Adam Falkowski

Which EFT

Zurich, 19 October 2020

Based on 1902.05936 with Riccardo Rattazzi

Plan

AA, Rattazzi
1902.05936

Two papers for a price of one!

- Part 1: General model-independent constraints on deformations of the cubic Higgs coupling in theories where new physics decouples (h^3 in SMEFT)
- Part 2: Physical difference between linearly and non-linearly realized electroweak symmetry breaking (h^3 in SMEFT vs. h^3 in HEFT)

This talk

Related paper: Chang, Luty
1902.05556

Linear vs non-linear

Two mathematical formulations for effective theories with SM spectrum

Linear	Non-linear
$SU(3)_C \times SU(2)_L \times U(1)_Y$	**$SU(3)_C \times U(1)_{em}$**

$$L \in SU(2)_L \quad R \in U(1)_Y$$

$$H \to LH \qquad\qquad U \to LUR^\dagger \qquad h \to h$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix} \qquad\qquad U = \exp\left(\frac{i\pi^a \sigma^a}{\mathrm{v}}\right)$$

125 GeV Higgs boson

Goldstone bosons eaten by W and Z

Higgs VEV $v \approx 246$ GeV

Expansion parameter $v \approx 246$ GeV

In general, the two formulations lead to two distinct effective theories

SMEFT $\subset$ **HEFT**

Linear vs non-linear: Higgs self-couplings

In the SM self-coupling completely fixed…

$$\mathscr{L}_{\rm SM} \supset m^2 |H|^2 - \lambda |H|^4$$

$$\to -\frac{1}{2} m_h^2 h^2 - \frac{m_h^2}{2{\rm v}} h^3 - \frac{m_h^2}{8{\rm v}^2} h^4$$

…but they can be deformed by BSM effects

SMEFT

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} - \frac{c_6}{\Lambda^2} |H|^6 + \mathscr{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{m_h^2}{2{\rm v}}(1+\delta\lambda_3) h^3 - \frac{m_h^2}{8{\rm v}^2}(1+\delta\lambda_4) h^4 - \frac{\lambda_5}{\rm v} h^5 - \frac{\lambda_6}{{\rm v}^2} h^6$$

$$\delta\lambda_3 = \frac{2 c_6 {\rm v}^4}{m_h^2 \Lambda^2}, \; \delta\lambda_4 = \frac{12 c_6 {\rm v}^4}{m_h^2 \Lambda^2}, \; \lambda_5 = \frac{3 c_6 {\rm v}^2}{4\Lambda^2}, \; \lambda_6 = \frac{c_6 {\rm v}^2}{8 \Lambda^2}$$

SMEFT: Predicts correlations between self-couplings as long as Λ >> v

HEFT

$$\mathscr{L}_{\rm HEFT} \supset - c_3 \frac{m_h^2}{2{\rm v}} h^3 - c_4 \frac{m_h^2}{8{\rm v}^2} h^4 - \frac{c_5}{\rm v} h^5 - \frac{c_6}{{\rm v}^2} h^6 + \ldots$$

HEFT: no correlations between self-couplings

Linear vs non-linear

SMEFT

HEFT

Higgs boson coupling to WW

$$\mathscr{L}_{\mathrm{SMEFT}} \supset m_W^2 W_\mu^+ W_\mu^- + 2m_W^2 \left(1 + c\frac{g_*^2 \mathrm{v}^2}{\Lambda^2}\right) \frac{h}{\mathrm{v}} W_\mu^+ W_\mu^-$$

free O(1) parameter

Parametric limit Λ→∞ where
Higgs boson couplings become SM-like

Higgs boson coupling to WW

$$\mathscr{L}_{\mathrm{HEFT}} \supset m_W^2 W_\mu^+ W_\mu^- + 2m_W^2 (1 + \delta) \frac{h}{\mathrm{v}} W_\mu^+ W_\mu^-$$

