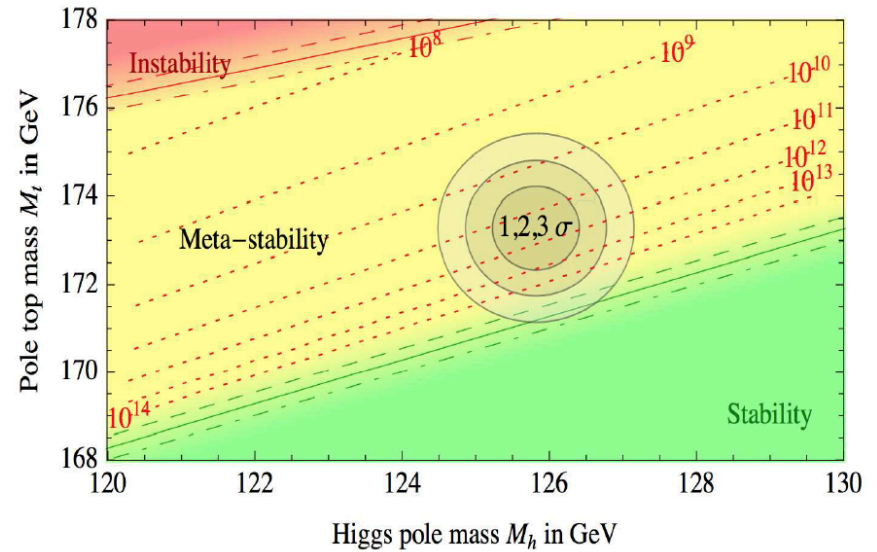
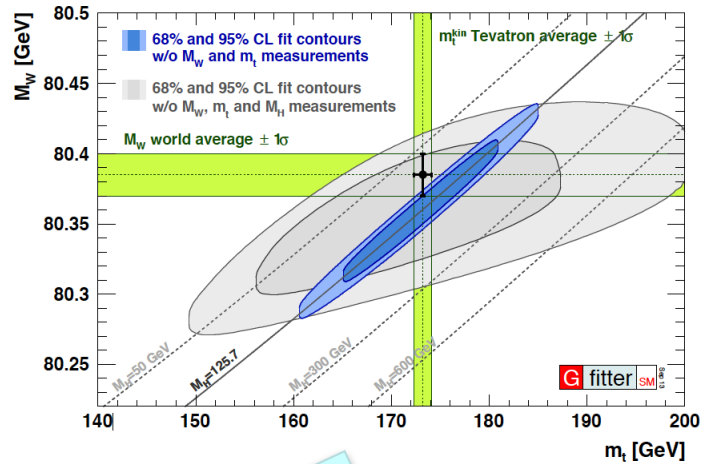
A small history on top mass reconstruction



- Many individual measurements with uncertainty below 1 GeV.
- Some discrepancies between LHC and Tevatron
- Reached <500MeV range.



Motivation



Aims: m_{top} wanted !

- Reduce error in m_{top} (MC)
- Clarify mass scheme m_{top} (MC)
- Improve / understand better MC

Main Top Mass Measurements Methods

LHC+Tevatron: Direct Reconstruction

kinematic mass determination

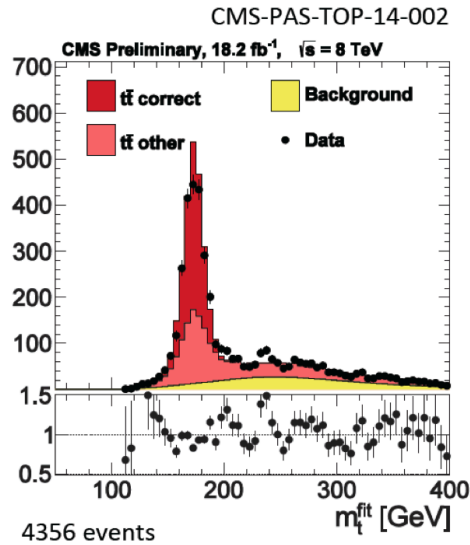
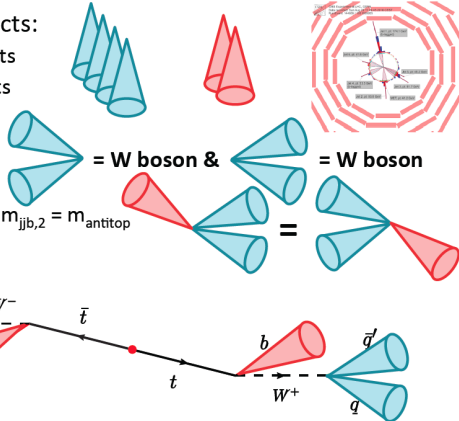
Kinematic Fit

Selected objects:

- 4 untagged jets
- 2 b-tagged jets

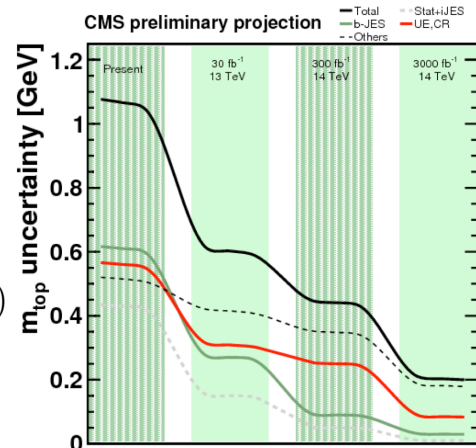
Constraints:

- $2 \times m_{jj} = m_W$
- $m_{top} = m_{jjb,1} = m_{jjb,2} = m_{antitop}$



Determination of the best-fit value of the Monte-Carlo top quark mass parameter

- $m_t^{MC} = 174.34 \pm 0.64$ (Tevatron final, 2014)
- $m_t^{MC} = 172.44 \pm 0.49$ (CMS Run-1 final, 2015)
- $m_t^{MC} = 172.84 \pm 0.70$ (ATLAS Run-1 final, 2016)



⊕ High top mass sensitivity

⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

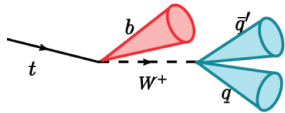
$\Delta m_t \sim 0.5 \text{ GeV}$

← $\Delta m_t \sim 200 \text{ MeV}$ (projection)

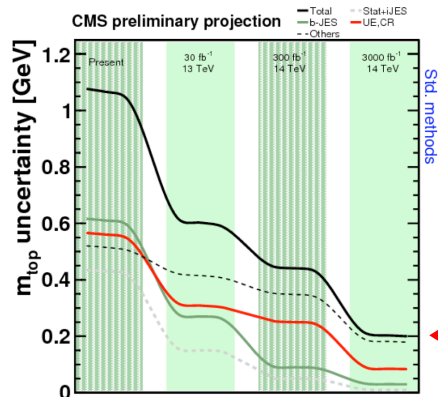
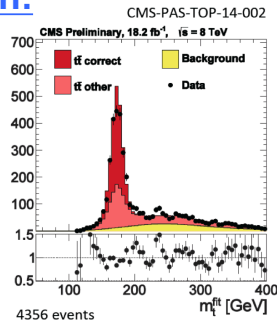
Top Mass Measurements Methods

LHC+Tevatron

Direct Reconstruction:



kinematic mass determination



⊕ High top mass sensitivity

⊖ Precision of MC ?

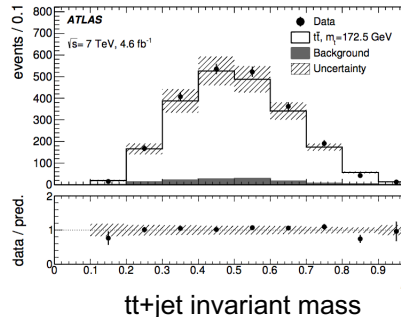
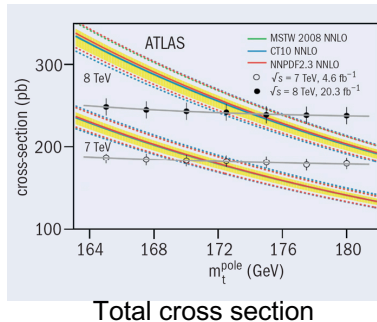
⊖ Meaning of m_t^{MC} ?

$$\Delta m_t \sim 0.5 \text{ GeV}$$

$$\Delta m_t \sim 200 \text{ MeV (projection)}$$

Indirect Mass Fit:

global mass dependence



⊕ pQCD calculations dominate

⊕ Control of mass scheme

⊖ Lower top mass sensitivity

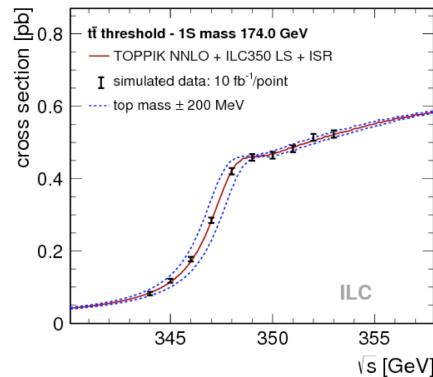
⊖ High sensitivity to norm errors

$$\Delta m_t \sim 1\text{-}2 \text{ GeV}$$

Future Linear Collider:

Top Pair Threshold:

kinematic mass determination
perturbative toponium



⊕ High top mass sensitivity

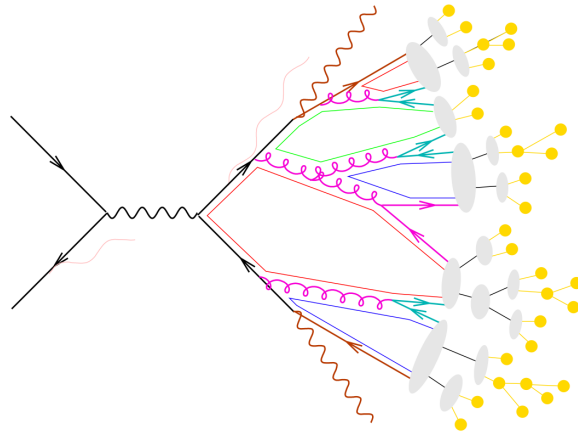
⊕ pQCD calculations dominate

⊕ Control of mass scheme

⊖ Available (maybe) > 2035

$$\Delta m_t \sim 100 \text{ MeV}$$

Monte-Carlo Event Generators



- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. $t \rightarrow bW$
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster \rightarrow hadrons
- hadronic decays

- 1) Matrix elements (LO/NLO)
- 2) Parton shower (LL)
- 3) Hadronization model

- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD \Leftrightarrow partly model
- Description power of data better than intrinsic theory accuracy.
- Top quark in parton shower: treated like a real particle ($m_t^{\text{MC}} \approx m_t^{\text{pole}} + ?$).
- Top quark in splitting function/matrix elements: $m_t^{\text{MC}} = m_t^{\text{pole}}$

BUT: parton showers sum (real & virtual !) perturbative corrections only above the shower cut and not pickup any corrections from below.

Uncertainty (a): But how precise is modelling? \rightarrow Part of exp. Analyses

Unvertainty (b): What is the meaning of MC QCD parameters? \rightarrow Calibration & Theory

Top Quark Mass Schemes

$$\text{---} + \text{---} \begin{array}{c} \text{wavy line} \\ \Sigma' \end{array} \text{---} = p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \dots \right] + \Sigma^{\text{fin}}(m^0, m^0, \mu)$$

MS scheme: $\bar{m}^0 = \bar{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \dots \right]$

Like running “strong coupling”

- $\bar{m}(\mu)$ is pure UV-object without IR-sensitivity
- Useful scheme for $\mu > m$
- Far away from a kinematic mass of the quark

Pole scheme: $m^0 = m^{\text{pole}} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \dots \right] - \Sigma^{\text{fin}}(m^{\text{pole}}, m^{\text{pole}}, \mu)$

Should not be used if uncertainties are below 1 GeV !

- Absorbes all self energy corrections into the mass parameter
- Close to the notion of the quark rest mass (kinematic mass)
- Renormalon problem: infrared-sensitive contributions from < 1 GeV that cancel between self-energy and all other diagrams cannot cancel.
- Has perturbative instabilities due to sensitivity to momenta < 1 GeV (Λ_{QCD})

Top Quark Mass Schemes

$$\text{---} + \text{---} \begin{array}{c} \text{wavy line} \\ \Sigma' \end{array} \text{---} = p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \dots \right] + \Sigma^{\text{fin}}(m^0, m^0, \mu)$$

MS scheme:

$$m_{\square}^0 = \bar{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \dots \right]$$

Pole scheme:

$$m^0 = m^{\text{pole}} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \dots \right] - \Sigma^{\text{fin}}(m^{\text{pole}}, m^{\text{pole}}, \mu)$$

MSR scheme:

$$m^{\text{MSR}}(R) = m^{\text{pole}} - \Sigma^{\text{fin}}(R, R, \mu)$$

Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart, AHH; arXiv:1704.01580

Jain, AH, Scimemi, Stewart (2008)

- Like pole mass, but self-energy correction from <R are not absorbed into mass
- Interpolates between MSbar and pole mass scheme

$$m_t^{\text{MSR}}(R = 0) = m^{\text{pole}}$$

$$m_t^{\text{MSR}}(R = \bar{m}(\bar{m})) = \bar{m}(\bar{m})$$

- More stable in perturbation theory.
- $m_t^{\text{MSR}}(R = 1 \text{ GeV})$ close to the notion of a kinematic mass, but without renormalon problem.

MSR Mass

Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart, AHH; arXiv:1704.01580

MS Scheme: $(\mu > \bar{m}(\bar{m}))$

$$\bar{m}(\bar{m}) - m^{\text{pole}} = -\bar{m}(\bar{m}) [0.42441 \alpha_s(\bar{m}) + 0.8345 \alpha_s^2(\bar{m}) + 2.368 \alpha_s^3(\bar{m}) + \dots]$$

MSR Scheme: $(R < \bar{m}(\bar{m}))$



$(m_b = m_c = 0)$

$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R [0.42441 \alpha_s(R) + 0.8345 \alpha_s^2(R) + 2.368 \alpha_s^3(R) + \dots]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \bar{m}(\bar{m})$$

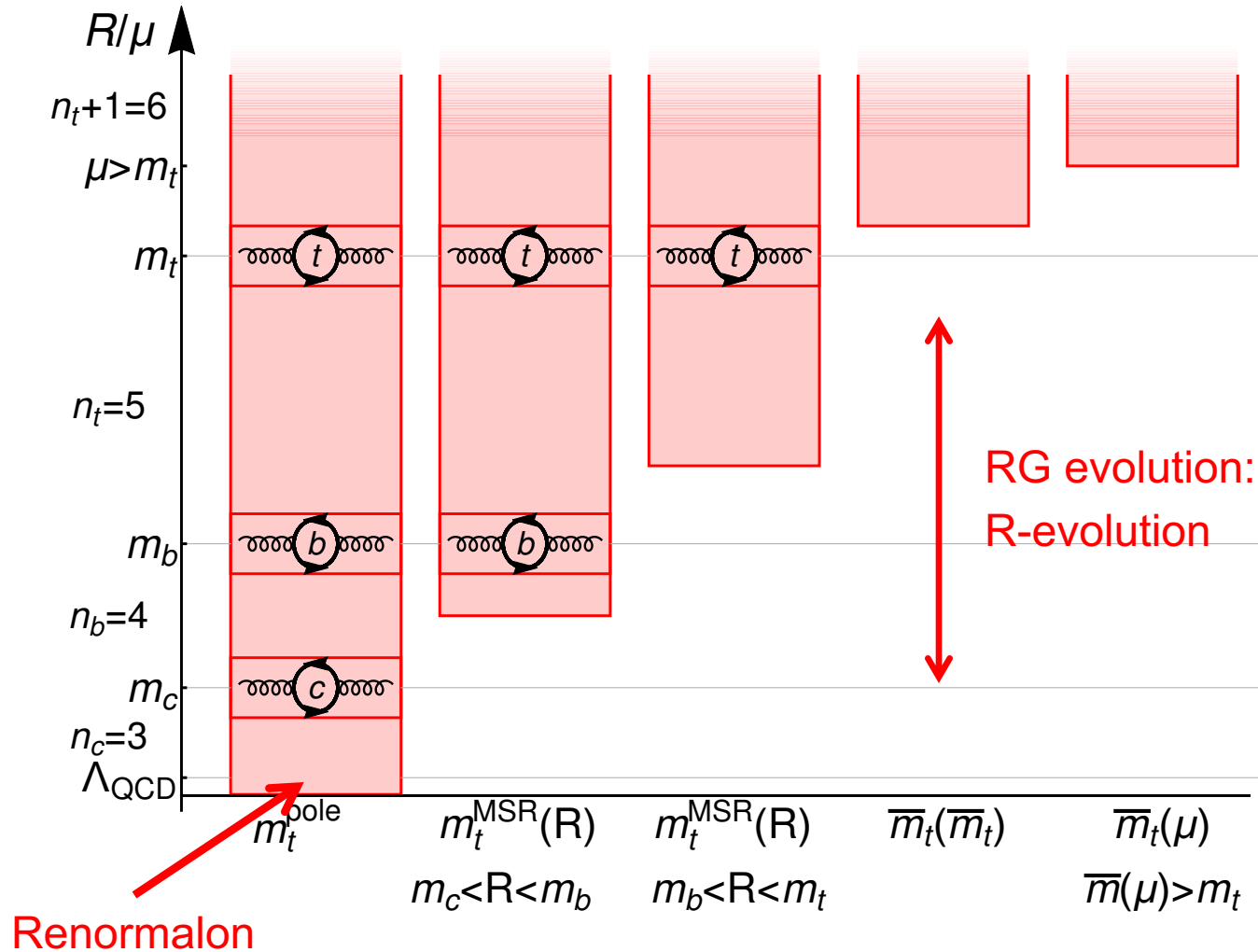
See Lepenik, Preisser, AHH; arXiv:1706.08526 for treatment of finite m_b, m_c

