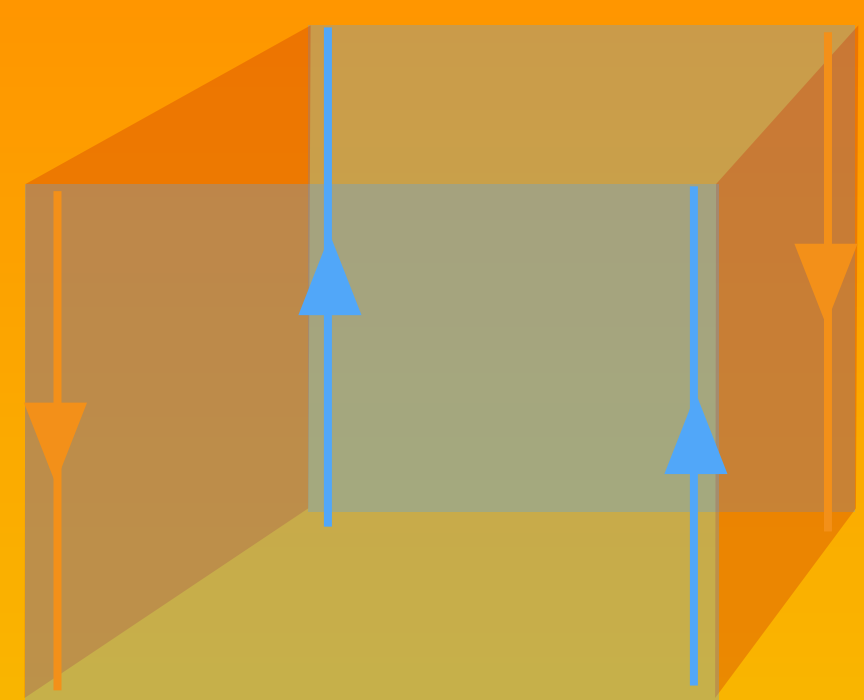