free O(1) parameter

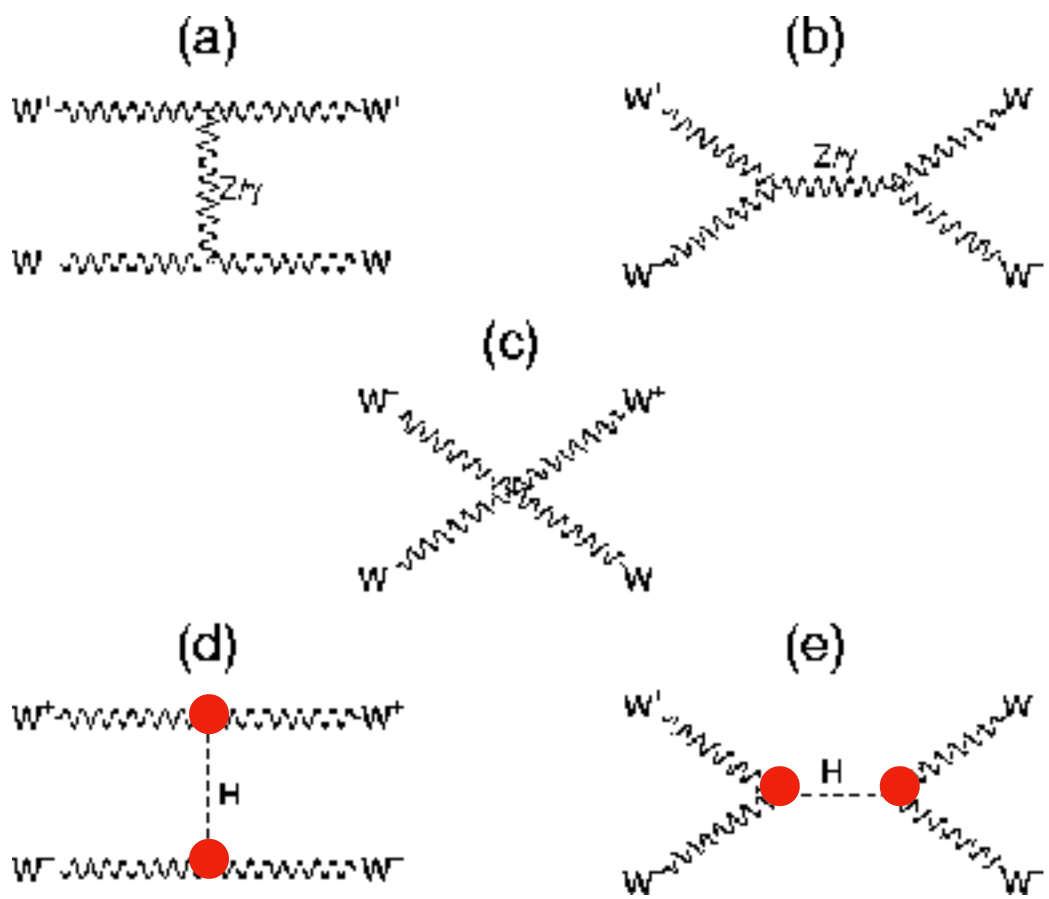
No parametric limit where
Higgs boson couplings become SM-like

Intuitively, no physical difference between SMEFT and HEFT if Λ~v

What is the difference between SMEFT with Λ>>v and HEFT with δ << 1 ?

HEFT in the decoupling limit?

$$\mathscr{L}_{\rm HEFT} \supset m_W^2 W_\mu^+ W_\mu^- + 2m_W^2 (1+\delta)\frac{h}{v} W_\mu^+ W_\mu^-$$



- One may think of a reason for δ << 1 in HEFT describing new physics is much heavier than EW scale
- As is well known, in the SM Higgs boson is crucial for unitarization of 2-to-2 WW scattering amplitudes
- For that to work, the Higgs boson coupling to WW has to be fixed such that δ = 0
- More generally, if δ << 1, tree level WW scattering avoids hitting strong coupling until far above the electroweak scale

Linear vs non-linear

Summary of the following ~10 slides:

- SMEFT and HEFT lead to a dramatically different phenomenology at the electroweak scale
- Choosing SMEFT or HEFT as our EFT above the electroweak scale implicitly entails an assumption about a class of BSM theories that we want to characterize
- SMEFT is appropriate to describe BSM theories which can be parametrically decoupled, that is to say, where the mass scale of the new particles depends on a free parameter(s) that can be taken to infinity
- Conversely, HEFT is appropriate to describe non-decoupling BSM theories, where the masses of the new particles vanish in the limit $v \to 0$

Example: cubic Higgs deformation

Consider toy EFT model where Higgs cubic (and only that) deviates from the SM

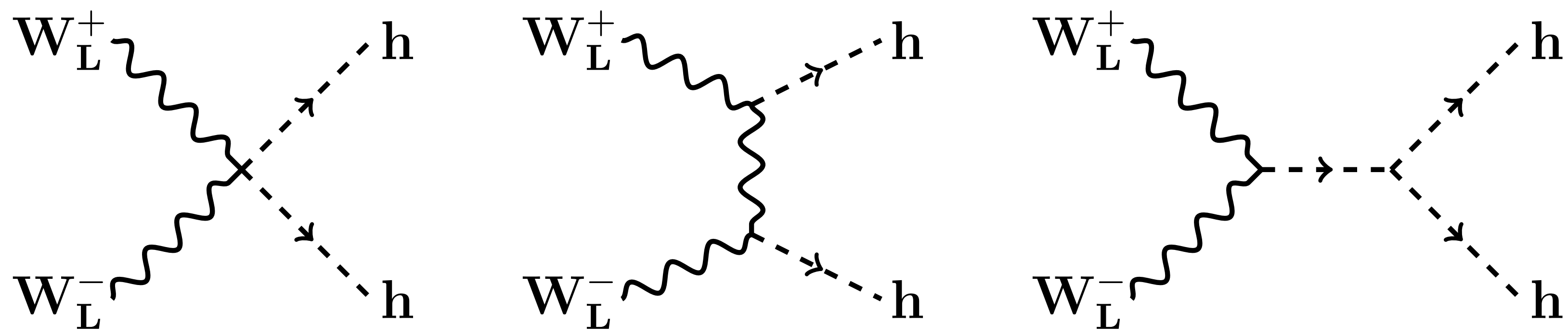
$$\mathscr{L} = \mathscr{L}_{\rm SM} - \Delta_3 \frac{m_h^2}{2v} h^3$$

$$\Downarrow$$

$$V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} \left(1 + \Delta_3\right) h^3 + \frac{m_h^2}{8v^2} h^4$$

This EFT belongs to HEFT but not SMEFT parameter space

Elastic channels



$\sim E^2$ - $\sim E^2$ + $\sim E^0$ = $\sim E^0$

In this model the Higgs cubic is modified, but not Higgs couplings to W bosons. In 2-to-2 scattering at tree level only the latter are important for unitarity

In this model, no problems at the level of tree-level 2-to-2 amplitudes

Non-analytic Higgs potential

$$V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2\mathrm{v}} \left(1 + \Delta_3\right) h^3 + \frac{m_h^2}{8\mathrm{v}^2} h^4 \qquad \textbf{(1)}$$

Given Lagrangian for Higgs boson h, one can always uplift it to manifestly SU(2)xU(1) invariant form replacing

$$h \to \sqrt{2H^\dagger H} - \mathrm{v}$$

After this replacement, Higgs potential contains terms non-analytic at H=0

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2} \left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}} \left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3 \qquad \textbf{(2)}$$

(1) and (2) are equal in the unitary gauge

$$H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathrm{v} + h \end{pmatrix}$$

<u>**Thus, (1) and (2) describe the same physics**</u>

Non-analytic Higgs potential

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2}\left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3$$

In the unitary gauge, the Higgs potential looks totally healthy and renormalizable…

