

High-Energy Limit of Mass-Suppressed Amplitudes

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Beauty and Magic of QCD at High Energy

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Topics discussed

● Introduction

- *high energy limit, Sudakov double logarithms*
- *power corrections and renormalization group*

● QCD at high energy to $\mathcal{O}(m_q^2/Q^2)$

- *non-Sudakov double logs in power suppressed amplitudes*
- *soft quark emission, eikonal color charge nonconservation*
- *factorization and resummation of non-Sudakov double logs*
- *light quark effects in Higgs production*
- *asymptotic formula for the quark form factors*

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- *factorization and resummation of non-Sudakov double logs*
- *light quark effects in Higgs production*
- *asymptotic formula for the quark form factors*

● *Moments of beauty and magic*

Based on

T. Liu, A.A. Penin, Phys.Rev.Lett. 119 (2017) 262001

previous work:

T. Liu, A.A. Penin and N. Zerf, Phys.Lett. B 771 (2017) 492

A.A. Penin and N. Zerf, Phys.Lett. B 760 (2016) 816

A.A. Penin, Phys.Lett. B 745 (2015) 69

Sudakov limit

● Sudakov limit

- *on-shell*
- *exclusive*
- *high energy*
- *fixed angle*

● Sudakov logarithms (*leading power in m_q^2/Q^2*)

- *each external line gets $e^{-\frac{\alpha}{4\pi} \frac{C_R}{2} \ln^2(Q^2)}$*

Sudakov (1956); Frenkel, Taylor (1976)

- *subleading logs exponentiate as well*

Mueller (1979); Collins (1980); Sen (1981), ...

● What about power suppressed logs?

High energy limit beyond Sudakov approximation

- Logarithmically enhanced power corrections
 - *phenomenologically crucial e.g. bottom loop in Higgs production*
 - *intriguing from QFT point of view*

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- Power corrections and renormalization group

$$\Delta\mathcal{L} = \sum_i C_i O_i, \quad \frac{d}{d \ln \mu} C_i = f_i(C_j)$$

- ✓ *OPE, large mass expansion* ⇨ *local composite operators*
- ✓ *threshold nonrelativistic limit* ⇨ *spatially nonlocal potentials*
- ✗ *Sudakov ultrarelativistic limit* ⇨ *light-cone Wilson lines, etc.*

High energy limit beyond Sudakov approximation

- Light quark mass effects in Higgs production
K.Melnikov and A.Penin, JHEP 1605 (2016) 172
- Next-to-eikonal soft gluon radiation
D.Bonocore, E.Laenen, L.Magnea, L.Vernazza, C.D.White, JHEP 1612, 121 (2016),
- Jettiness
R. Boughezal, X. Liu and F. Petriello, JHEP 1703, 160 (2017)
- Event shapes
I.Moult, I.W.Stewart, G.Vita, H.X.Zhu, JHEP 1808 (2018) 013
- Threshold resummation (*small $1 - z$*)
M.Beneke et al., arXiv:1809.10631 [hep-ph]
- *many other recent studies ...*

Tools

● SCET



Tools

- SCET



- *A concrete hard problem without known solution*



Tools

- SCET



- *A concrete hard problem without known solution*



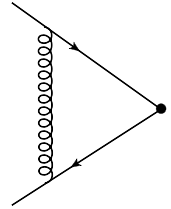
- Asymptotic expansion
Sudakov method
Ward identities



Sudakov logarithms

● Origin of double logs (*Dirac form factor*)

- *only vertex correction contribute in a covariant gauge*
- *gluon is soft and collinear to p_i*
- *quarks are eikonal*



$$S = \frac{\not{p}_i - \not{l} + m_q}{(p_i - l)^2 - m_q^2} \approx -\frac{\not{p}_i}{2p_i l} \approx \frac{\gamma_{\pm}}{|l|(\cos \theta \pm 1)}$$