⇒ $m_{\text{MSR}}(R)$ Short-distance mass that smoothly interpolates all R scales
≈ “pole mass subtraction for momentum scales larger than R”

• Precision in relation to any other short-distance mass: $\lesssim 20 \text{ MeV} @ O(\alpha_s^4)$

Top Quark Mass Schemes

Lepenik, Preisser, AHH; arXiv:1706.08526



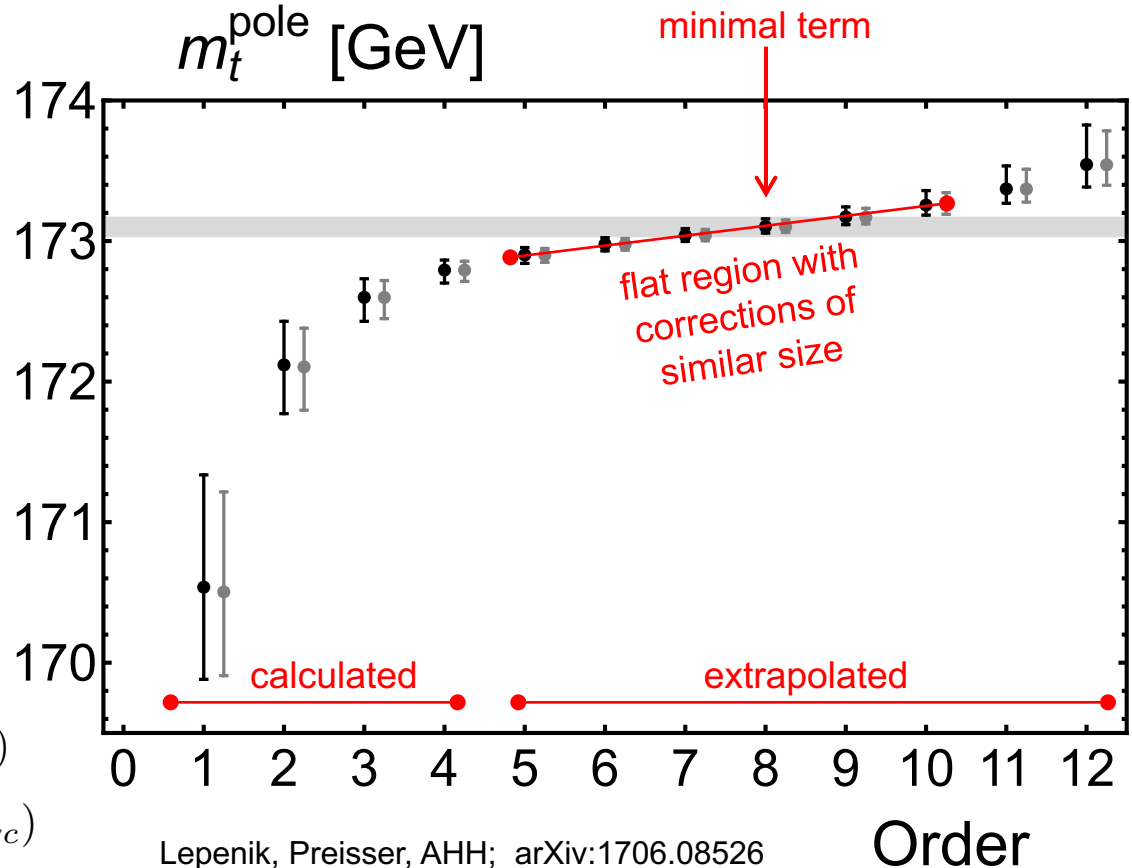
Pole Mass Renormalon Problem

- Asymptotic series
- Bad convergence
- Scale dependence underestimates higher order corrections
- Flat region defines best estimated for pole mass

m_t^{pole} from $\bar{m}_t(\bar{m}_t)$

Scale variation: $\bar{m}_t(\bar{m}_t)/2 < \mu < 2 \bar{m}_t(\bar{m}_t)$

$m_b = m_c = 0$, $\bar{m}_t(\bar{m}_t) = 163 \text{ GeV}$



Beneke, Marquard, Nason, Steinhauser
arXiv:1605.03609

- Claim: “Minimal term determines best estimate and ambiguity”

$$\Delta m_t^{\text{pole}} = 70 \text{ MeV} \quad (m_b = m_c = 0)$$

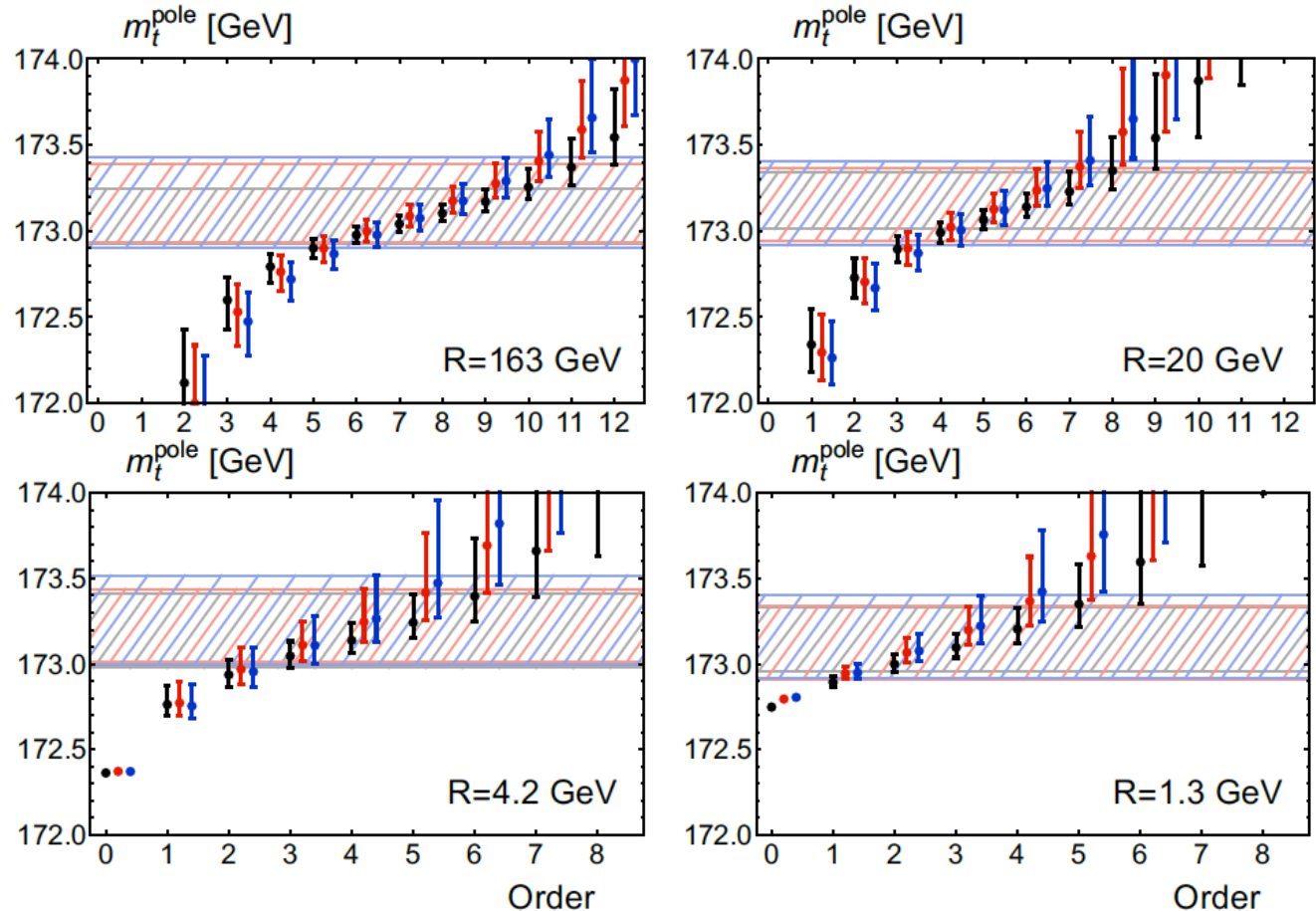
$$\Delta m_t^{\text{pole}} = 110 \text{ MeV} \quad (\text{finite } m_b, m_c)$$

Pole Mass Renormalon Problem

Lepenik, Preisser, AHH; arXiv:1706.08526

Our approach:

- Method respects **heavy quark symmetry**
- Ambiguity independent of value of top quark mass (R values)
- Summation of logarithms of m_t , m_b , m_c
- Flat region accounted for
- Low scale determination approach pole mass at much lower orders.

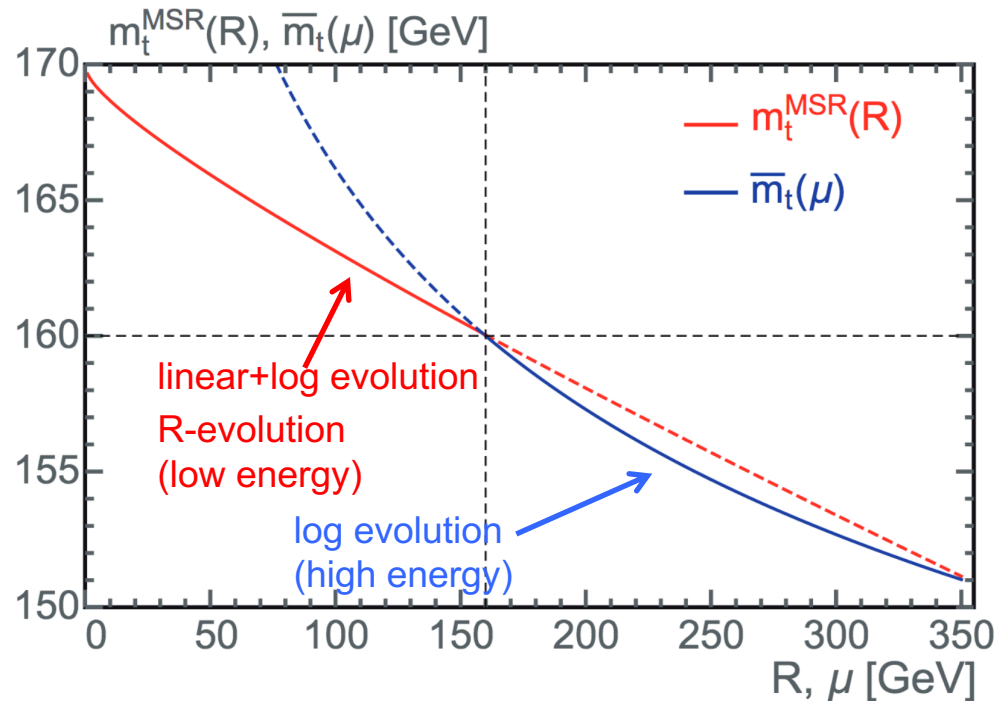


$$\Delta m_t^{\text{pole}} = 180 \text{ MeV} \quad (m_b = m_c = 0)$$

$$\Delta m_t^{\text{pole}} = 250 \text{ MeV} \quad (\text{finite } m_b, m_c)$$

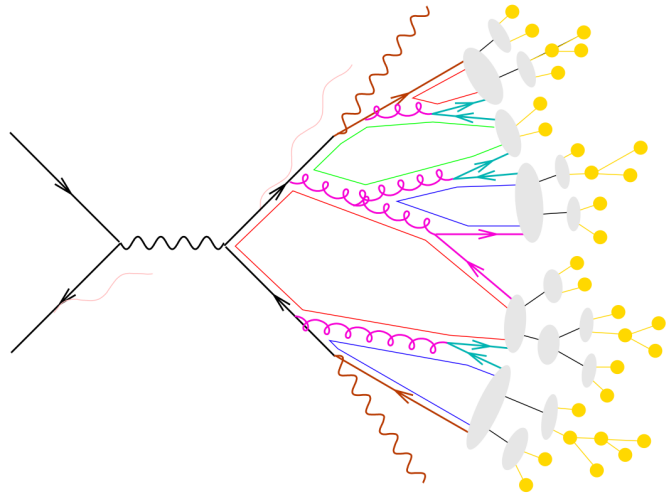
MC Top Quark Mass (for reconstruction)

Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart, AHH; arXiv:1704.01580



- MSR mass is the extension of the $\overline{\text{MS}}$ mass for scales below the mass.

MC Top Quark Mass (for reconstruction)



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- 1) Matrix elements (LO/NLO)
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$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) \sim \mathcal{O}(1 \text{ GeV})$$

Stewart, AHH, 2008 AHH, 2014

- small size of $\Delta_{t,\text{MC}}$
- Renormalon-free
- little parametric dependence on other parameters

Calibration of the MC Top Mass

Method:

- ✓ 1) **Strongly mass-sensitive hadron level observable** (as closely as possible related to reconstructed invariant mass distribution !)
- ✓ 2) Accurate **hadron level QCD predictions** at \geq NLL/NLO with **full control over the quark mass scheme dependence**.
- ✓ 3) QCD masses as function of m_t^{MC} from **fits** of observable.
- 4) Cross check observable independence / universality

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) = \bar{\Delta} + \delta\Delta_{\text{MC}} + \delta\Delta_{\text{pQCD}} + \delta\Delta_{\text{param}}$$

Experimental systematics

Monte Carlo dependence:

- different tunings
- parton showers
- color reconnection
- Intrinsic error, ...

QCD errors:

- perturbative error
- scale uncertainties
- electroweak effects

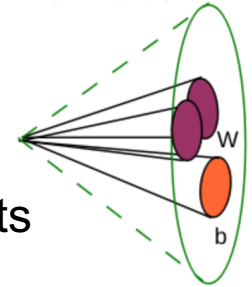
Parametric errors:

- strong coupling α_s
- Non-perturbative parameters

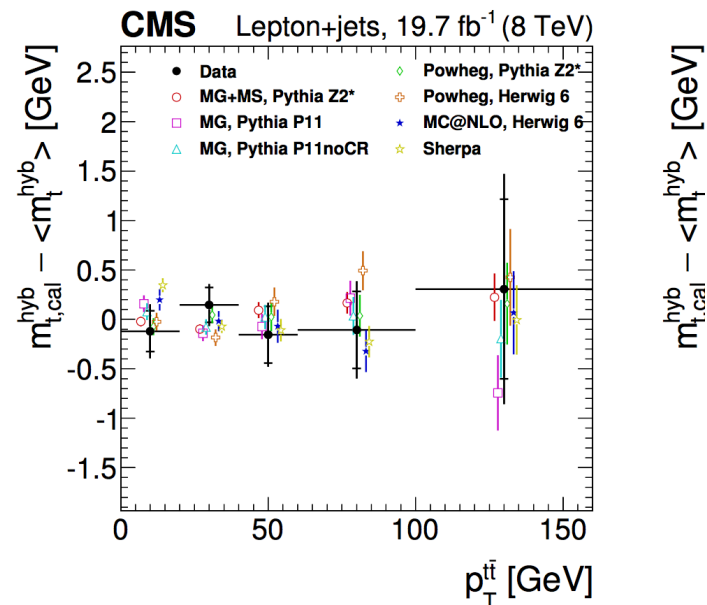
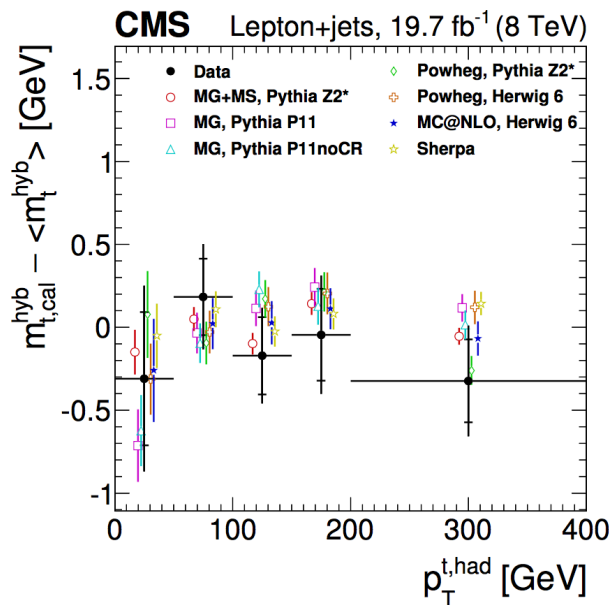
Treated in our analysis

Boosted Top Quarks

First simplification: $Q - 2p_T \gg m_t$



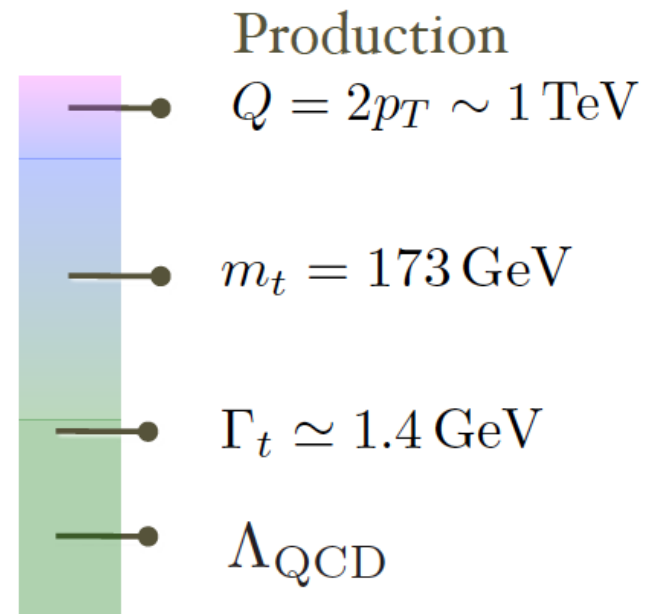
- Enables us to be inclusive w.r. to the hard-collinear decay products



- Top mass from reconstruction of boosted tops consistent with low p_T results.
- More precise studies possible with more statistics from Run-2.
- Meaning of m_t^{MC} for boosted tops and slow top quarks consistent.