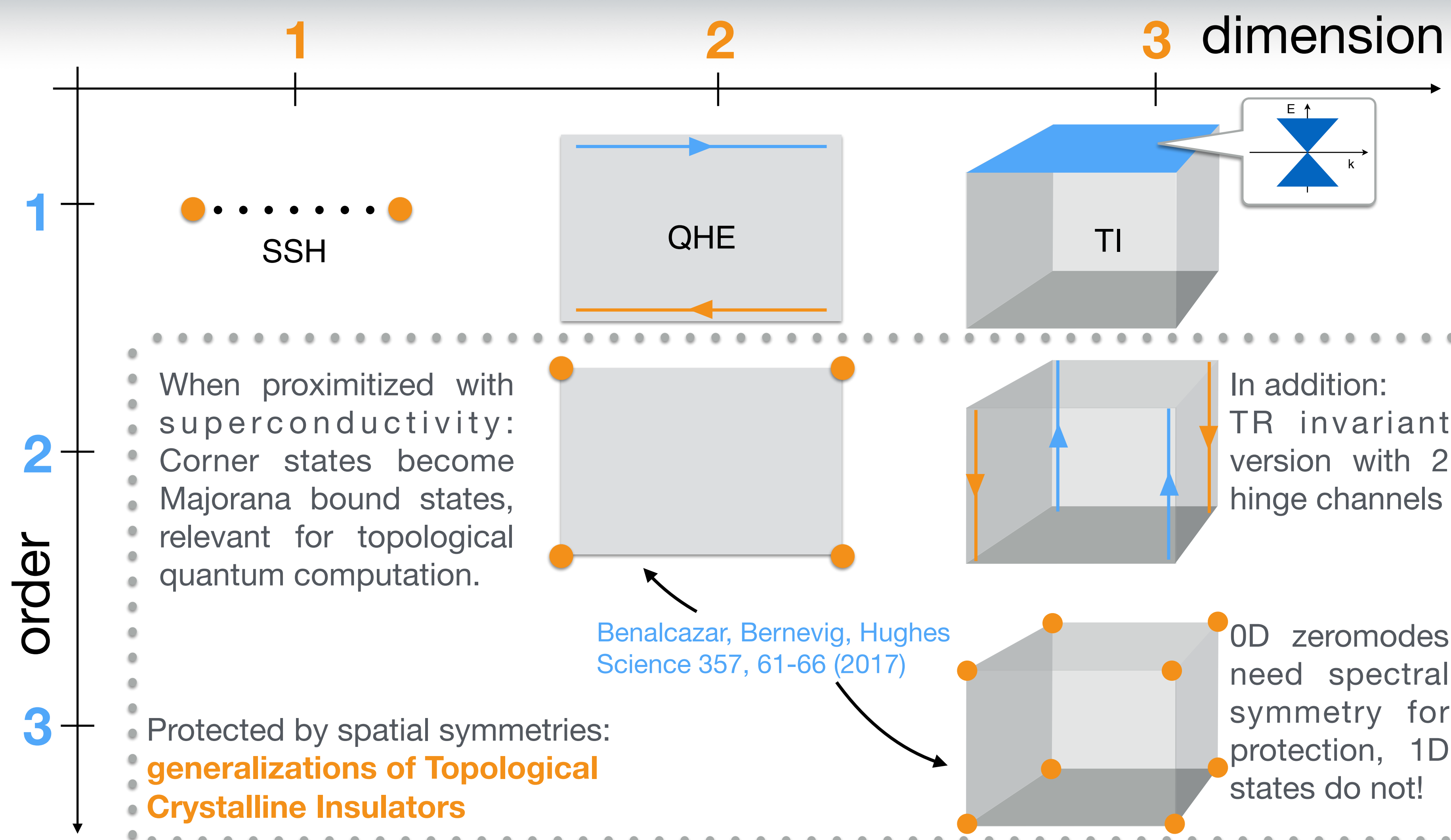