● *Soft and collinear divergences*

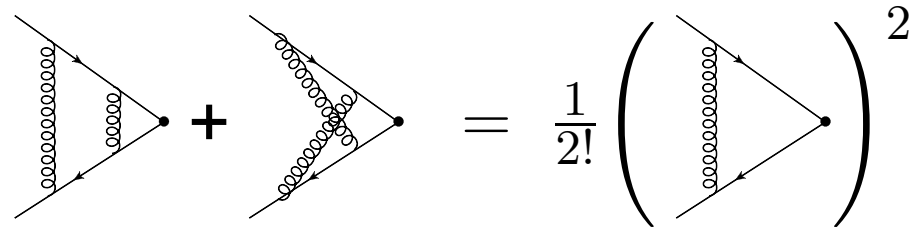
$$\int \frac{d|l|}{|l|} \rightarrow -\frac{1}{\varepsilon} + \ln(Q^2/\mu^2) \quad \int \frac{d \cos \theta}{\cos \theta \pm 1} \rightarrow \ln(Q^2/m_q^2)$$

Sudakov logarithms

- Eikonal factorization

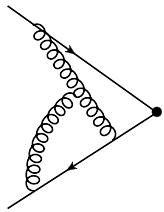
$$\frac{1}{p_i l_1} \frac{1}{p_i (l_1 + l_2)} + \frac{1}{p_i (l_1 + l_2)} \frac{1}{p_i l_2} = \frac{1}{p_i l_1} \frac{1}{p_i l_2}$$

- Exponentiation (QED)



$$\text{n-loop} = \frac{(1\text{-loop})^n}{n!}$$

- in QCD



cancels the non-Abelian part by Ward identity

- Sudakov asymptotic result

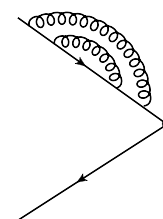
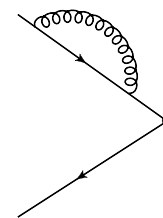
$$F_D \sim e^{-C_F x}$$

$$\left(\mu = m_q, \quad x = \frac{\alpha_s}{4\pi} \ln^2(Q^2/m_q^2) \right)$$

Sudakov logarithms

Alternative picture: physical gauge

- forward radiation suppressed by helicity conservation
⇒ only on-shell self-energies contribute
- universality of Sudakov DL
- ordering of virtual momenta ⇒ rainbow diagrams
- trivial color structure of Sudakov DL



Alternative picture: evolution equation

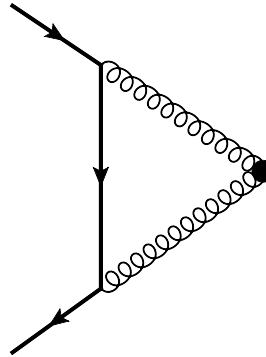
$$\frac{\partial}{\partial \ln Q} F = [-\Gamma_{cusp} + \dots] F$$

- Cusp anomalous dimension

$$\Gamma_{cusp} = -\frac{C_F \alpha_s}{\pi} \ln(m_q^2/Q^2)$$

Power-suppressed amplitude

- Quark scattering by $(G_{\mu\nu}^a)^2$ operator



- *helicity flip* \Rightarrow *mass suppression*

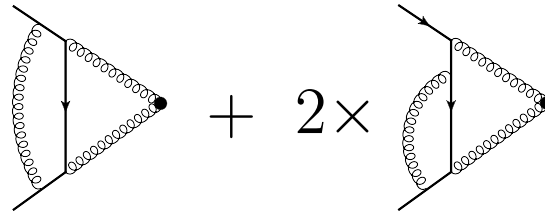
- *soft quark* $S \approx m_q/l^2$ *eikonal gluons* $D \approx g_{\mu\nu}/(2p_i l)$