Theory Issues for $pp \rightarrow t \bar{t} X$

- jet observable
- suitable top mass for jets
- initial state radiation
- final state radiation
- underlying event
- color reconnection
- beam remnant
- parton distributions
- sum large logs $Q \gg m_t \gg \Gamma_t$



Theory Issues for $pp \rightarrow t \bar{t} X$

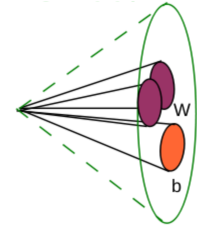
- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- underlying event
- color reconnection (★) ←
- beam remnant
- parton distributions
- sum large logs $Q \gg m_t \gg \Gamma_t$ ★

First
 $e^+ e^- \rightarrow t \bar{t} X$
and the issues ★

Only final-final state
color reconnection

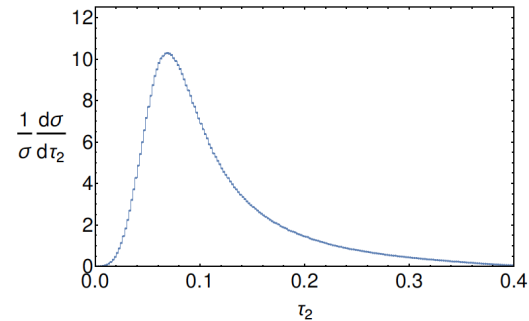
Thrust Distribution

Observable: 2-jettiness in e^+e^- for $Q = 2p_T \gg m_t$ (boosted tops)



$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

$$\tau_2 \xrightarrow{\text{peak}} \approx \frac{M_1^2 + M_2^2}{Q^2}$$

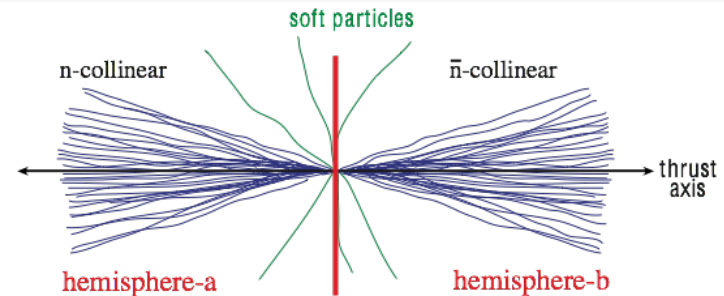


Invariant mass distribution in the resonance region of wide hemisphere jets !

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q, \mu) \int dl J_0(Ql, \mu) S_0(Q\tau - l, \mu)$$

Excellent mass sensitivity:

$$\tau_2^{\text{peak}} = 1 - \sqrt{1 - \frac{4m_t^2}{Q^2}} \quad (\text{tree level})$$



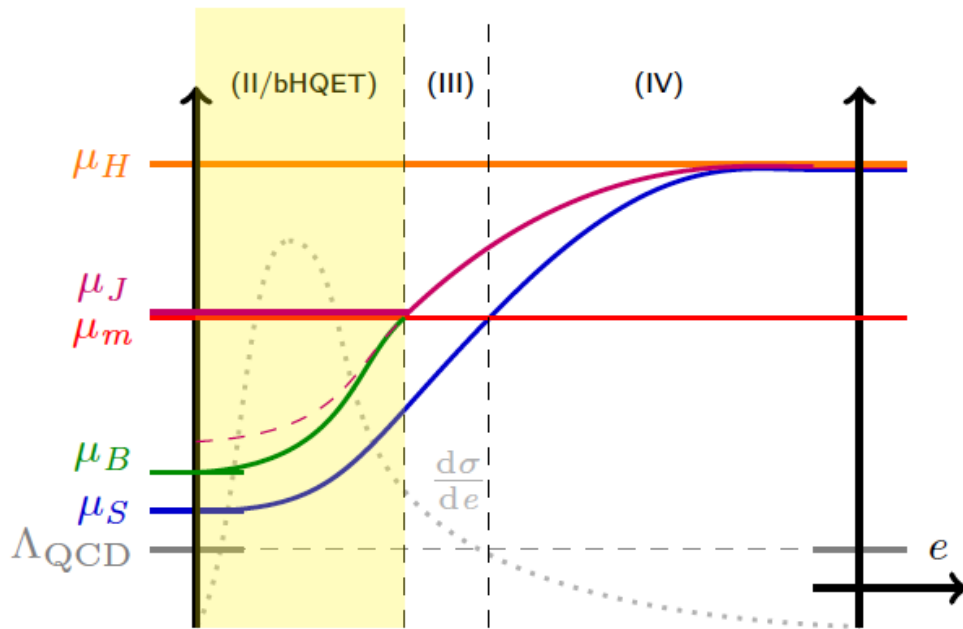
Factorization: EFT Treatment

- Boosted top jets

[Fleming, Hoang, Mantry, Stewart 2007]

$$n_f = n_\ell + 1$$

$$\frac{d\sigma^{\text{bHQET}}}{d\tau} = Q H(Q, m, \mu_H) U_H^{(n_f)}(Q, \mu_H, \mu_m) H_m^{(n_f)}(Q, \mu_m) U_m^{(n_\ell)}(Q, m, \mu_m, \mu_B) \\ \times \int ds dl B_e^{(n_\ell)}(s, m, \mu_B) U_S^{(n_\ell)}(\ell, \mu_B, \mu_S) S_e^{(n_\ell)}\left(Q(\tau - \tau_{\text{min}}) - \frac{s}{Q} - \ell, \mu_S\right)$$



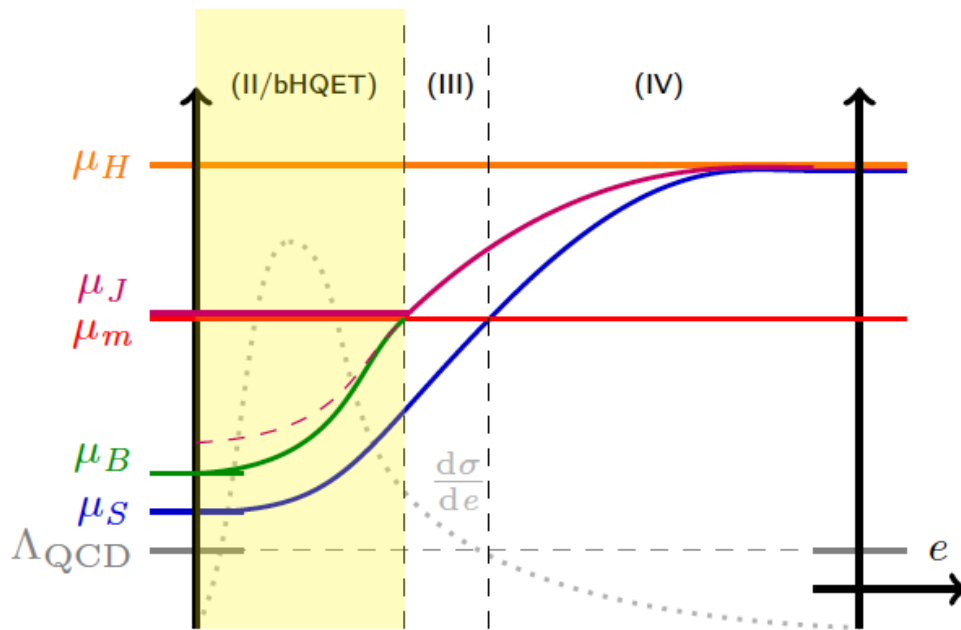
Factorization: EFT Treatment

- Developments:

- ▶ VFNS for final state jets (with massive quarks)

[Gritschacher, Hoang, Jemos, Mateu, Pietrulewicz '13 '14]

[Butenschön, Dehnadi, Hoang, Mateu '16 (to appear)]



- ▶ Non-perturbative power-corrections are included via a shape function

[Korchensky, Serman 1999]

[Hoang, Stewart 2007]

[Ligeti, Stewart, Tackmann 2008]

$$\frac{d\sigma}{d\tau} = \frac{d\sigma^{\text{part}}}{d\tau} \otimes F_{\text{mod}}(\Omega_1, \Omega_2, \dots)$$

- ▶ Gap-scheme

- ▶ MSR mass & R-evolution

[Hoang, Jain, Scimemi, Stewart 2010]

Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart, AHH :1704.01580

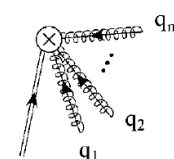
➔ NNLL + NLO + non-singular
+ hadronization
+ renormalon-subtraction
+ top quark decay ☐☐

- Good convergence
- Reduction of scale variation (NLL vs. NNLL)

Why the Observed Pole is not at the Pole Mass

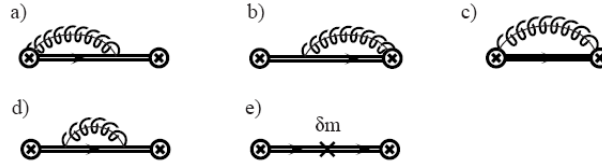
Jet function: $B_+(2v_+ \cdot k) = \frac{-1}{8\pi N_c m} \text{Disc} \int d^4x e^{ik \cdot x} \langle 0 | T \{ \bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x) \} | 0 \rangle$

- perturbative, any mass scheme
- depends on m_t, Γ_t
- Breit-Wigner at tree level
- Gauge-invariant off-shell top quark dynamics



$$W = \sum_{m=0}^{\infty} \sum_{\text{perms}} \frac{(-g)^m}{m!} \frac{\bar{n} \cdot A_{n,q_1}^{a_1} \dots \bar{n} \cdot A_{n,q_m}^{a_m}}{\bar{n} \cdot q_1 \bar{n} \cdot (q_1 + q_2) \dots \bar{n} \cdot (\sum_{i=1}^m q_i)} T^{a_m} \dots T^{a_1}$$

Singular functions encode information about where the physical pole is located



$$\hat{s} = \frac{M^2 - m_t^2}{m_t}$$

$$B_{\pm}^{\Gamma=0}(\hat{s}, \mu, \delta m) = \delta(s) + \frac{\alpha_s(\mu) C_F}{\pi} \left\{ \frac{2}{m\mu} \left[\frac{\theta(z) \ln(z)}{z} \right]_+ - \frac{1}{m\mu} \left[\frac{\theta(z)}{z} \right]_+ + \delta(s) \left[1 - \frac{\pi^2}{8} \right] \right\} - \frac{2\delta m}{m} \delta'(\hat{s})$$

$$B_{\pm}(\hat{s}, 0, \mu, \delta m) = -\frac{1}{\pi m} \frac{1}{\hat{s} + i0} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left[4 \ln^2 \left(\frac{\mu}{-\hat{s} - i0} \right) + 4 \ln \left(\frac{\mu}{-\hat{s} - i0} \right) + 4 + \frac{5\pi^2}{6} \right] \right\} - \frac{1}{\pi m} \frac{2\delta m}{(\hat{s} + i0)^2}$$

Fleming, AHH, Mantry, Stewart 2007

Why the Observed Pole is not at the Pole Mass

Is the pole mass determining the top single particle pole?

NO !

$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t}$$

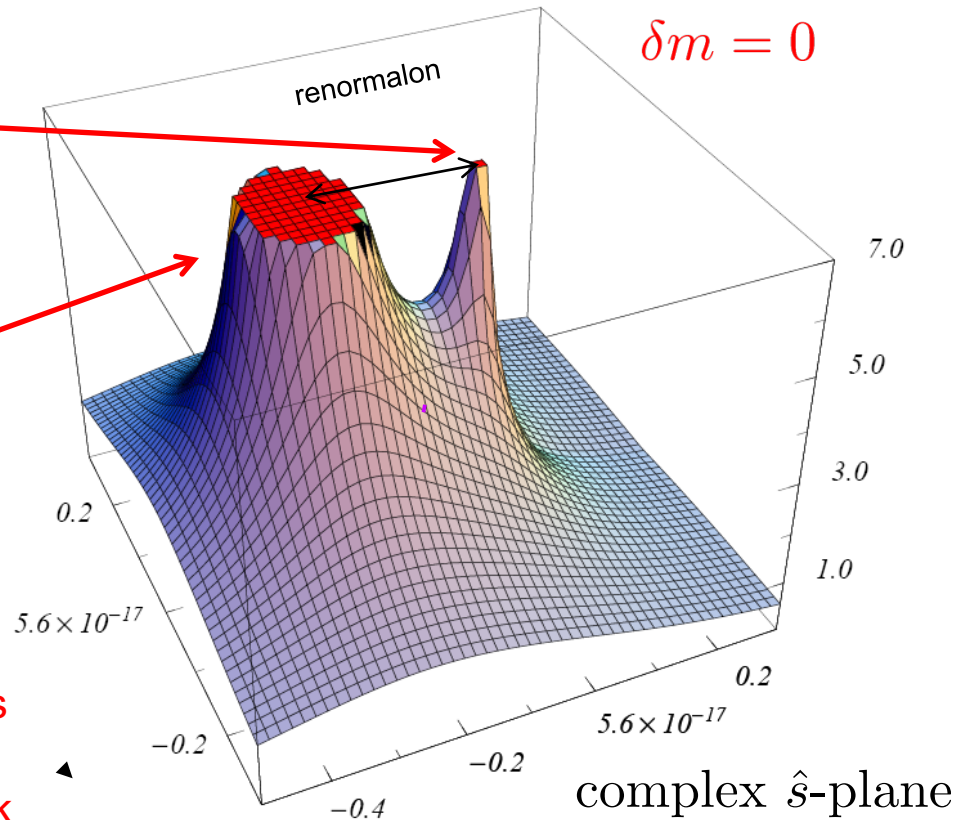
$$|\mathcal{B}_{\pm}(\hat{s}, \Gamma_t, \mu)|^2$$

$$\delta m = 0$$

pole mass peak
Invisible for $\Gamma_t > 0.5$
GeV

observable peak

- pole mass and observable peak separated by non-converging series
- pole mass peak (residue) decreases with order
- MSR mass close to observable peak

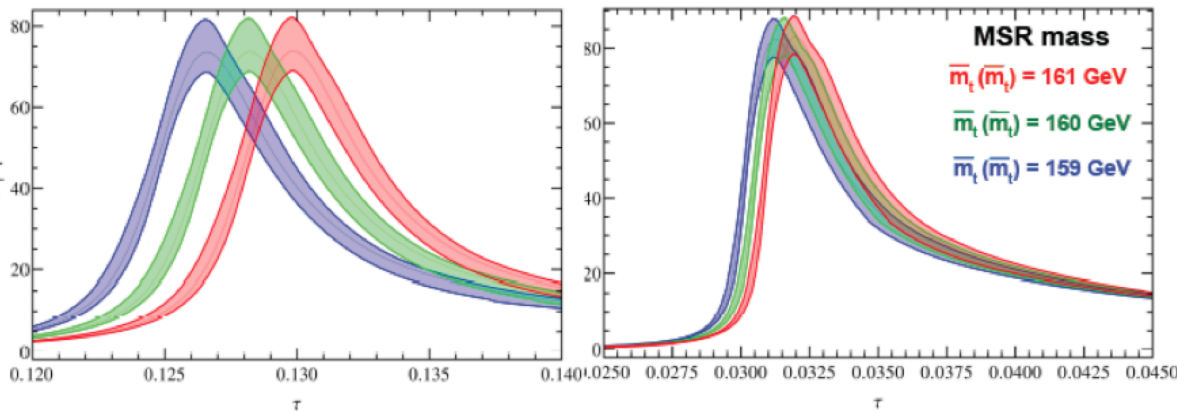


2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t}_{\text{renorm. scales finite lifetime}})$$

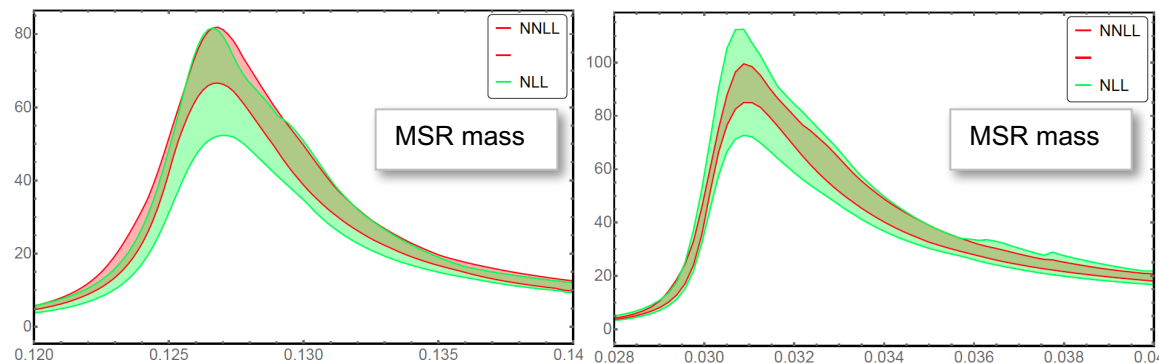
Q=700 GeV ($p_T = 350$ GeV)

Q=1400 GeV ($p_T = 700$ GeV)