Higher-Order Topological Phases

1st-order topological insulators exhibit the bulk-boundary correspondence: A topologically nontrivial gapped d -dimensional bulk gives rise to gapless $(d-1)$ -dimensional boundary states when terminated.

n th-order topological insulators in d dimensions instead feature $(d-n)$ dimensional gapless states at their terminations. Importantly, this allows for the dissipationless 1D quantum Hall and quantum spin Hall edge states to be realized in intrinsically 3D crystals, for which there are many more materials.

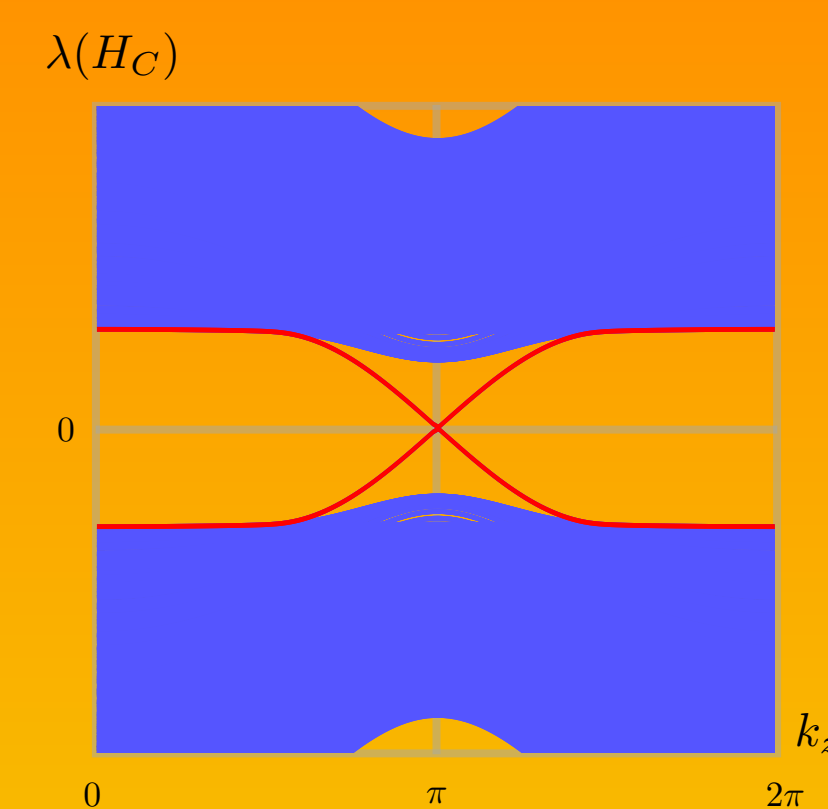


Toy model for a 2nd-order 3D TI with C_4T symmetry

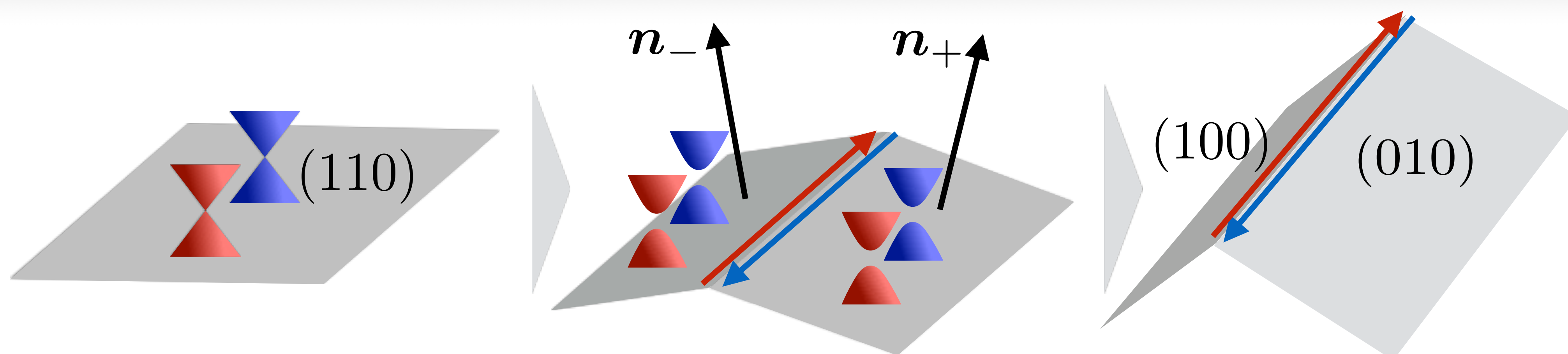
$$\mathcal{H}_c(\vec{k}) = \left(M + t \sum_i \cos k_i \right) \tau_z \sigma_0 + \Delta_1 \sum_i \sin k_i \tau_x \sigma_i + \Delta_2 (\cos k_x - \cos k_y) \tau_y \sigma_0$$