- Power suppressed double logs

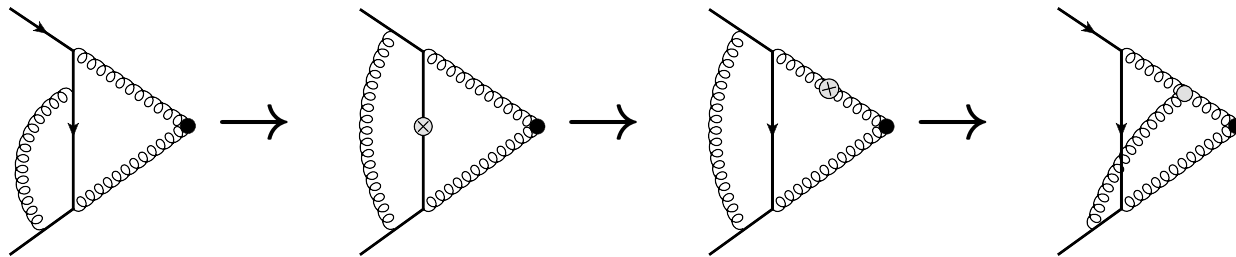
- *soft fermion exchange*
- *eikonal color charge nonconservation*

Power-suppressed amplitude

- Radiative corrections (QED)



- Ward identities

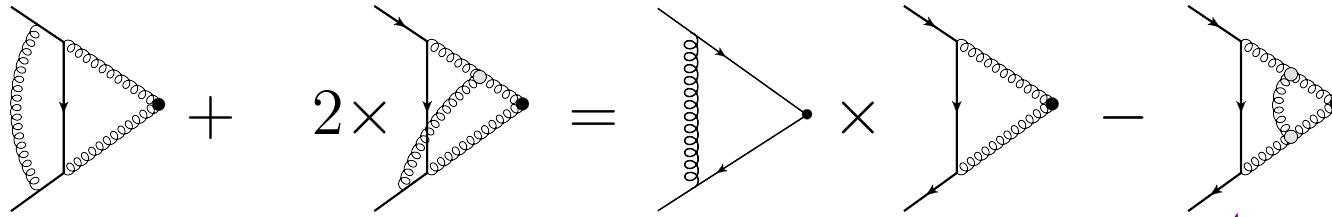


- crossed vertex: $S(l) \rightarrow S(l) - S(l + l_g^+)$

- 3-photon vertex: $2e_q p_1^\mu$, where e_q is the quark charge

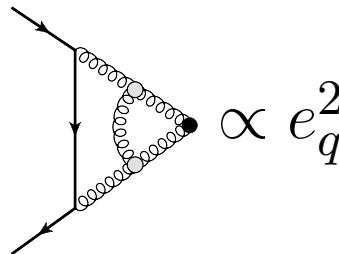
Power-suppressed amplitude

- Sudakov logs factorization



missing diagram to complete Sudakov factorization

- Non-Sudakov logs



- *multiple soft photons exponentiate*

Power-suppressed amplitude

- In QCD

$$e_q^2 \rightarrow C_F - C_A$$

➔ *measures the eikonal color charge nonconservation*

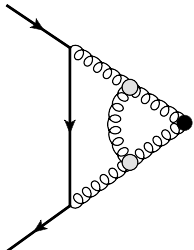
- Factorization formula

$$\mathcal{G} = Z_q^2 g(-z) \mathcal{G}^{(0)}$$

- *quark Sudakov factor* $Z_q^2 = \exp \left[-C_F \left(\frac{\alpha_s}{2\pi} \frac{\ln(m_q^2/Q^2)}{\varepsilon} + x \right) \right]$

- *non-Sudakov double-logarithmic function* $g(-z)$,
where $z = (C_A - C_F)x$, $x = \frac{\alpha_s}{4\pi} \ln^2(Q^2/m_q^2)$, $g(0) = 1$

Evaluation of $g(z)$

$$g(-z) \sim \text{diagram}$$
A Feynman diagram representing a triangle loop with a gluon loop inside. The triangle is formed by three solid lines with arrows pointing clockwise. The top and bottom lines are straight, while the right side is a curved line representing a gluon loop. Inside the triangle, there is a smaller loop of curly lines representing a gluon loop. The diagram is connected to external lines on the left.