Q=700 GeV

Q=1400 GeV



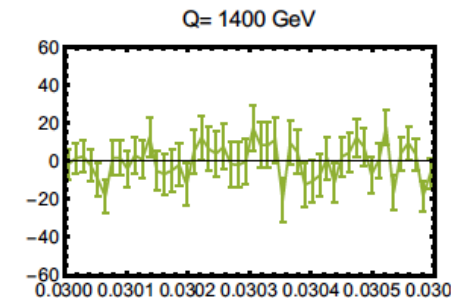
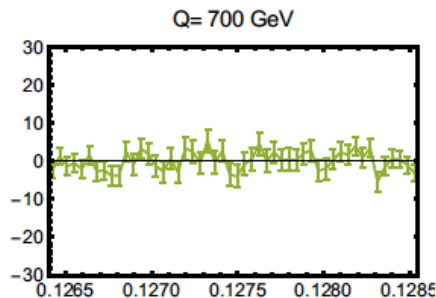
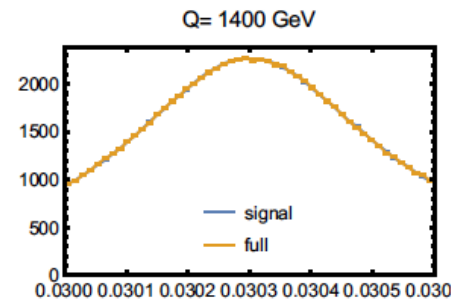
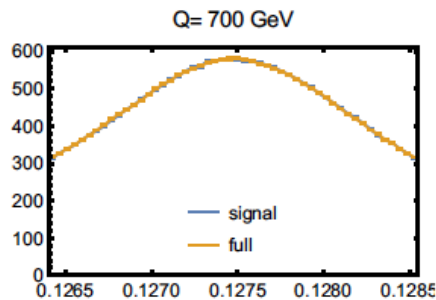
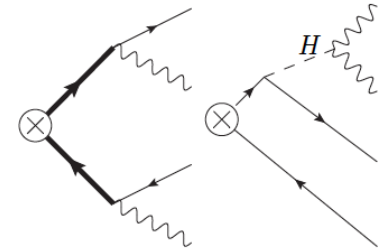
- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: $\Omega_{1,2,\dots}$
- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution

Signal $t\bar{t}b\bar{b}$ vs full $ee \rightarrow WWbb$

MadGraph 5 study:

- Non-resonant contributions are irrelevant for τ_2 distribution

- ▶ PYTHIA (or similar MCs) will give a good description of the production process at LO
- ▶ hemisphere invariant mass \sim top invariant mass (no pollution from background)

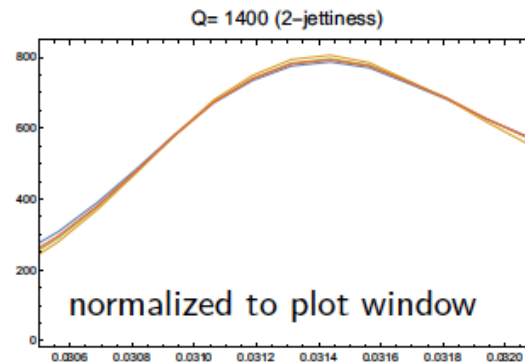
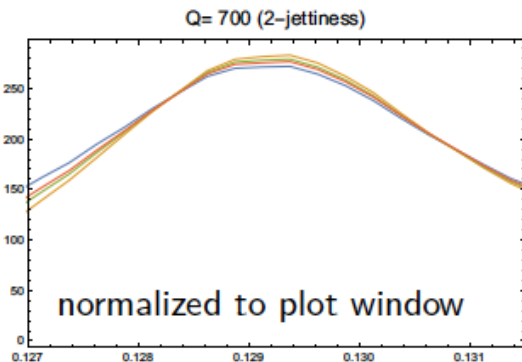
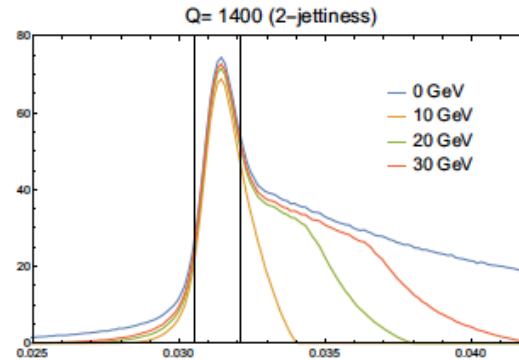
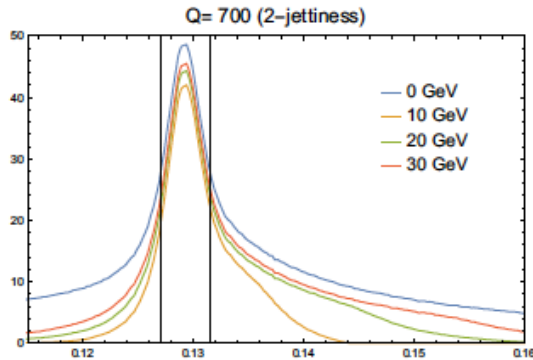


Pythia Study: Hemisphere Mass Cuts

- In our theory description we treat the top decay as inclusive w.r.t. hemisphere
 - ▶ violated by decay products which cross to the other hemisphere
 - ▶ no differential impact in resonance region (irrelevant when normalized to signal region)

Cuts on hemisphere invariant mass above and below:

$$M_i^{\text{cut}} = m_t^{\text{MC}} \pm \Delta^{\text{cut}}$$



Fit Procedure Details

Butenschön, Dehnadi, Mateu, Preisser, Stewart,AH; PRL 117 (2016) 153

- $\frac{d\sigma}{d\tau} = f(m_t^{\text{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \dots, \mu_H, \mu_J, \mu_S, \mu_M, R, \Gamma_t)$

any scheme non-perturbative renorm. scales finite lifetime

- Generating PYTHIA Samples: (PYTHIA 8.205)

at different energies: $Q = 600, 700, 800, \dots, 1400$ GeV

- ▶ masses: $m_t^{\text{MC}} = 170, 171, 172, 173, 174, 175$ GeV
- ▶ width: $\Gamma_t = 1.4$ GeV
- ▶ Statistics: 10^7 events for each set of parameters
- ▶ Tune 7 (Monash□)

- Feed MC data into **Fitting Procedure**: all ingredients are there

Fit parameters: $m_t^{\text{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \dots$

- ▶ Take $\alpha_s(M_Z)$ as input from world average.
(Sensitivity to strong coupling very weak.)

- ▶ standard fit based on χ^2 minimization
 - ▶ analysis with 500 sets of profiles (τ_2 dependent renorm. scales) for the each MC sample
 - ▶ different Q-sets: 7 sets with energies between 600 - 1400 GeV
 - ▶ different n-sets: 3 choices for fitranges - (xx/yy)% of maximum peak height
- } 21 fit setups

Fit Result: Pythia 8.205 vs. Theory

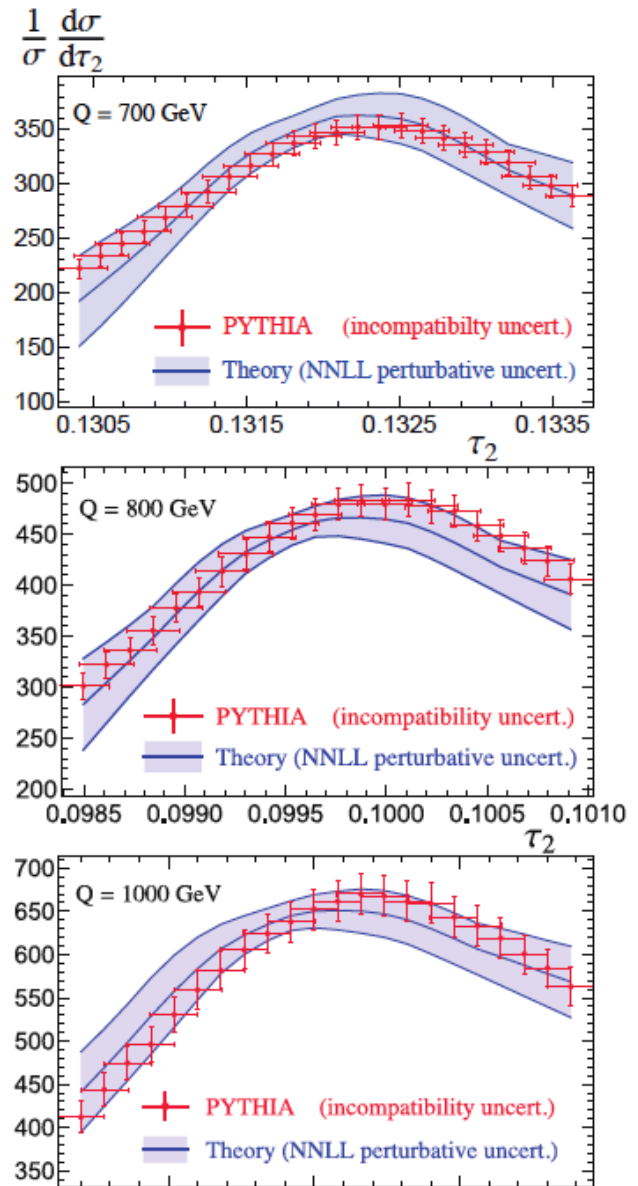
$\Gamma_t = 1.4$ GeV, tune 7,

$m_t^{\text{MC}} = 173$ GeV

$\Omega_1 = 0.44$ GeV,

$m_t^{\text{MSR}(1\text{GeV})} = 172.81$ GeV

- Good agreement of PYTHIA with NNLL/NLO theory predictions
- **Perturbative uncertainties** of theory predictions based on scale uncertainties (profiles)
- **MC uncertainties:**
 - Vertical: rescaled statistical error (PDF rescaling method) \rightarrow independent on statistics
 - Horizontal: fit coverage from 21 fit setups (**incompatibility uncertainty**)



Convergence & Stability: MSR vs. Pole Mass

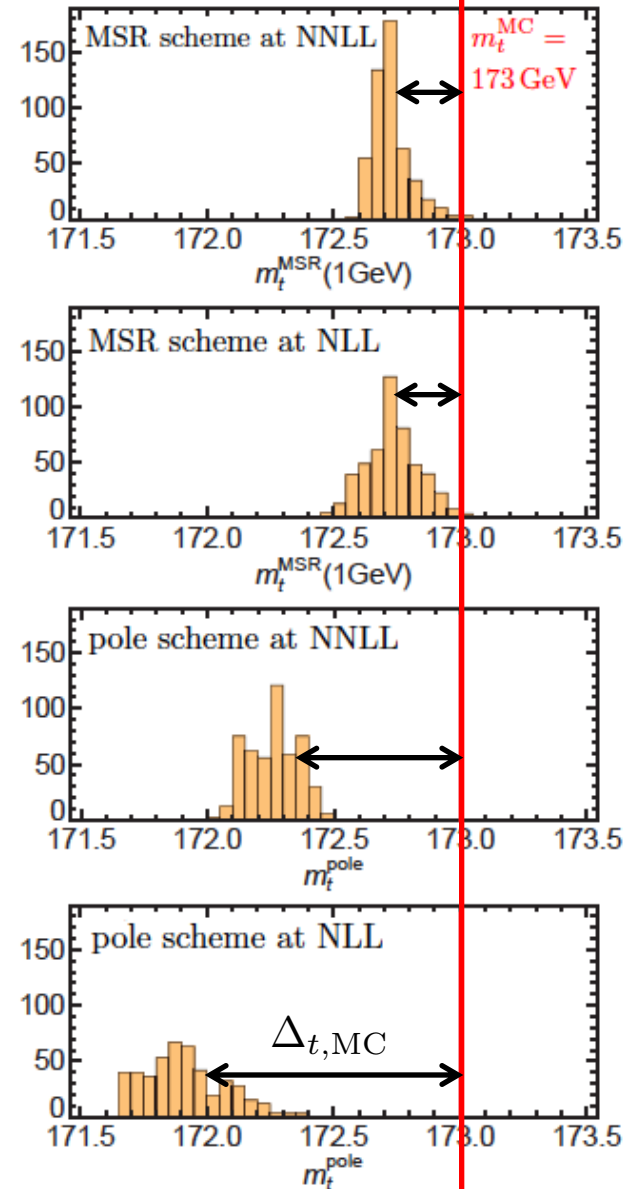
500 profiles; $\alpha_s = .118$; $\Gamma_t = 1.4$ GeV; tune 7;
 $Q = 700, 1000, 1400$ GeV; peak(60/80)%

Input: $m_t^{\text{MC}} = 173$ GeV

fit to find $m_t^{\text{MSR}}(1\text{GeV})$ or m_t^{pole}

- Good convergence & stability for MSR mass
- Mass $m_t^{\text{MSR}}(1\text{GeV})$ mass definition closest to the MC top mass m_t^{MC} .
- Pole mass shows worse convergence.
- Pole mass not compatible with MC mass within errors
- 1100/700 MeV difference at NLL/NNLL
- $m_t^{\text{pole}} \neq M_t^{\text{Pythia 8.2}}$

Similar analyses from the 20 other Q-set and n-range setups.



MSR Mass Tune Dependence

500 profiles; $\Gamma_t = 1.4, -1$ GeV; tune 1, 3, 7;
diff. Q-sets; peak(60/80)%

$$m_t^{\text{PYTHIA}} = 173 \text{ GeV}$$

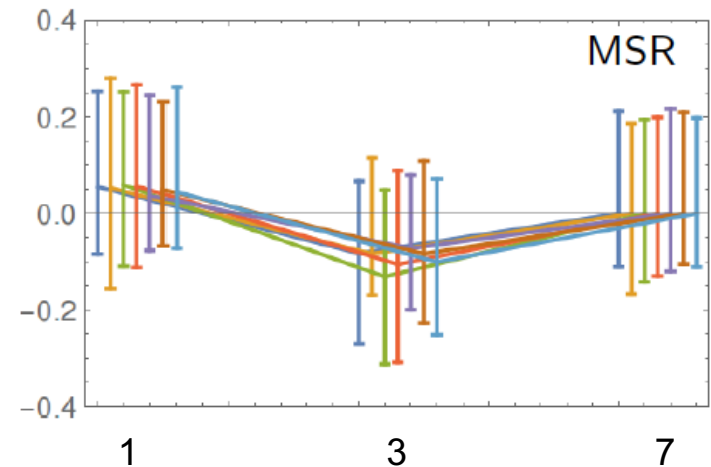
- tune dependence:

$$m^{\text{MSR}}[\text{tune}] - m^{\text{MSR}}[7]$$

- clear sensitivity to tune
- m^{MC} will depend on tune
- tune dependence is not a calibration uncertainty:

(different tune \Rightarrow different MC $\Rightarrow m_t^{\text{MC}}$)

- Tune dependence partially cancels in the calibration procedure to the extent they affect the observable(s) used for the calibration.



Final Result for $m_t^{\text{MSR}}(1 \text{ GeV})$

- All investigated MC top mass values show consistent picture
- MC top quark mass is indeed closely related to MSR mass

within uncertainties:

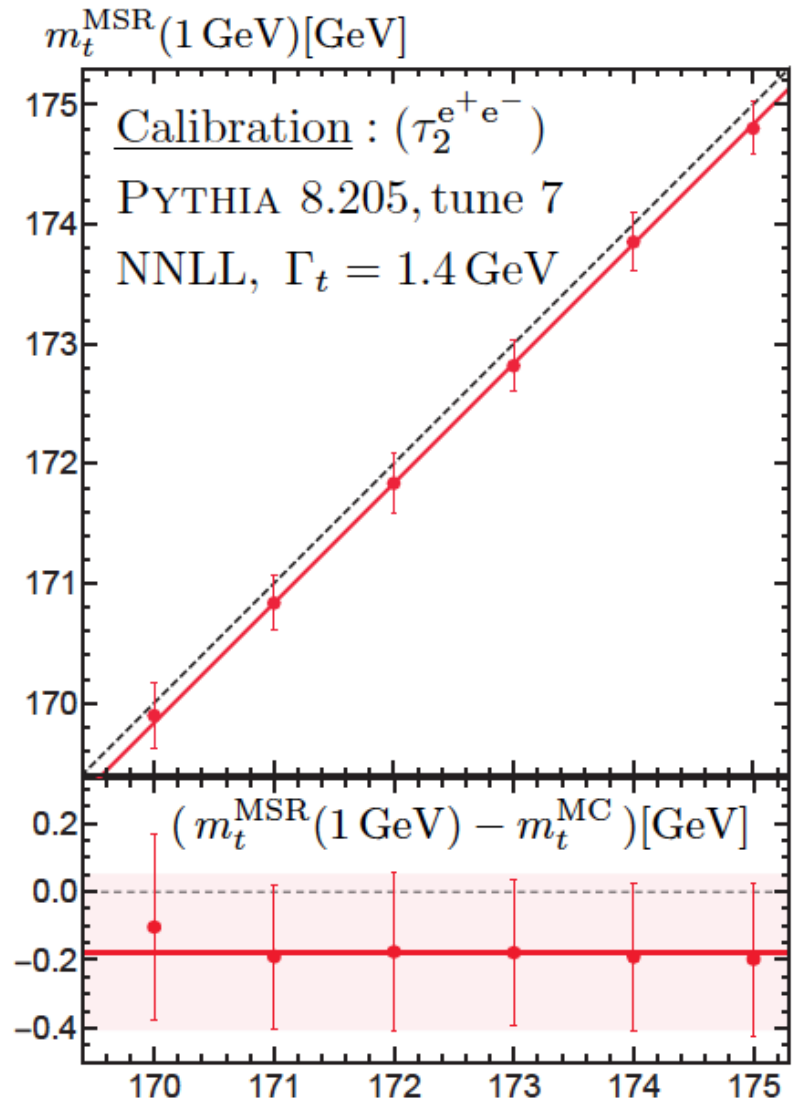
$$m_t^{\text{MC}} \simeq m_t^{\text{MSR}}(1 \text{ GeV})$$

$$m_t^{\text{MC}} = 173 \text{ GeV} \quad (\tau_2^{e^+e^-})$$

mass	order	central	perturb.	incompatibility	total
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	NLL	172.80	0.26	0.14	0.29
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	N ² LL	172.82	0.19	0.11	0.22
m_t^{pole}	NLL	172.10	0.34	0.16	0.38
m_t^{pole}	N ² LL	172.43	0.18	0.22	0.28

↓
Spread of results
from 21 fit setups

$$m_t^{\text{MSR}}(1 \text{ GeV}) = 172.82 \pm 0.22 \text{ GeV}$$



$$\Omega_1^{\text{PY}} = 0.41 \pm 0.07 \pm 0.02 \text{ GeV at NLL} \quad m_t^{\text{MC}} [\text{GeV}]$$

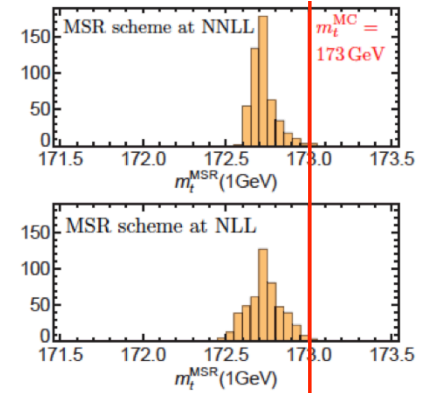
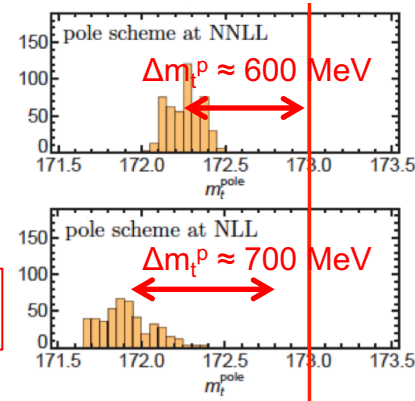
$$\Omega_1^{\text{PY}} = 0.42 \pm 0.07 \pm 0.03 \text{ GeV at N}^2\text{LL}$$

Pole Mass Determination

1) Pole mass implemented in code:

$m_t^{\text{MC}} = 173 \text{ GeV } (\tau_2^{e^+e^-})$					
Mass	Order	Central	Perturb.	Incompatibility	Total
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	NLL	172.80	0.26	0.14	0.29
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	NNLL	172.82	0.19	0.11	0.22
m_t^{pole}	NLL	172.10	0.34	0.16	0.38
m_t^{pole}	NNLL	172.43	0.18	0.22	0.28

Pole mass
smaller than
MSR mass



2) Pole mass determined from MSR mass:

$$\alpha_s(M_Z) = 0.118$$

$$n_f = 5$$

Pole mass
larger than
MSR mass

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(1 \text{ GeV}) = 0.173 + 0.138 + 0.159 + 0.23 \text{ GeV}$$

- Calibration in terms of the pole mass involves large higher-order perturbative corrections
- Additional uncertainty on pole mass: $(m_t^{\text{pole}})_{\text{NLL}} = 172.45 \pm 0.52 \text{ GeV}$,
(added quadratically) $(m_t^{\text{pole}})_{\text{NNLL}} = 172.72 \pm 0.41 \text{ GeV}$
- Theoretical ambiguity of the top quark pole mass: **250 MeV**
Pole mass should be abandoned once uncertainties reach 0.5 GeV.