3D TI

T, C_4 breaking term



Construction of a 2nd-order 3D TI with **time-reversal** and **mirror** symmetry



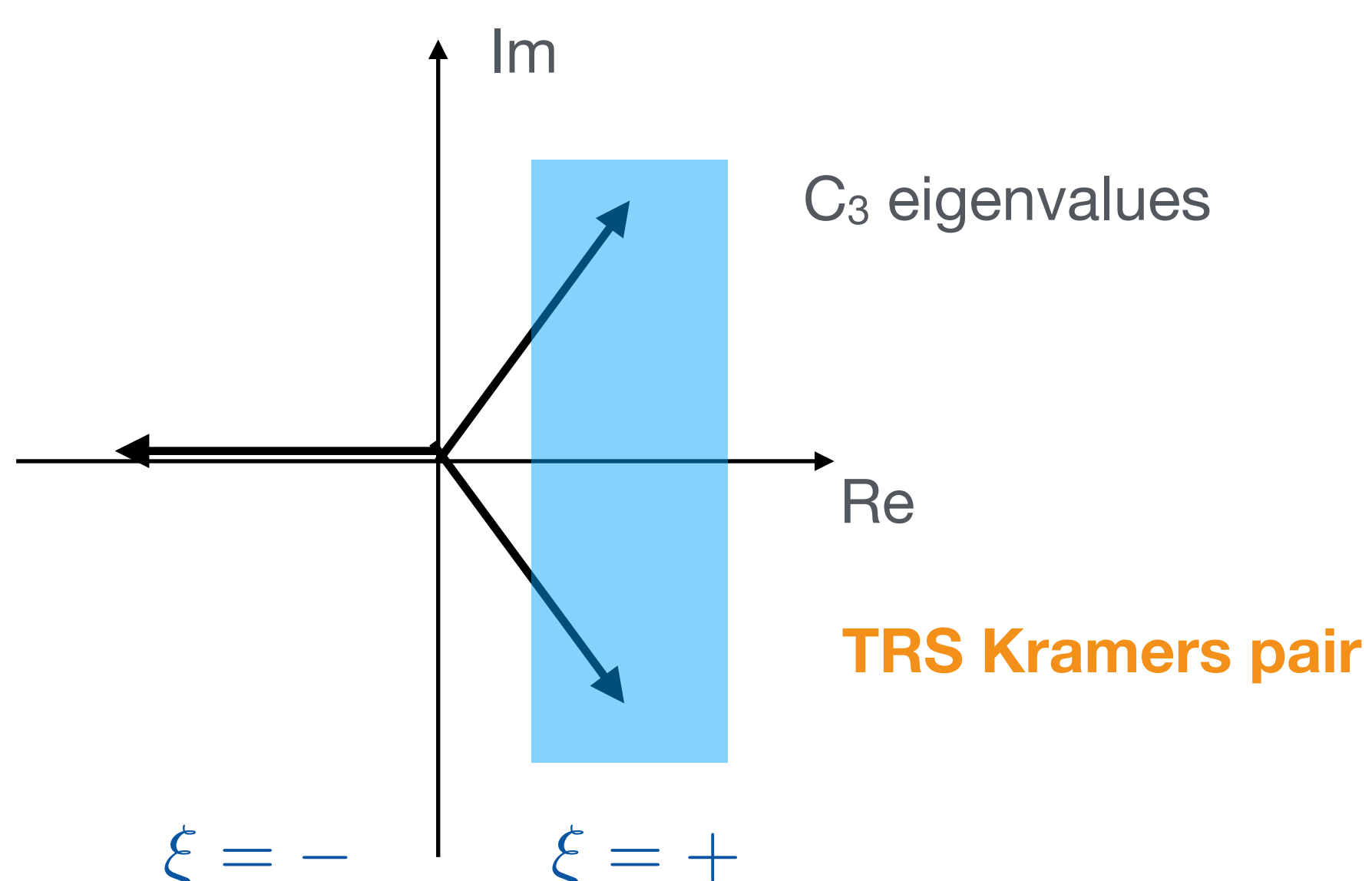
Time-reversal symmetric 2nd-order TIs can arise from nontrivial mirror Chern numbers. For $C_m = 2$ we have 2 Dirac cones on mirror preserving surfaces, when introducing a kink these gap out and leave a helical Kramers pair of modes behind.