• New variables

• Sudakov parameters $l = up_1 + vp_2 + l_\perp$

• hypercube coordinates

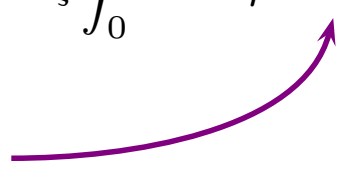
$$\eta = \ln v / \ln(m_q^2/Q^2), \quad \xi = \ln u / \ln(m_q^2/Q^2), \quad 0 < \eta, \xi < 1$$

• strict ordering $\eta_i < \eta_j, \dots$, onshell condition $\eta_i + \xi_i < 1$

• Integral representation

$$g(z) = 2 \int_0^1 d\xi \int_0^{1-\xi} d\eta e^{2z\eta\xi}$$

soft gluon loop exponent



Result

- Exact formula

$$g(z) = {}_2F_2(1, 1; 3/2, 2; z/2)$$

- Asymptotic behavior at $z \rightarrow +\infty$

$$g(-z) \sim \frac{\ln(2z) + \gamma_E}{z}, \quad g(z) \sim \left(\frac{2\pi e^z}{z^3} \right)^{1/2},$$

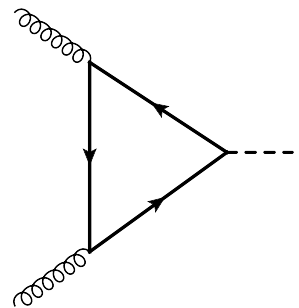
- The amplitude $\mathcal{G} \propto g(-z)$

- in QCD $z > 0 \Rightarrow$ power suppressed

- in QED $z < 0 \Rightarrow$ exponentially enhanced

Bottom loop in $gg \rightarrow H$

● Leading contribution



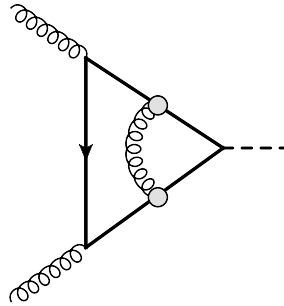
The diagram shows a triangle loop of bottom quarks. Two incoming gluons (represented by wavy lines) enter from the left, and a Higgs boson (represented by a dashed line) exits to the right. The loop is formed by two solid lines with arrows pointing clockwise, representing the bottom quark propagators.

$$\propto \ln^2(m_H^2/m_b^2) \frac{m_b^2}{m_H^2}$$

- *effective expansion parameter* $\alpha_s \ln^2(m_H^2/m_b^2) \sim 40\alpha_s$
- ➔ *resummation is mandatory (main source of uncertainty)*
- *factorization structure similar to \mathcal{G}*

Moment of beauty

- Non-Sudakov logs



- *inverted color flow*: $C_F \leftrightarrow C_A, z \rightarrow -z$

- Factorization formula

$$\mathcal{M}_{gg \rightarrow H}^b = Z_g^2 g(z) \mathcal{M}_{gg \rightarrow H}^{b(0)}$$

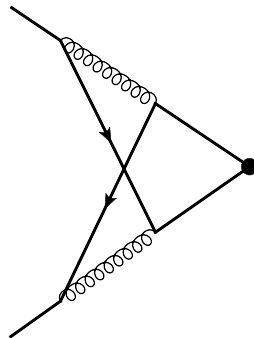
- *gluon Sudakov factor* $Z_g^2 = \exp \left[-\frac{C_A}{\epsilon^2} \frac{\alpha_s}{2\pi} \frac{\mu^{2\epsilon}}{m_H^{2\epsilon}} \right]$
- $g(z)$ *exponentially enhanced in QCD*