Lepenik, Preisser, AHH; arXiv:1706.08526

Conversion to the Top $\overline{\text{MS}}$ Mass

1) Approach: $m_t^{\text{MC}} \sim m_t^{\text{MSR}}(1 \text{ GeV})$

No detailed analysis!

No electroweak corrections!

$$m_t^{\text{pMSR}}(1 \text{ GeV}) = 172.82 \pm 0.022, \text{ GeV}$$

$$\begin{aligned} m_t^{\text{pMSR}}(1 \text{ GeV}) - m_t^{\text{pMSR}}(163.018 \text{ GeV}) &= \\ &= 8.913 + 0.906 + 0.052 - 0.070 \pm 0.035 \text{ GeV} \\ &= 9.802 \pm 0.035 \text{ GeV} \end{aligned}$$

$$\overline{m}_t(\overline{m}_t) = 163.020 \pm 0.230 \text{ GeV}$$

→ Can be improved by next order

2) Approach: $m_t^{\text{MC}} \sim m_t^{\text{Pole}}$

$$m_t^{\text{pole}} = 172.72 \pm 0.410 \text{ GeV}$$

$$\begin{aligned} m_t^{\text{pole}} - m_t^{\text{nMSR}}(163 \text{ GeV}) &= 7.505 + 1.581 + 0.481 + 0.193 \\ &+ 0.111 + 0.079 + 0.066 + 0.064 + 0.071 + \dots \text{ GeV} \end{aligned}$$

$$\overline{m}_t(\overline{m}_t)_{2\text{loop}} = 163.634 \pm 0.890 \text{ GeV}$$

$$\overline{m}_t(\overline{m}_t)_{3\text{loop}} = 163.153 \pm 0.475 \text{ GeV}$$

$$\overline{m}_t(\overline{m}_t)_{4\text{loop}} = 162.960 \pm 0.430 \text{ GeV} \quad \overline{m}_t(\overline{m}_t)_{8\text{loop}} = 162.640 \pm 0.430 \text{ GeV}$$

→ Proper interpretation hard due to renormalon problem.

Theory Issues for $pp \rightarrow t \bar{t} X$

- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- underlying event
- color reconnection ★
- beam remnant
- parton distributions
- sum large logs $Q \gg m_t \gg \Gamma_t$ ★

Can apply this to current measurements if we trust Pythia extrapolation for remaining items

Theory Issues for $pp \rightarrow t \bar{t} X$

- jet observable ★ ★ Jet Mass in Jet of radius R
- suitable top mass for jets ★
- initial state radiation ★ Better: factorization
for pp
- final state radiation ★
- underlying event ← Note: no star here
- color reconnection ★
- beam remnant ★ Jet veto
- parton distributions ★ multiple channels
- sum large logs $Q \gg m_t \gg \Gamma_t$ ★

Jet Mass of Boosted Top Quarks

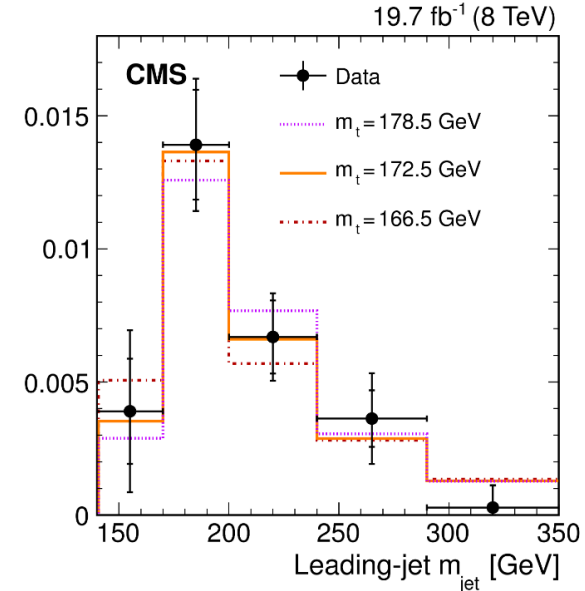
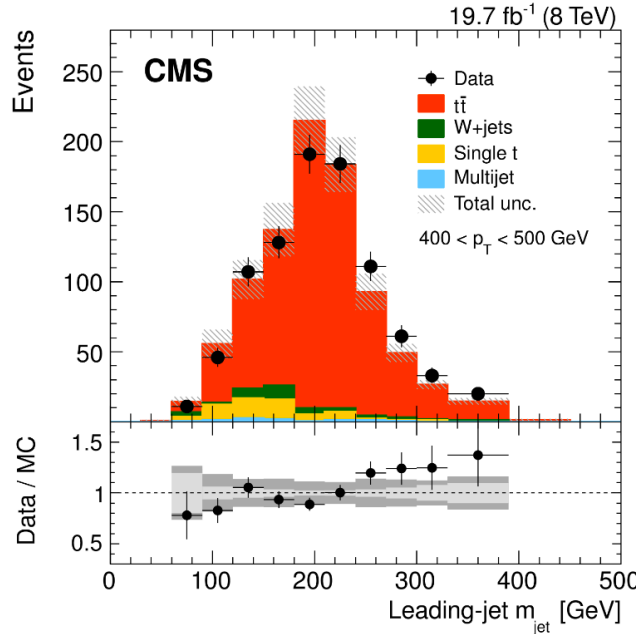
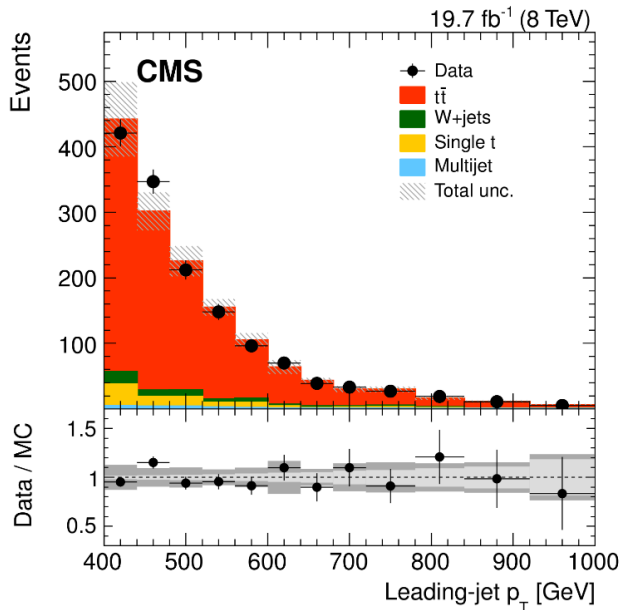


<http://arxiv.org/abs/1703.06330>

Top mass from boosted jet mass

Cambridge-Aachen jet with distance parameter $R = 1.2$, and $p_T > 400$ GeV.

$$m_t = 170.8 \pm 9.0 \text{ GeV}$$



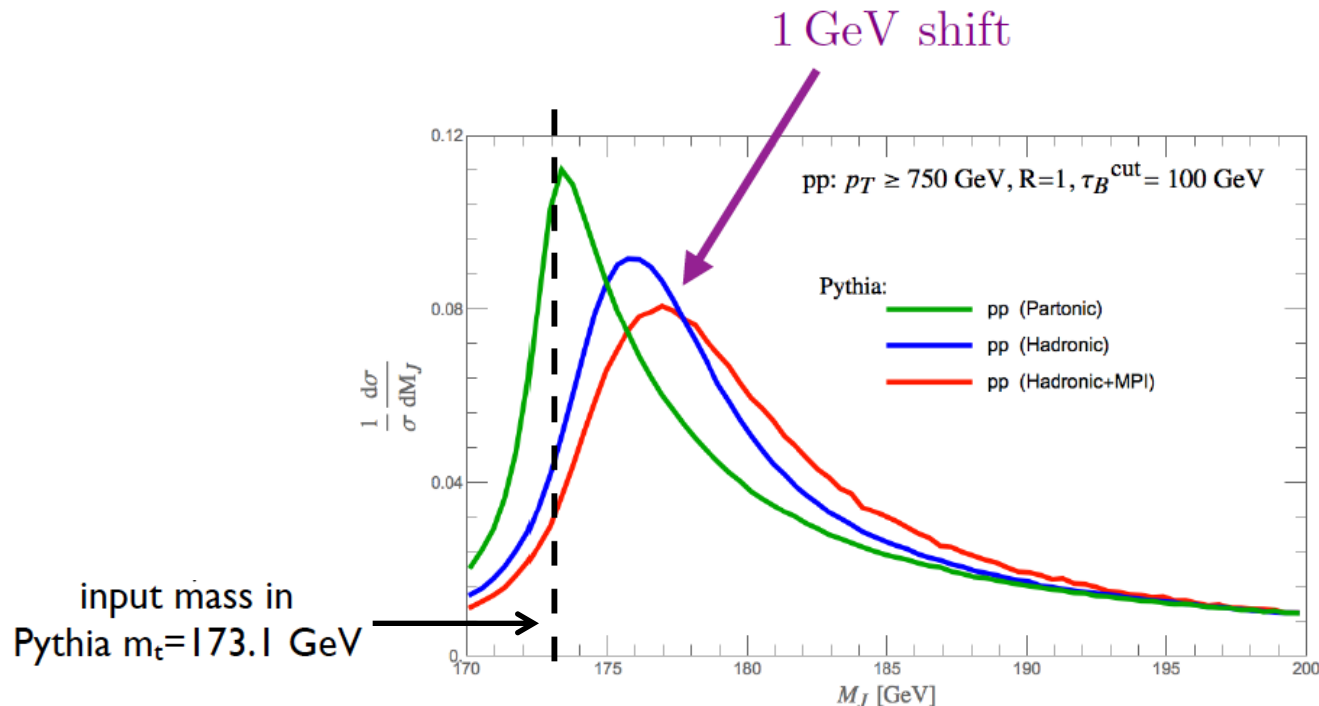
Theory Issues for $pp \rightarrow t \bar{t} X$

Extension to pp (in principle) straightforward: (e.g. N-jettiness & X-Cone jets)

$$\frac{d^2\sigma}{dM_{J_1}^2 dM_{J_2}^2 d\mathcal{T}^{\text{cut}}} = \text{tr} \left[\hat{H}_{Q_m} \hat{S}(\mathcal{T}^{\text{cut}}, R, \dots) \otimes F \right] \otimes J_B \otimes J_B \otimes \mathcal{I} \otimes f f$$

Issue is that UE / MPI is significant:

Same jet functions as e^+e^-



Theory Issues for $pp \rightarrow t \bar{t} X$

Extension to pp (in principle) straightforward: (e.g. N-jettiness & X-Cone jets)

$$\frac{d^2\sigma}{dM_{J_1}^2 dM_{J_2}^2 d\mathcal{T}^{\text{cut}}} = \text{tr}[\hat{H}_{Q_m} \hat{S}(\mathcal{T}^{\text{cut}}, R, \dots) \otimes \mathbf{F}] \otimes J_B \otimes J_B \otimes \mathcal{I} \otimes f f$$

Issue is that UE / MPI is significant:

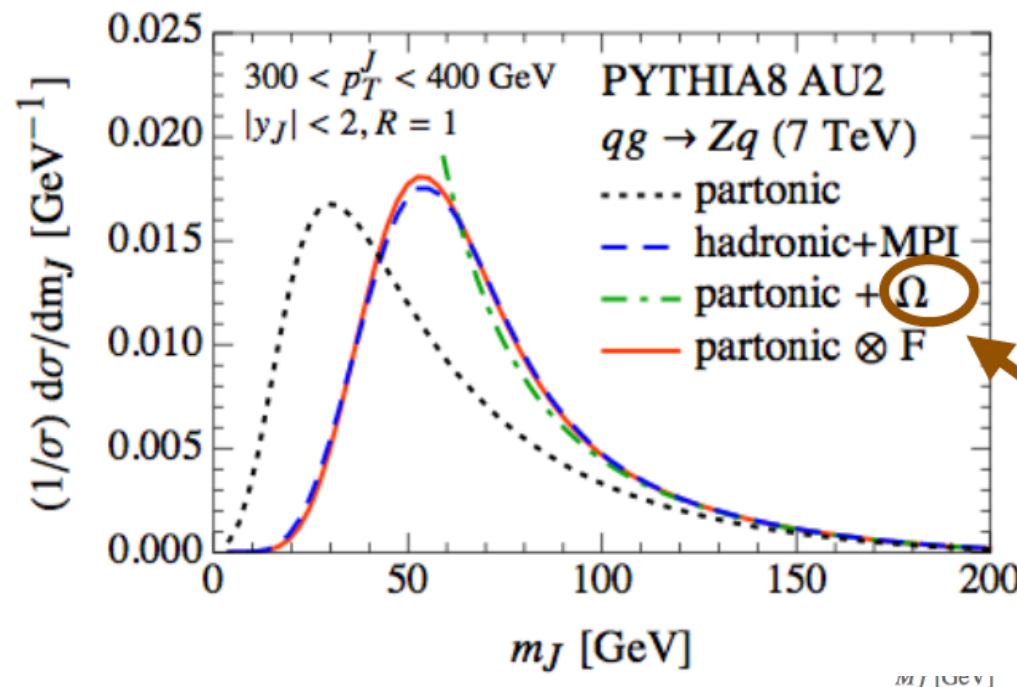
Same jet functions as e^+e^-

BUT control of Underlying Event is model dependent.