Going away from the unitary gauge:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix}$$

$$V \supset \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{(h+\mathrm{v})^2 + G^2} - \mathrm{v}\right)^3$$

$$G^2 \equiv \sum_i G_i^2$$

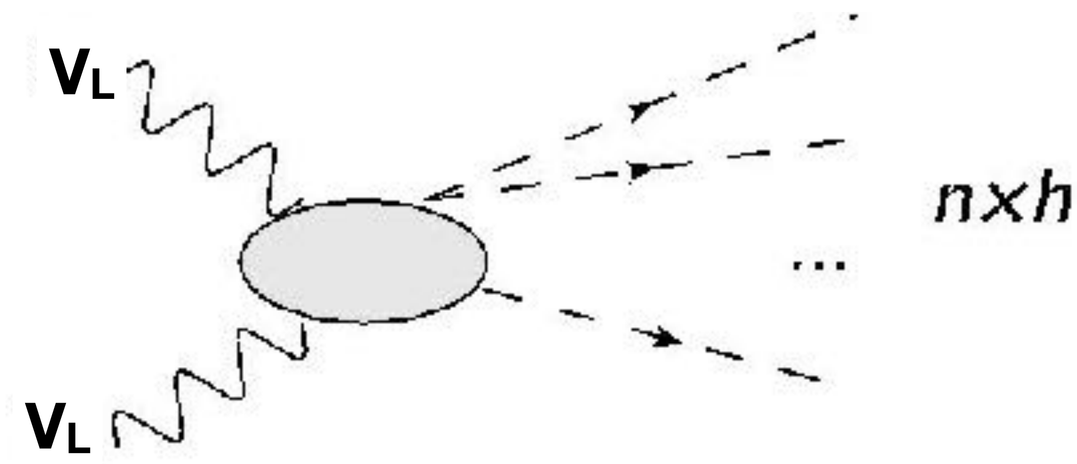
Away from the unitary gauge, it becomes clear that the Higgs potential contains non-renormalizable interactions suppressed only by the EW scale v

$$V \supset \Delta_3 \frac{3m_h^2}{4\mathrm{v}} \frac{G^2 h^2}{h + \mathrm{v}} + \mathcal{O}(G^4) = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty} \left(\frac{-h}{\mathrm{v}}\right)^n + \mathcal{O}(G^4)$$

Multi-Higgs production

Consider VBF production of n ≥ 2 Higgs bosons: $$V_L V_L \to n \times h$$

By equivalence theorem, at high energies the same as $$GG \to n \times h$$



Expanded V contains interactions

$$V \supset = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty} \left(\frac{-h}{\mathrm{v}} \right)^n$$

leading to interaction vertices with arbitrary number of Higgs bosons

s-wave isospin-0 amplitude for GG→h^n is momentum-independent constant proportional to the non-analytic deformation

$$\mathscr{M}([GG]^{l=0}_{I=0} \to \underbrace{h\ldots h}_{n}) \approx \frac{(-1)^{n+1}}{4\sqrt{\pi}} \Delta_3 \frac{3\sqrt{3}n!m_h^2}{2\mathrm{v}^n}$$

Amplitudes for multi-Higgs production in W/Z boson fusion are only suppressed by scale v and not decay with growing energy, leading to unitarity loss at some scale above v

Unitarity primer

S matrix unitarity $S^\dagger S = 1$

symmetry factor for n-body final state

implies relation between forward scattering amplitude, and elastic and inelastic production cross sections

$$2\mathrm{Im}\mathscr{M}(p_1p_2 \to p_1p_2) = S_2 \int d\Pi_2 |\mathscr{M}^{\mathrm{elastic}}(p_1p_2 \to k_1k_2)|^2 + \sum S_n \int d\Pi_n |\mathscr{M}^{\mathrm{inelastic}}(p_1p_2 \to k_1\ldots k_n)|^2$$

Equation is "diagonalized" after initial and final 2-body state are projected into partial waves

$$a_l(s) = \frac{S_2}{16\pi}\sqrt{1-\frac{4m^2}{s}}\int_{-1}^{1} d\cos\theta P_l(\cos\theta)\mathcal{M}(s,\cos\theta),$$

$$2\mathrm{Im}a_l = a_l^2 + \sum S_n \int d\Pi_n |\mathscr{M}_l^{\mathrm{inelastic}}|^2$$