Dirac form factor beyond the leading power

- High-energy expansion

$$F_D = Z_q^2 \left(1 + \frac{m_q^2}{Q^2} F_D^{(1)} + \dots \right),$$

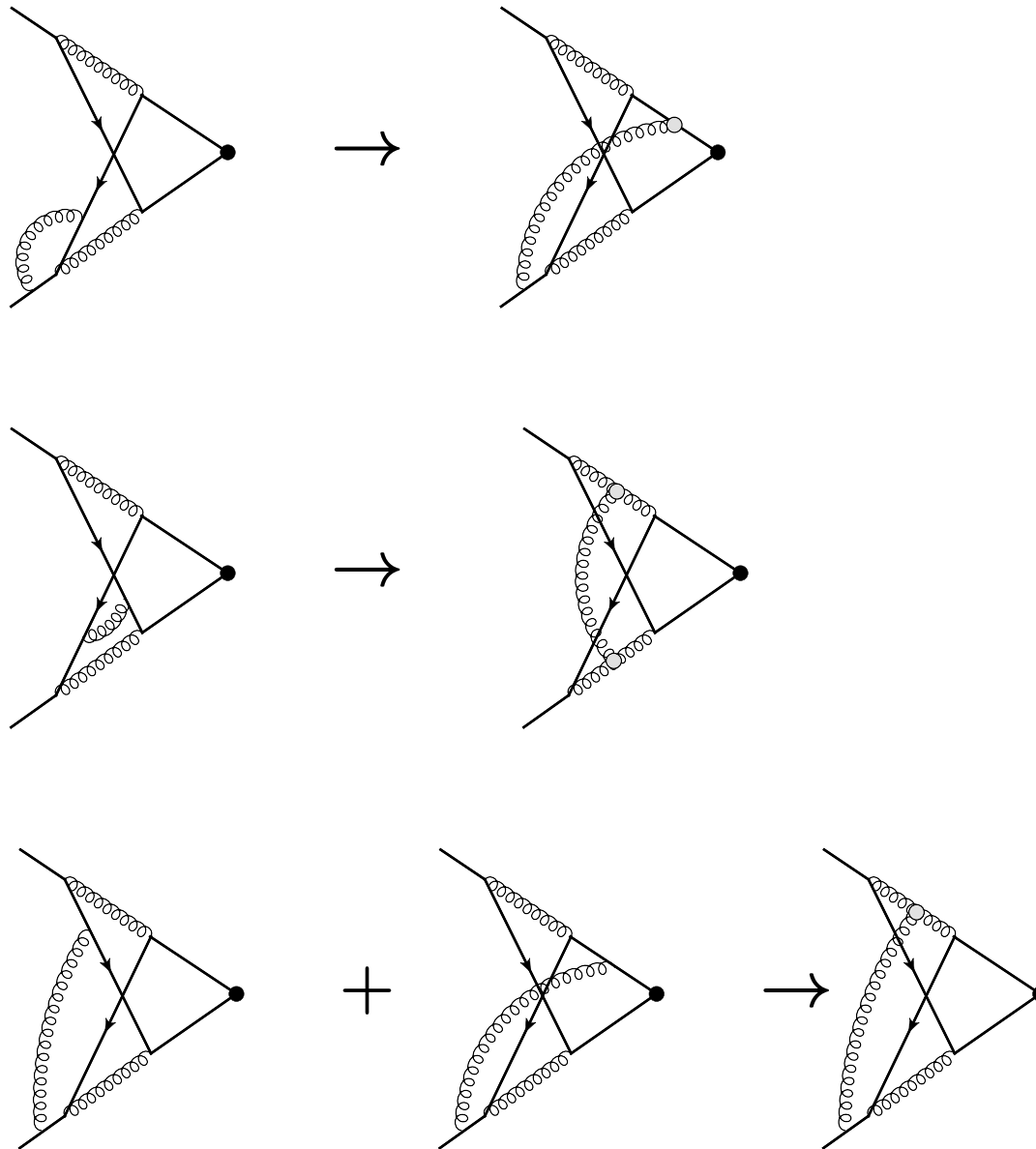
- Double logarithms in $F_D^{(1)}$



- *induced by soft quark pair exchange*

A.A. Penin, Phys.Lett. B 745 (2015) 69

Diagram transformation



Dirac form factor beyond the leading power

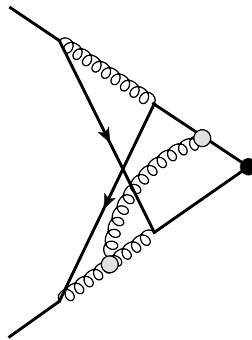
- All-order result

$$F_1^{(1)} = \left[\frac{C_F(C_A - 2C_F)}{6} x^2 \right] f(-z)$$

two-loop contribution

- non-Sudakov double-logarithmic function

$$f(-z) \sim$$



+ *symmetric*

Result for $f(z)$

Integral representation

$$f(z) = 12 \int_0^1 d\eta_1 \int_{\eta_1}^1 d\eta_2 \int_0^{1-\eta_2} d\xi_2 \int_{\xi_2}^{1-\eta_1} d\xi_1 e^{2z\eta_1(\xi_1-\xi_2)} e^{2z\xi_2(\eta_2-\eta_1)}$$

Taylor series $f(z) = \sum_n c_n z^n$

high-order behavior $c_n \sim \frac{\ln n}{n! 2^n n^{5/2}}$

Asymptotic behavior at $z \rightarrow +\infty$

$$f(-z) \sim C_- \left(\frac{\ln z}{z} \right)^2, \quad f(z) \sim C_+ \ln z \left(\frac{e^z}{z^5} \right)^{1/2}$$

$$C_- = 3.6 \dots, \quad C_+ = 14.8 \dots$$

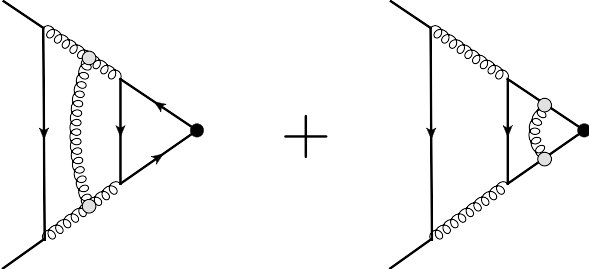
→ exponentially enhanced in QED, power suppressed in QCD

Moment of magic

- Scalar form factor