Same model used for Hadronization can describe UE by (primarily) tuning one parameter Ω .

$$\Omega = \int dk k F(k)$$

Stewart, Tackmann, Waalewijn, 2015



Grooming with SoftDrop

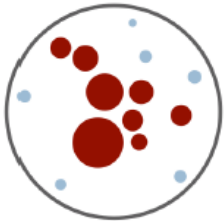
- Grooms soft radiation from the jet

Larkowski, Marzani, Soyez, Thaler, 2014

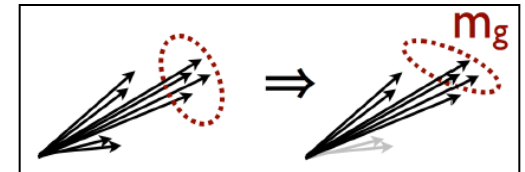
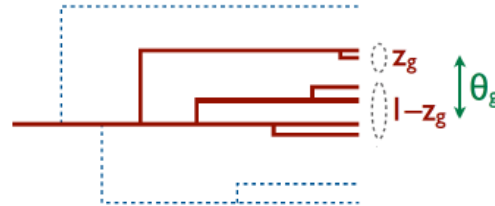
$$\frac{\min(p_{Ti}, p_{Tj})}{p_{Ti} + p_{Tj}} > z_{\text{cut}} \left(\frac{\Delta R_{ij}}{R_0} \right)^\beta \quad z > z_{\text{cut}} \theta^\beta$$

two grooming parameters

Groomed Jet



Groomed Clustering Tree



More Grooming

$\beta \rightarrow -\infty$

$\beta < 0$

$\beta = 0$

$\beta > 0$

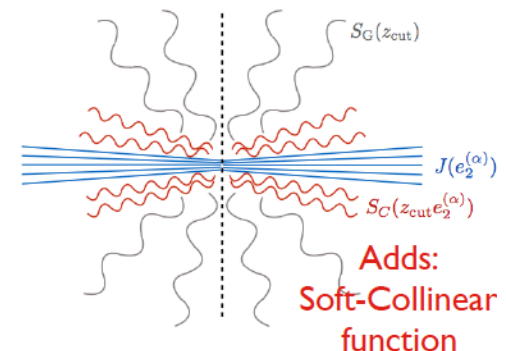
Less Grooming

$\beta \rightarrow \infty$

- Allows for factorization calculations

Frye, Larkowski, Schwartz, Yan, 2016

Mode separation: additional soft-collinear modes



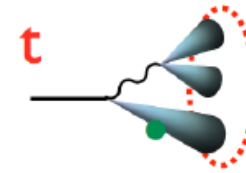
Theory Set Up with SoftDrop

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

$$p_T \gg m_t \gg \Gamma_t > \Lambda_{\text{QCD}}$$

- **Boosted Tops** $p_T \gg m_t$ retain top decay products

- **Fat Jets** $R \gg \frac{m_t}{p_T}$

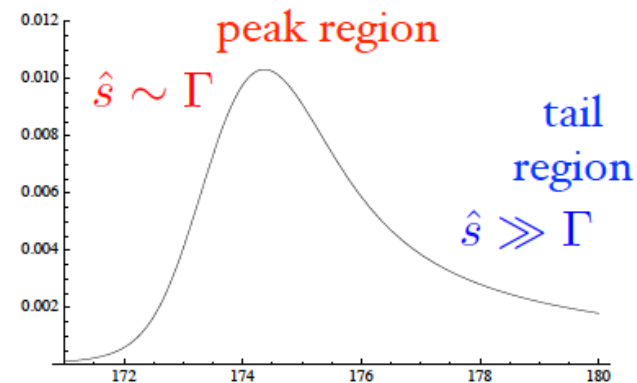


- **Sensitivity** $\hat{s} \sim \Gamma_t$ for measurement of jet-mass m_J

$$\hat{s} = \frac{m_J^2 - m_t^2}{m_t}$$

- **Grooming** z_{cut}, β

- **Jet Veto** \mathcal{T}^{cut} or p_T^{cut}

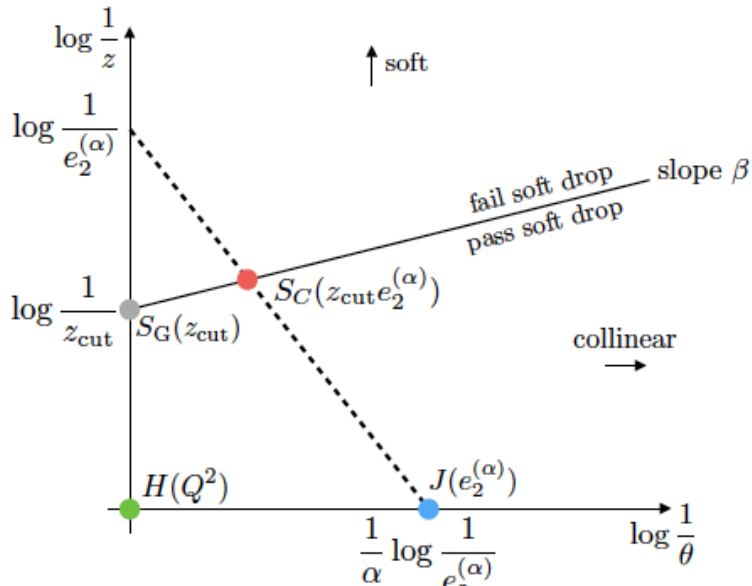


(Perturbative and Nonperturbative effects give $\Gamma > \Gamma_t$)

Theory Set Up with SoftDrop

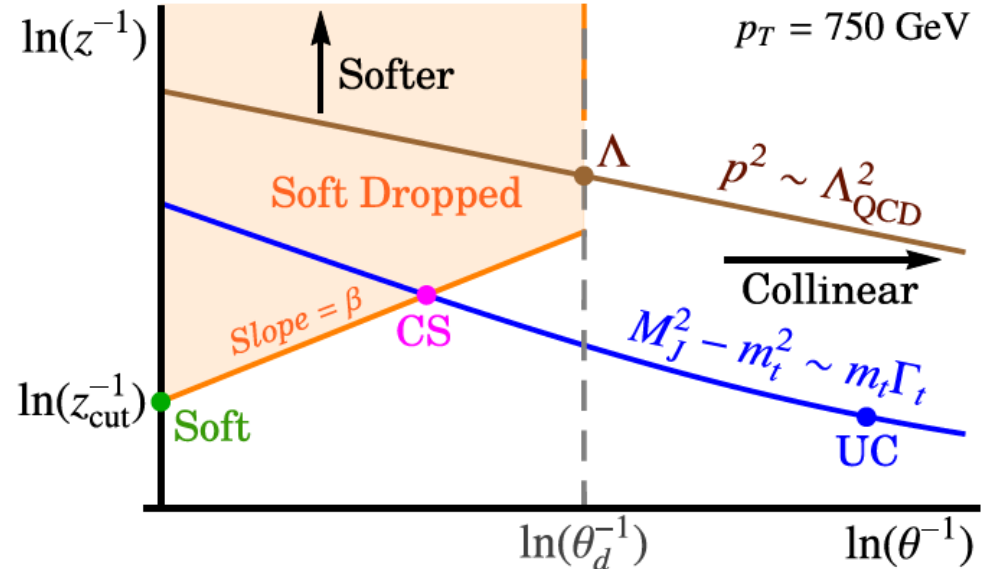
Modes:

massless quarks:

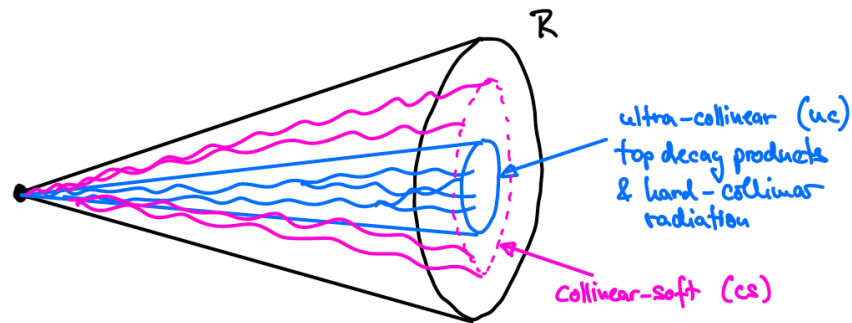


Frye, Larkowski, Schwartz, Yan, 2016

top quarks:



Mantry, Pathak, Stewart, AHH; arXive:1708.02586



Theory Set Up with SoftDrop

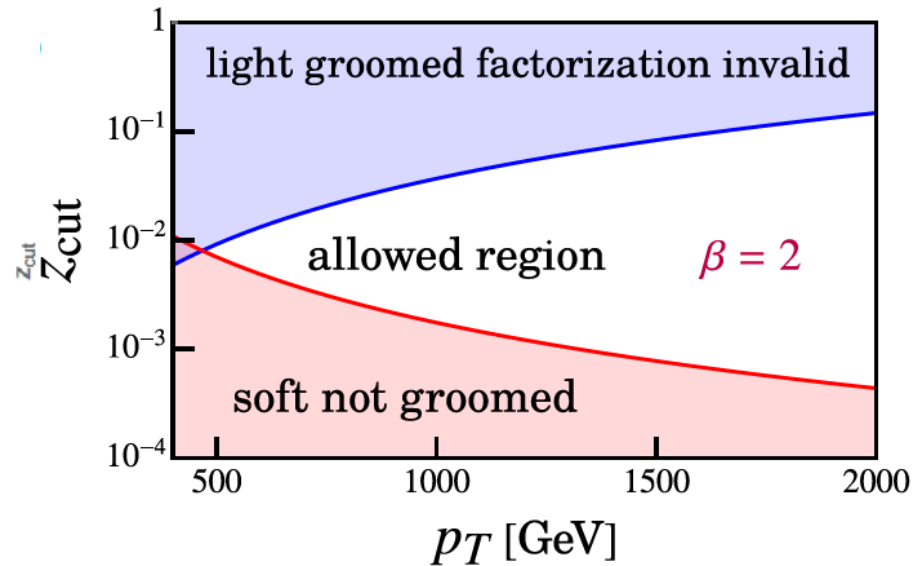
AH, Mantry, Pathak, Stewart; arXiv:1708.02586

Can only apply a “light soft drop” for tops:

$$\frac{\Gamma_t}{m} \left(\frac{Q}{2m} \right)^\beta \gg z_{\text{cut}} \gg \frac{2m\Gamma_t}{Q^2}$$

Ensure soft drop
does not touch J_B

Ensure soft drop removes global
soft radiation from measurement



Factorization with Soft Drop on one jet:

$$\frac{d^2\sigma}{dM_j^2 dT^{\text{cut}}} = \text{tr} \left[\hat{H}_{Qm} \hat{S}(T^{\text{cut}}, Qz_{\text{cut}}, \beta, \dots) \otimes F \right] \otimes J_B \otimes \mathbb{I} \otimes f f$$

$$\times \left\{ \int dldk J_B \left(\hat{s}_t - \frac{Q\ell}{m}, \Gamma_t, \delta m \right) S_C \left[\left(\ell - k \left(\frac{k}{Q_{\text{cut}}} \right)^{\frac{1}{1+\beta}} \right) Q_{\text{cut}}^{\frac{1}{1+\beta}}, \beta, \mu \right] F_C(k) \right\}$$

(“high- p_T factorization”)

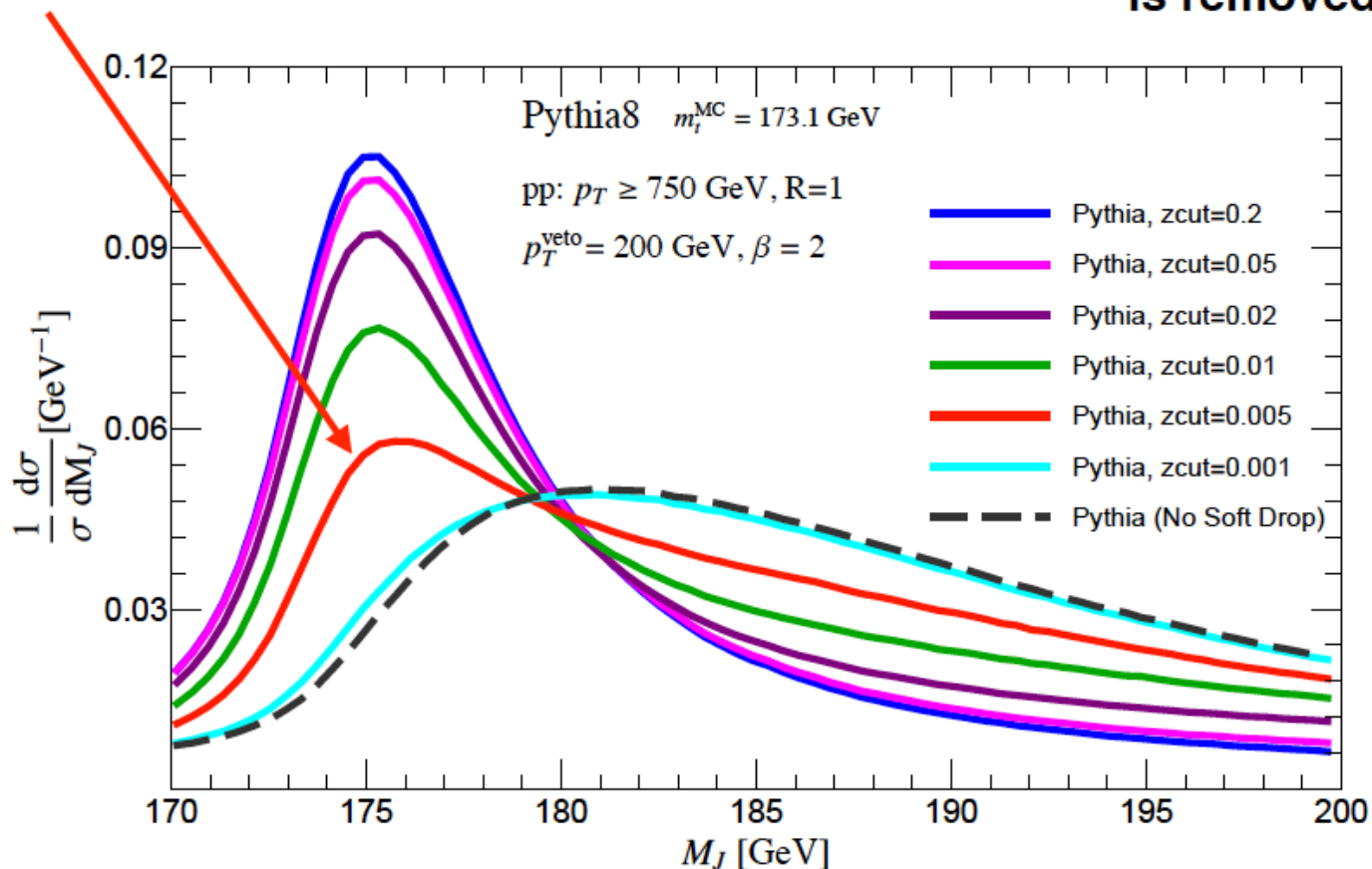
Preliminary Studies of SoftDrop Effects

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

z_{cut} dependence

predict transition for “light Soft Drop” ✓

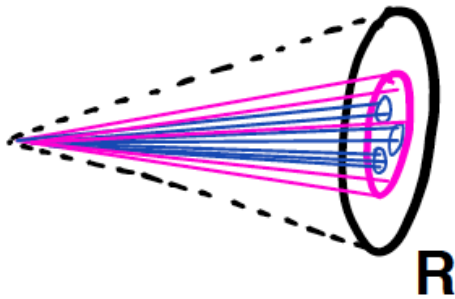
most contamination
is removed



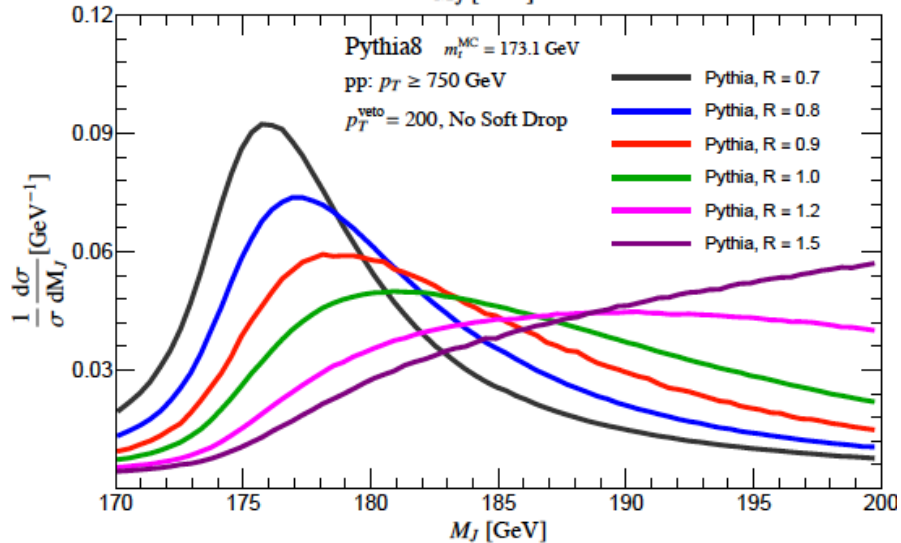
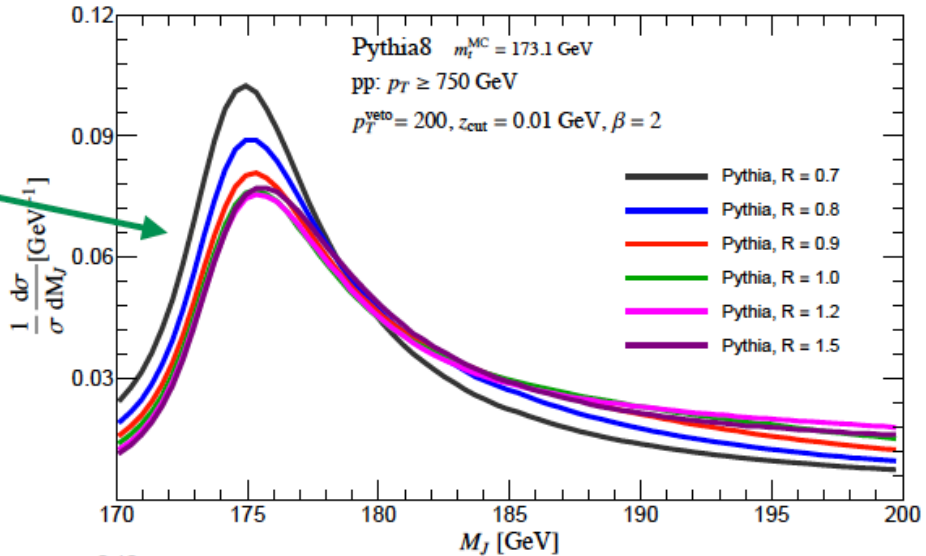
Preliminary Studies of SoftDrop Effects