Material realization of this kind of physics: **Tin Telluride (SnTe)**

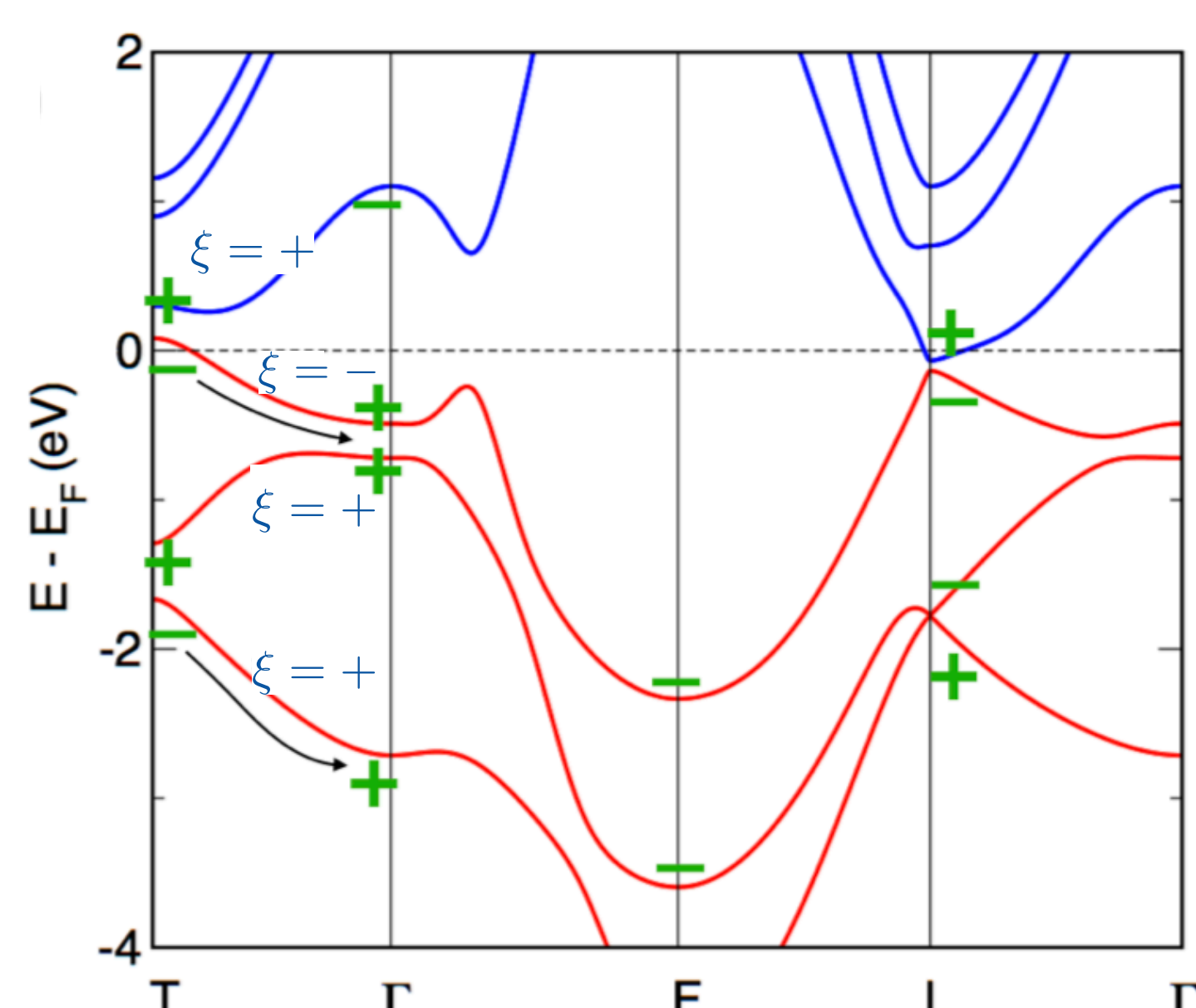
Reference: *Higher-order topological insulators*, Frank Schindler, Ashley M. Cook, Maia G. Vergniory, Zhijun Wang, Stuart S. P. Parkin, B. Andrei Bernevig and Titus Neupert, **Science Advances** 01 Jun 2018: Vol. 4, no. 6, eaat0346