This can be rewritten as the Argand circle equation

$$(\mathrm{Re}a_l)^2 + (\mathrm{Im}a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n |\mathscr{M}_l^{\mathrm{inelastic}}|^2$$

Unitarity primer

Argand circle equation

$$(\mathrm{Re}\,a_l)^2 + (\mathrm{Im}\,a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n \,|\,\mathscr{M}_l^{\text{inelastic}}\,|^2$$

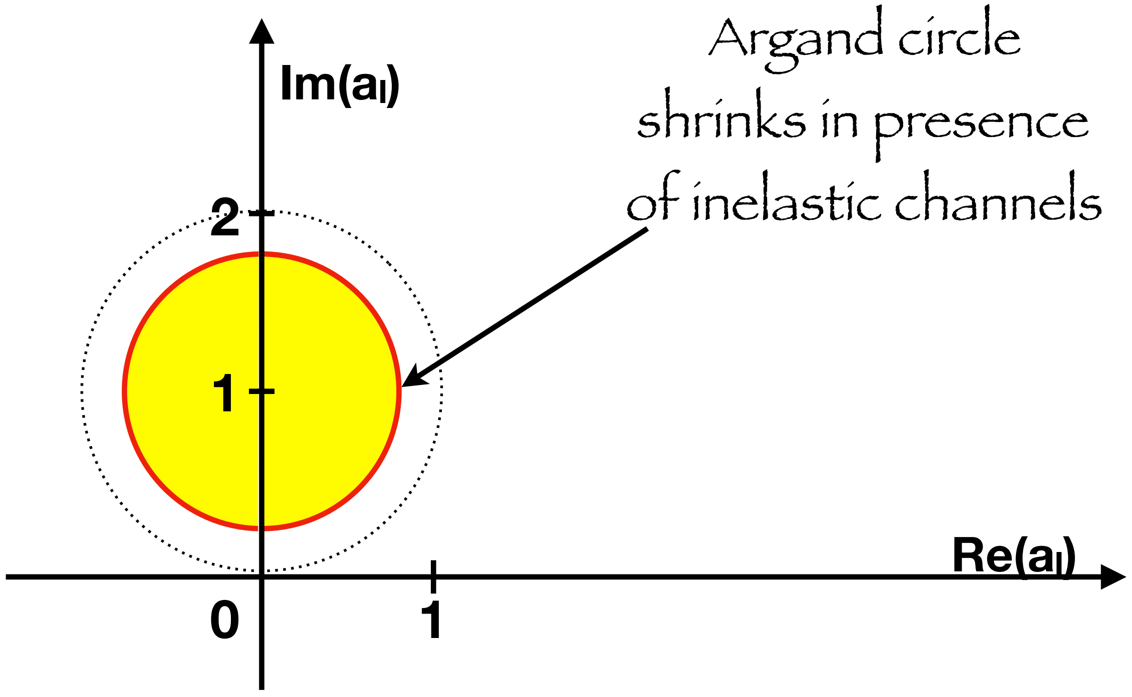
implies constraints on <u>both</u> elastic and inelastic amplitudes