$$F_S^{(1)} = \left[-\frac{C_F T_F}{3} x^2 \right] f_S(-z)$$

- non-Sudakov double-logarithmic function

$$f_S(-z) \sim$$


- *Amazing universality*

$$f_S(z) = f(z)$$

➔ *the same function for axial and pseudoscalar form factors*

Higgs phenomenology

- Higgs production in gluon fusion

- *no jets, veto on soft emission* $\sim m_b$

$$\delta\sigma_{gg\rightarrow H} = -3\rho \ln^2 \rho \left(1 + \frac{z}{6} + \frac{z^2}{45} + \frac{z^3}{420} + \dots \right) \sigma_{gg\rightarrow H}$$

$$z = \frac{\alpha_s}{4\pi} (C_A - C_F) \ln^2 \rho \approx 1.2 \quad 3\rho \ln^2 \rho \approx 20\% \quad \rho = m_b^2/m_H^2$$

- *three-loop correction approximately* -0.6%

Summary

- The first LL resummation beyond leading power in QCD

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 - *relations between different amplitudes and gauge groups*

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- Non-Sudakov logs
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 - *eikonal charge nonconservation*
 - *exponential enhancement*
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 - *universality* \Leftrightarrow *two functions $g(z)$ and $f(z)$*
 - *relations between different amplitudes and gauge groups*
- Bottom mass effect in $gg \rightarrow H$
 - -0.6% *to exclusive cross section*