Elementary Bismuth as a HOTI

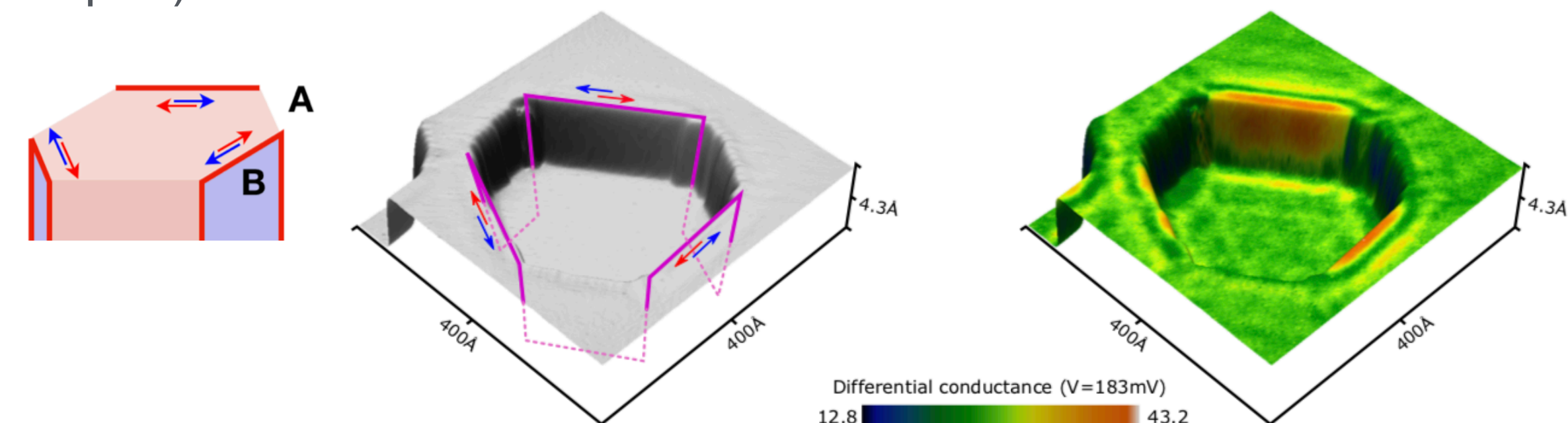
With time-reversal symmetry, there are two classes of C_3 eigenvalues



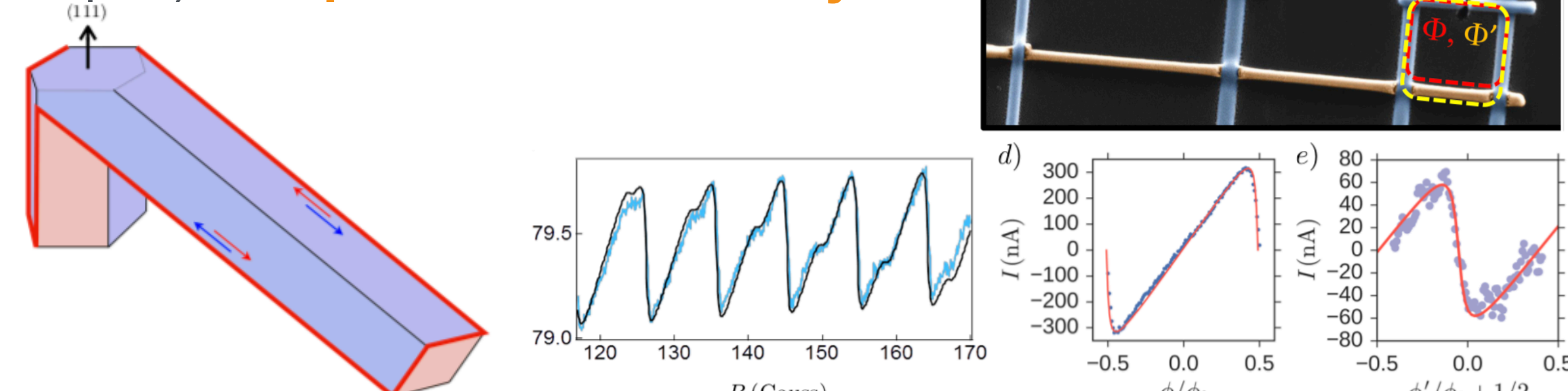
Bismuth has a single band inversion in each subspace



Exp. 1) **STM**



Exp. 2) **Josephson interferometry**



Reference: *Higher-order topology in bismuth*, Frank Schindler, Zhijun Wang, Maia G. Vergniory, Ashley M. Cook, Anil Murani, Shamashis Sengupta, Alik Yu. Kasumov, Richard Deblock, Sangjun Jeon, Ilya Drozdov, Hélène Bouchiat, Sophie Guéron, Ali Yazdani, B. Andrei Bernevig, Titus Neupert, **arXiv:1802.02585** (acc. in **Nature Physics**)