Often used

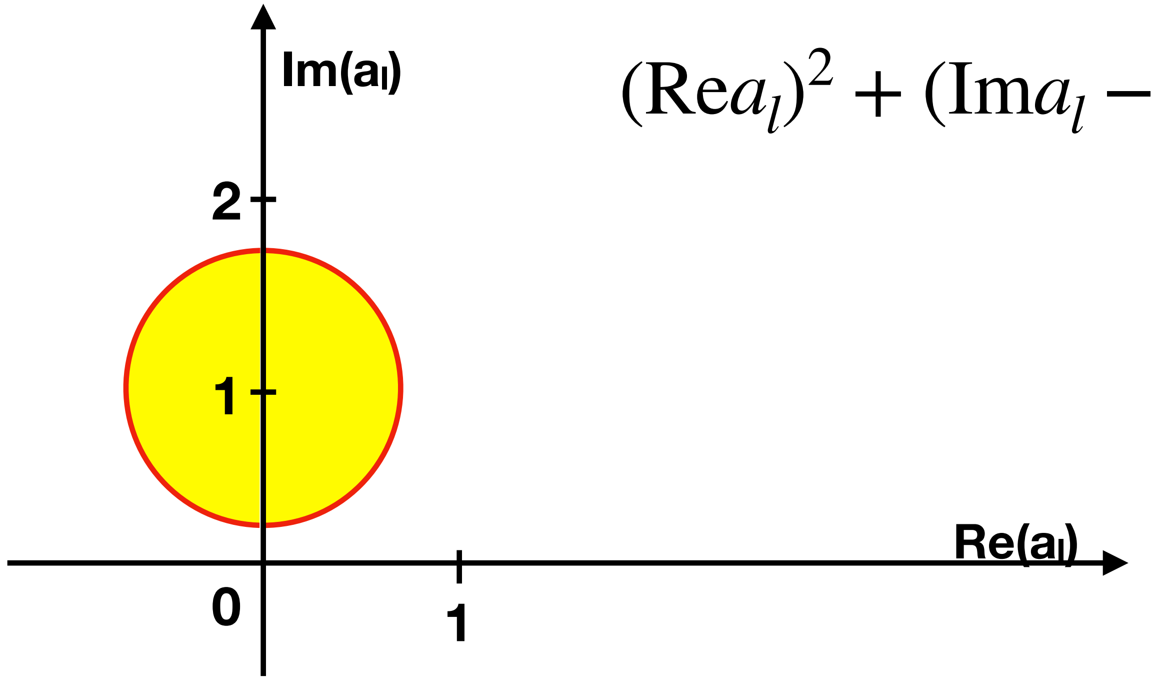
$$|\,\mathrm{Re}\,a_l\,| \leq 1$$

$$\sum S_n \int d\Pi_n \,|\,\mathscr{M}_l^{\text{inelastic}}\,|^2 \leq 1$$

Often forgotten



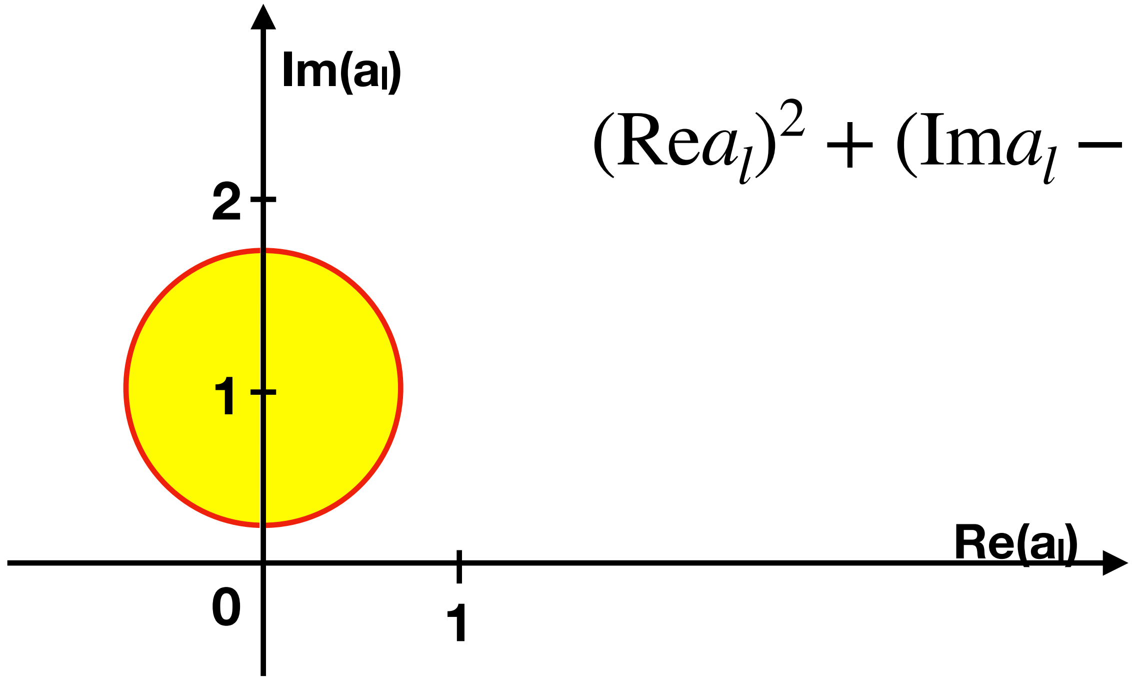
Unitarity primer



$$(\mathrm{Re}\,a_l)^2 + (\mathrm{Im}\,a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n \, |\mathscr{M}_l^{\text{inelastic}}|^2$$

- In a unitary theory, all partial wave amplitudes must lie on the boundary of the Argand circle
- Amplitudes calculated in perturbation theory may violate this condition, which signals that higher order corrections are non-negligible
- This goes under the name of perturbative unitarity violation
- New degrees of freedom must appear around the scale of perturbative unitarity violation, either as a UV completion of the effective theory, or as a strong coupling transition

Unitarity primer



$$(\mathrm{Re}a_l)^2 + (\mathrm{Im}a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n |\mathscr{M}_l^{\mathrm{inelastic}}|^2$$

Scale Λ_u where perturbative predictions are no longer reliable

$$(\mathrm{Re}a_l)^2 + \sum S_n \int d\Pi_n |\mathscr{M}_l^{\mathrm{inelastic}}|^2 |_{\sqrt{s}=\Lambda_u} = 1$$

Estimated scale Λ_* where new degrees of freedom must appear

$$(\mathrm{Re}a_l)^2 + \sum S_n \int d\Pi_n |\mathscr{M}_l^{\mathrm{inelastic}}|^2 |_{\sqrt{s}=\Lambda_*} \sim \pi^2$$

Unitarity constraints on inelastic channels