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

predict:
independent of
Jet Radius



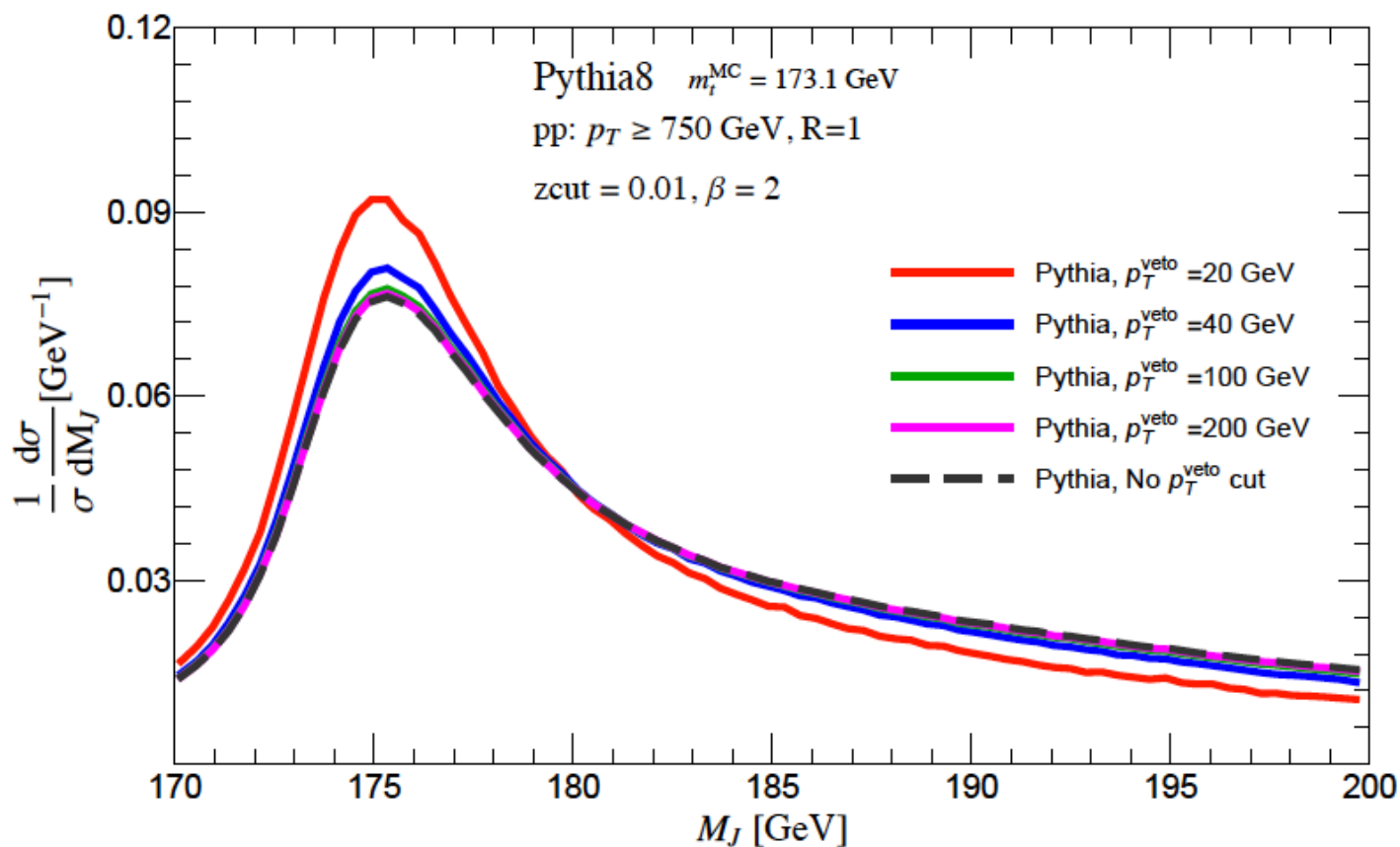
Without
Soft Drop
(huge):



Preliminary Studies of SoftDrop Effects

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

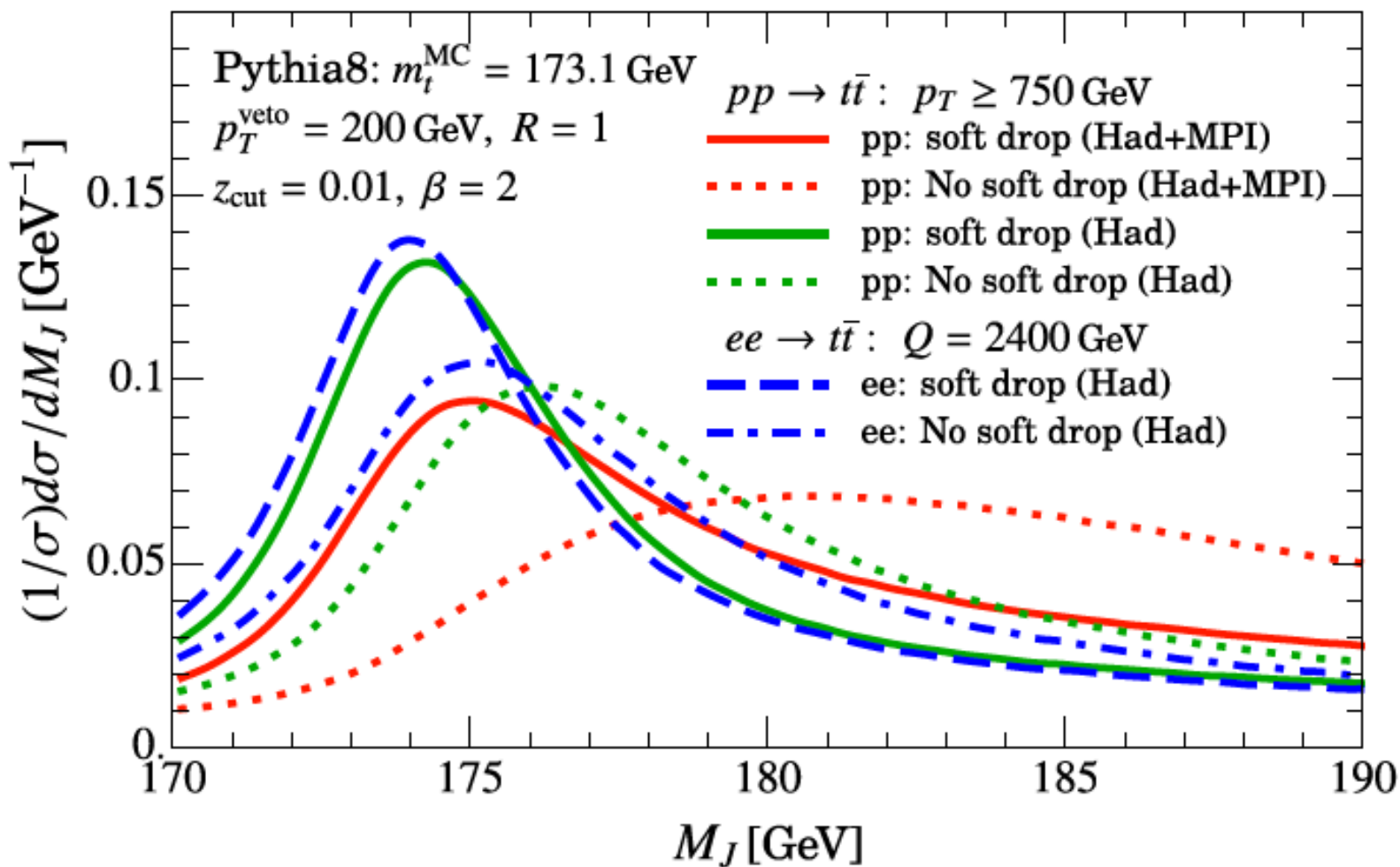
**Predict independent of cutoff
on radiation outside the jet (“jet veto”):**



Preliminary Studies of SoftDrop Effects

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

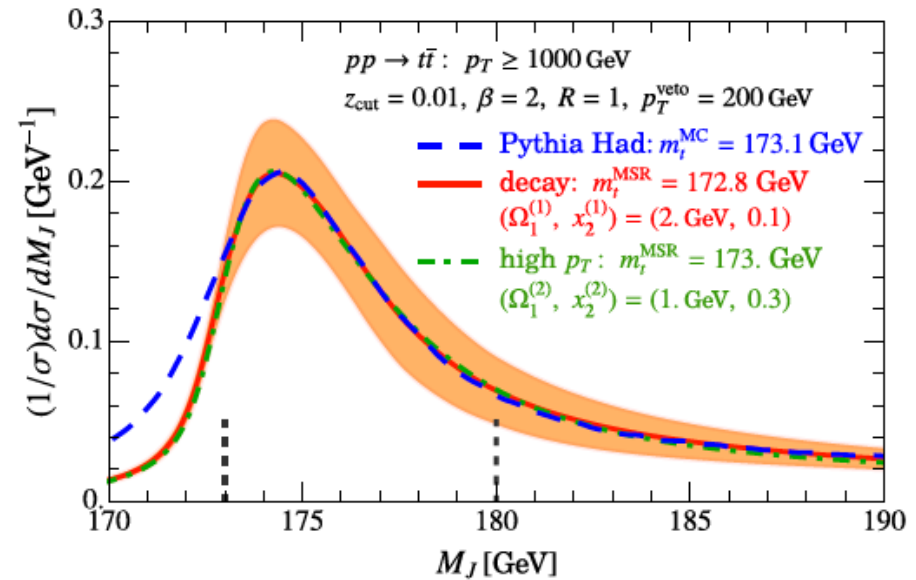
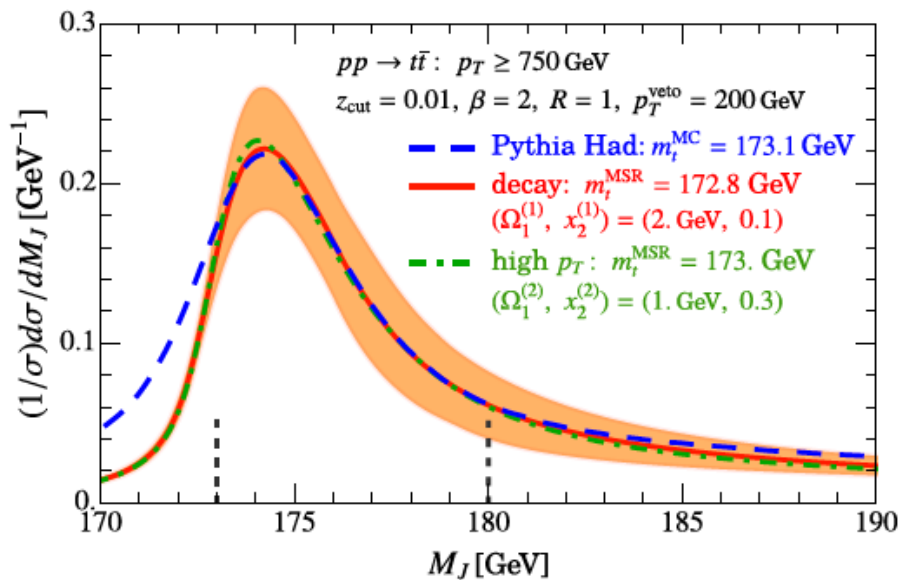
Soft Drop prediction: Same Result for e^+e^- and pp collisions



Top Mass Fits (to Pythia 8 output)

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

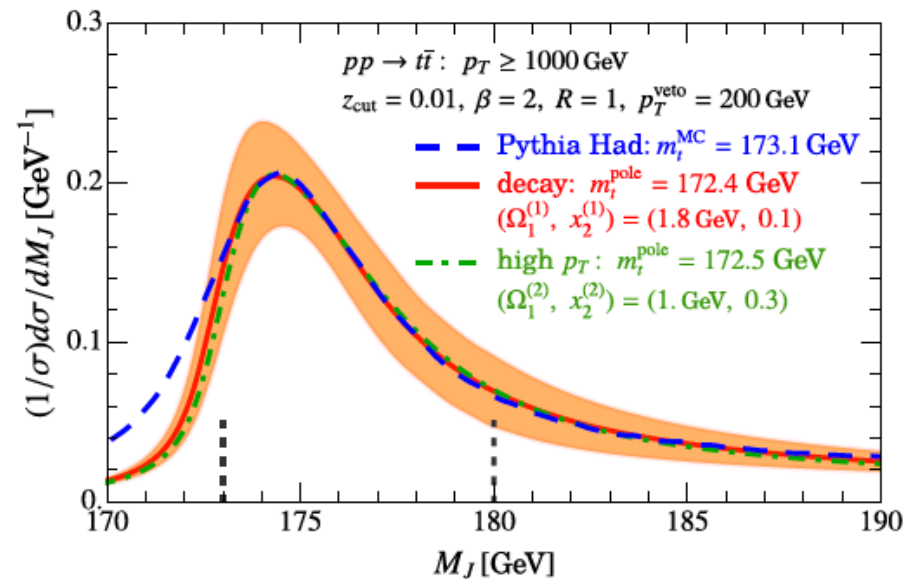
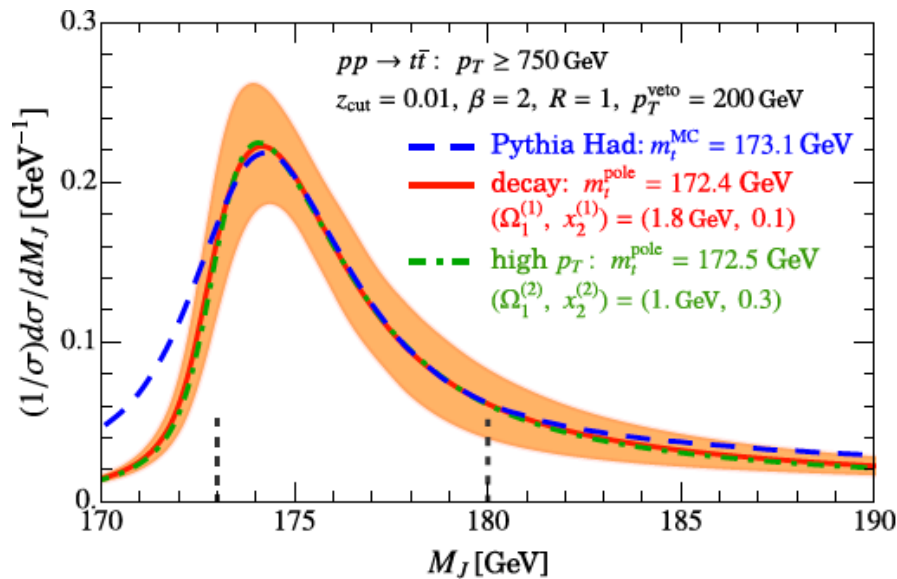
Hadronization only: MC mass and MSR mass compatible



Top Mass Fits (to Pythia 8 output)

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

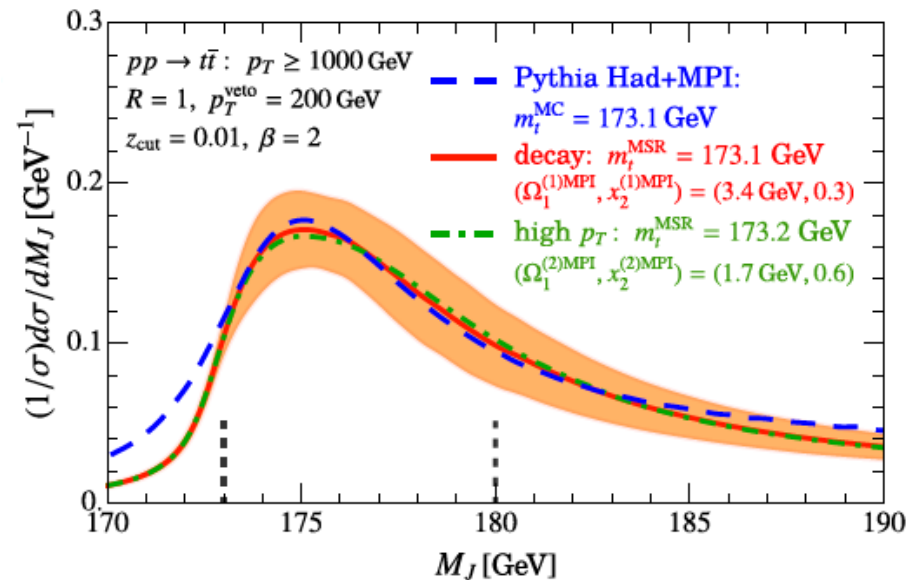
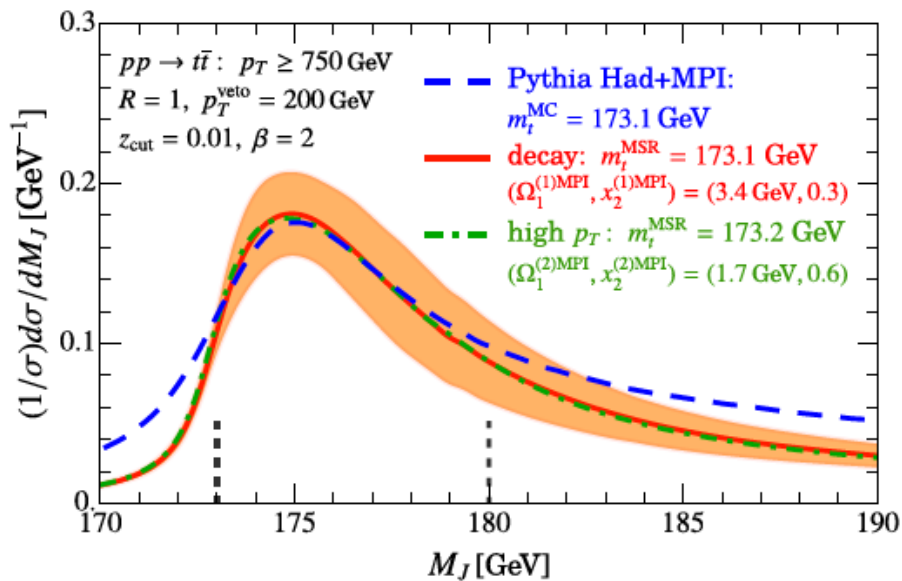
Hadronization only: MC mass and pole mass have larger discrepancy



Top Mass Fits (to Pythia 8.2 output)

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

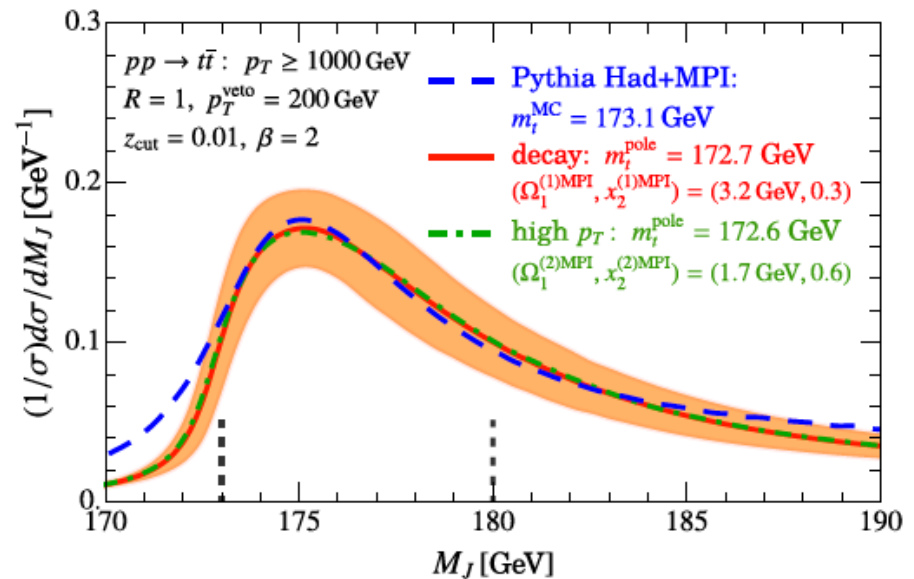
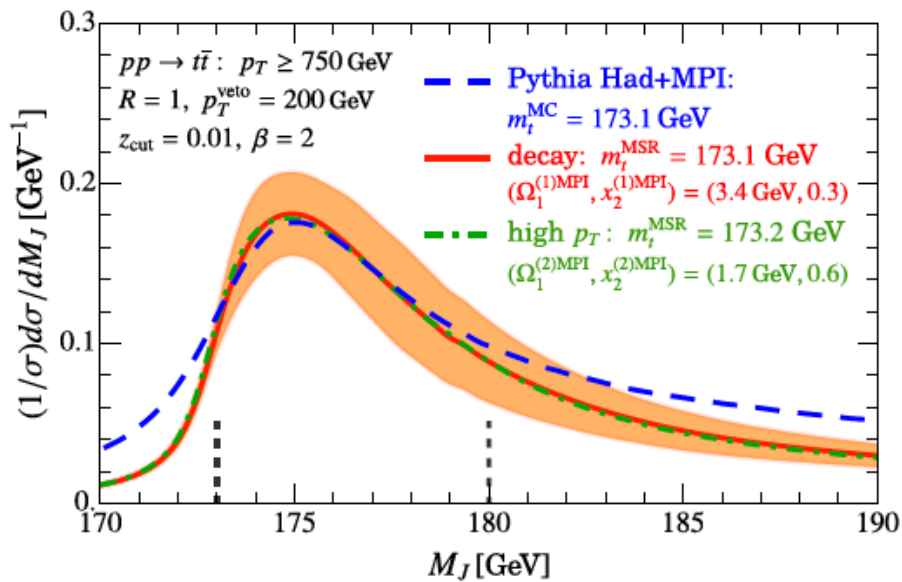
Hadronization + MPI: MC mass and MSR mass compatible



Top Mass Fits (to Pythia 8.2 output)

AH, Mantry, Pathak, Stewart; arXiv:1708.02586

Hadronization + MPI: MC mass and pole mass have larger discrepancy



Summary

- First systematic MC top quark mass calibration based on e^+e^- 2-jettiness (large p_T): [related to observables dominating the reconstruction method](#)

$$\blacktriangleright \square m_t^{\text{Pythia8.2}} = 173 \text{ GeV}$$

$$\blacktriangleright \square m_t^{\text{MSR}(1\text{GeV})} = 172.82 \pm 0.22 \text{ GeV}$$

$$\blacktriangleright \square (m_t^{\text{pole}})_{\text{NLO}} = 172.71 \pm 0.41 \text{ GeV}$$

- NNLL+NLO QCD calculations based on an extension of the SCET approach concerning massive quark effects (all large logs incl. $\text{Ln}(m)$'s summed systematically) [describing boosted top quarks](#).

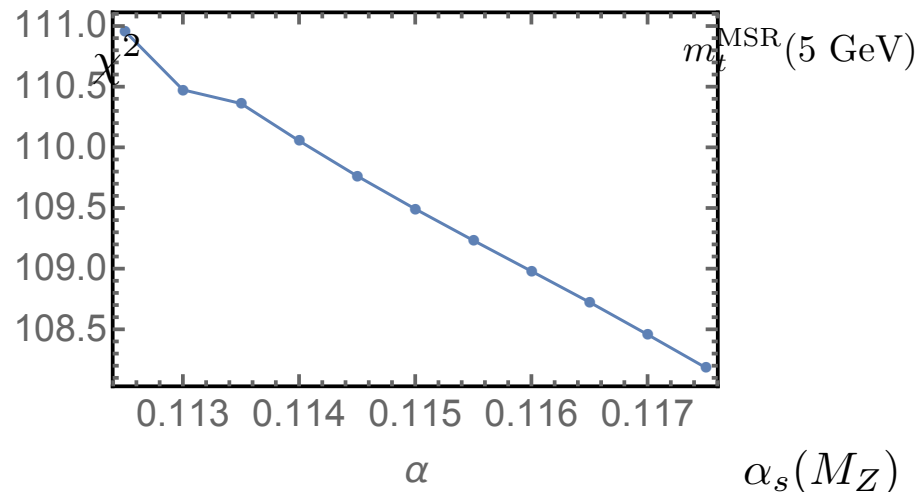
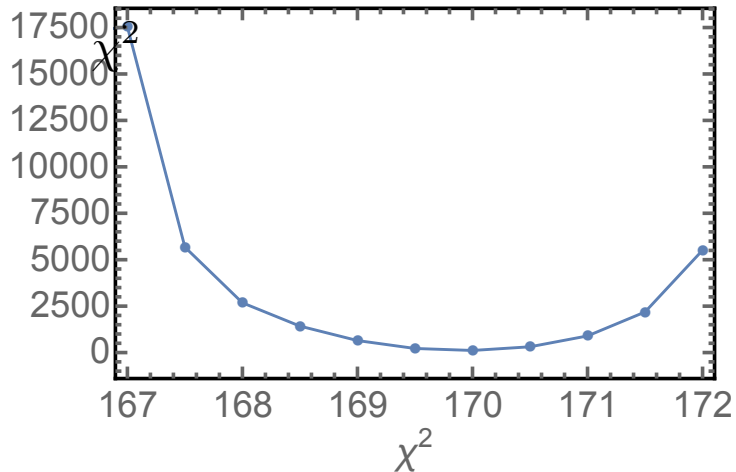
Future: consolidation & extension to pp collisions & MC studies

- [Extension to pp collisions](#) looks very promising with [SoftDrop](#) grooming to suppress MPI effects (boosted top quarks essential as well).
- Provides new ways to test and improve MC event generators.
- Plans:
 - Public code for calibration (CALIPER)
 - Other e^+e^- eventshapes (C-parameter, HJM)
 - NNLL for e^+e^-
 - pp with SoftDrop (at NNLL)
 - Electroweak corrections
- Theory of the MC top quark mass: parton shower, hadronization model, NLO matching

Backup Slides

Peak Fits Parameter Sensitivity

Default renormalization scales; $\Gamma_t=1.4$ GeV, tune 7, $\Omega_{1,\text{smear}}=2.5$ GeV, $m_t^{\text{Pythia}}=171$ GeV, $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%



→ $\chi^2_{\text{min}} \sim O(100)$

- Very strong sensitivity to m_t
- Low sensitivity to strong coupling
- Take PDF strong coupling as input: $\alpha_s(M_Z) = 0.1181(13)$ (error irrelevant for m_t^{MSR} , m_t^{pole})
- χ^2_{min} and δm_t^{stat} do not have any physical meaning
- PDF rescaling method: $(\chi^2_{\text{min}})^{\text{rescale}} = 1$ can be used to define an incompatibility uncertainty

MSR/MS Parametric Dependence on α_s

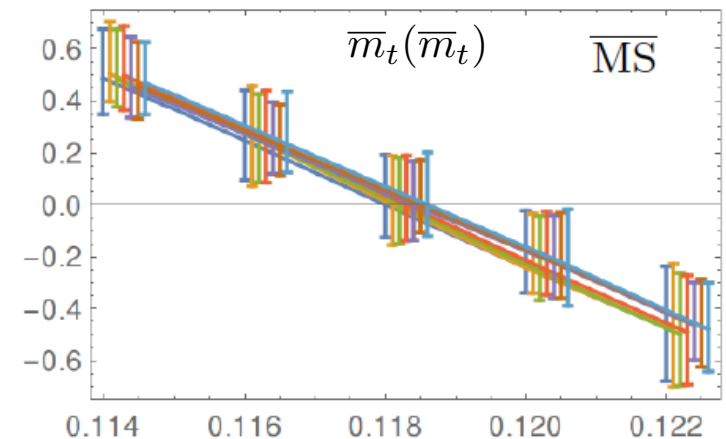
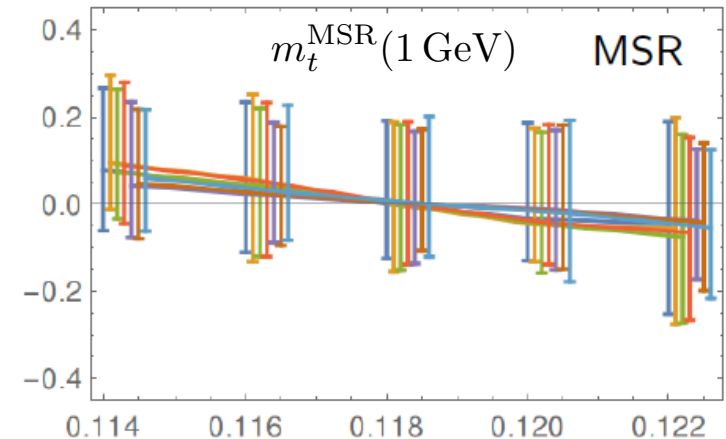
500 profiles; $\Gamma_t = 1.4, -1$ GeV; tune 7;
diff. Q-sets; peak(60/80)%

$$m_t^{\text{PYTHIA}} = 173 \text{ GeV}$$

- α_s dependence:

$$m^{\text{scheme}}[\alpha_s] - m^{\text{scheme}}[.118]$$

- small dependence of MSR mass on α_s
 ~ 50 MeV error ($\delta\alpha_s = .002$)
- large sensitivity of $\overline{\text{MS}}$ mass on α_s
- not an error:
calculated from MSR



Top Mass Reconstruction Error Budget

Lepton+jets channel	m_t fit type			
	2D δm_t^{2D} (GeV)	δ JSF	1D δm_t^{1D} (GeV)	hybrid δm_t^{hyb} (GeV)
Experimental uncertainties				
Method calibration	0.04	0.001	0.04	0.04
Jet energy corrections				
– JEC: Intercalibration	<0.01	<0.001	+0.02	+0.01
– JEC: In situ calibration	–0.01	+0.003	+0.24	+0.12
– JEC: Uncorrelated non-pileup	+0.09	–0.004	–0.26	–0.10
– JEC: Uncorrelated pileup	+0.06	–0.002	–0.11	–0.04
Lepton energy scale	+0.01	<0.001	+0.01	+0.01
E_T^{miss} scale	+0.04	<0.001	+0.03	+0.04
Jet energy resolution	–0.11	+0.002	+0.05	–0.03
b tagging	+0.06	< 0.001	+0.04	+0.06
Pileup	–0.12	+0.002	+0.05	–0.04
Backgrounds	+0.05	< 0.001	+0.01	+0.03
Modeling of hadronization				
JEC: Flavor-dependent				
– light quarks (u d s)	+0.11	–0.002	–0.02	+0.05
– charm	+0.03	<0.001	–0.01	+0.01
– bottom	–0.32	<0.001	–0.31	–0.32
– gluon	–0.22	+0.003	+0.05	–0.08
b jet modeling				
– b fragmentation	+0.06	–0.001	–0.06	<0.01
– Semileptonic b hadron decays	–0.16	<0.001	–0.15	–0.16
Modeling of perturbative QCD				
PDF	0.09	0.001	0.06	0.04
Ren. and fact. scales	+0.17 ± 0.08	–0.004 ± 0.001	–0.24 ± 0.06	–0.09 ± 0.07
ME-PS matching threshold	+0.11 ± 0.09	–0.002 ± 0.001	–0.07 ± 0.06	+0.03 ± 0.07
ME generator	–0.07 ± 0.11	–0.001 ± 0.001	–0.16 ± 0.07	–0.12 ± 0.08
Top quark p_T	+0.16	–0.003	–0.11	+0.02
Modeling of soft QCD				
Underlying event	+0.15 ± 0.15	–0.002 ± 0.001	+0.07 ± 0.09	+0.08 ± 0.11
Color reconnection modeling	+0.11 ± 0.13	–0.002 ± 0.001	–0.09 ± 0.08	+0.01 ± 0.09
Total systematic	0.59	0.007	0.62	0.48
Statistical	0.20	0.002	0.12	0.16
Total	0.62	0.007	0.63	0.51

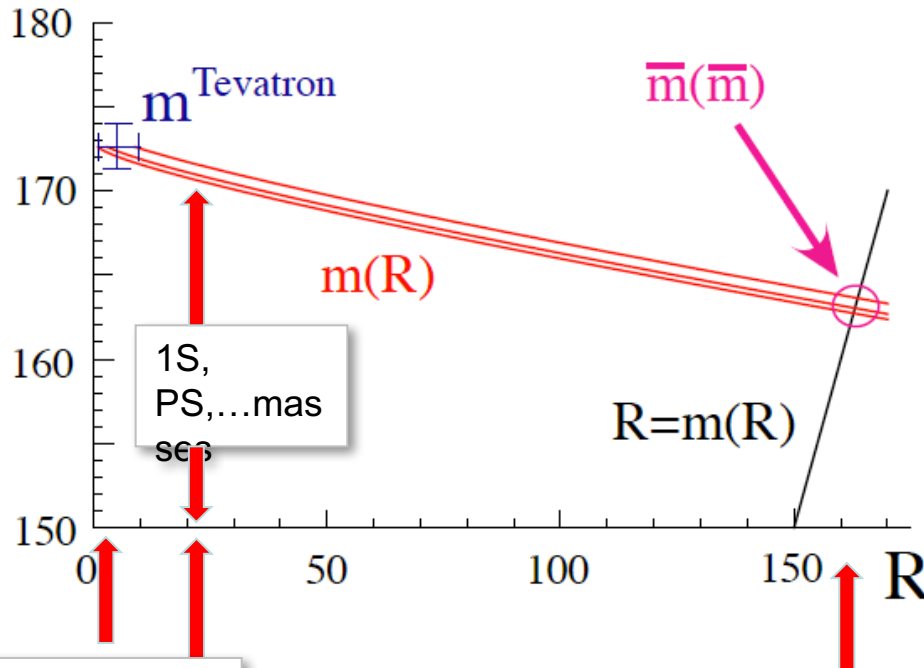
$m_t^{\text{MC}} = 172.44 \pm 0.49$
 (CMS Run-1 final, 2015)
 arXiv:1509.04044

← NLO ME corrections

MSR Mass Definition

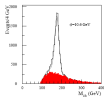
AH, Stewart: arXiv:0808.0222

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(3_{-2}^{+6} \text{ GeV}) = m_t^{\text{MSR}}(3 \text{ GeV})_{-0.3}^{+0.6}$$



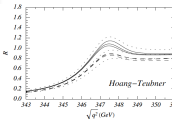
Good choice for R:

Of order of the typical scale of the observable used to measure the top mass.



Peak of invariant mass distribution, endpoints

Top-antitop threshold at the ILC



Total cross section, e.w. precision obs., Unification, MSbar mass

Masses Loop-Theorists Like to use

Total cross section (LHC/Tev):

$$m_t^{\text{MSR}}(R = m_t) = \bar{m}_t(\bar{m}_t)$$

$$M_t = M_t^{(O)} + M_t(0)\alpha_s + \dots$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

Threshold cross section (ILC):

$$m_t^{\text{MSR}}(R \sim 20 \text{ GeV}), m_t^{1S}, m_t^{\text{PS}}(R)$$

$$M_t = M_t^{(O)} + \langle p_{\text{Bohr}} \rangle \alpha_s + \dots$$

$$\langle p_{\text{Bohr}} \rangle = 20 \text{ GeV}$$

Inv. mass reconstruction (ILC/LHC):

$$m_t^{\text{MSR}}(R \sim \Gamma_t), m_t^{\text{jet}}(R)$$

$$M_t = M_t^{(O)} + \Gamma_t \alpha_s + \dots$$

$$\Gamma_t = 1.3 \text{ GeV}$$

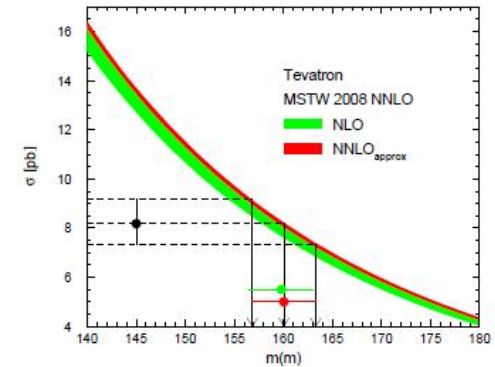
- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections



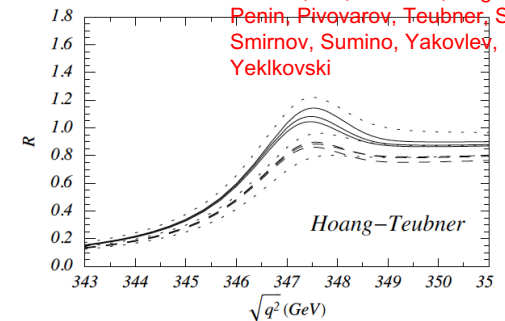
Mass schemes related to different computational methods

Relations computable in perturbation theory

Langenfeld, Moch, Uwer



Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Fleming, AH, Mantry, Stewart