Unitarity (strong coupling) constraint on inelastic multi-Higgs production

$$\sum_{n=2}^{\infty} \frac{1}{n!} \int d\Pi_n \, |\mathcal{M}([GG]_{I=0}^{l=0} \to h^n)|^2{}_{\sqrt{s}=\Lambda_*} = \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\Lambda_*) \, |\mathcal{M}([GG]_{I=0}^{l=0} \to h^n)|^2 \sim \pi^2$$

Volume of phase space in massless limit: $V_n(\sqrt{s}) = \int d\Pi_n = \dfrac{s^{n-2}}{2(n-1)!(n-2)!(4\pi)^{2n-3}}$

In a unitary theory, 2 → n amplitude must decay as $1/s^{n/2-1}$ in order to maintain unitarity up to arbitrary high scales

Process	*Unitary behavior*
2 → 2	**1**
2 → 3	**$1/s^{1/2}$**
2 → 4	**1/s**
…	…

Unitarity constraints on HEFT

Unitarity equation

$$\sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s}) \, |\mathcal{M}(GG \to h^n)|^2 \lesssim \mathcal{O}(\pi^2)$$

Our amplitude

$$\mathcal{M}(GG \to \underbrace{h \ldots h}_{n}) \sim \Delta_3 \frac{n! m_h^2}{\mathrm{v}^n}$$

$$\mathcal{O}(1) \gtrsim \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s}) \, |\mathcal{M}(GG \to h^n)|^2 \sim \sum_{n=2}^{\infty} \frac{1}{n!} \frac{s^{n-2}}{(n!)^2 (4\pi)^{2n}} \Delta_3^2 \frac{(n!)^2 m_h^4}{\mathrm{v}^{2n}} \sim \frac{\Delta_3^2 m_h^4}{s^2} \exp\left[\frac{s}{(4\pi \mathrm{v})^2}\right]$$

In model with deformed Higgs cubic, multi-Higgs amplitude do not decay with energy leading to unitarity loss at a finite value of energy

$$\Lambda \lesssim (4\pi \mathrm{v}) \log^{1/2}\left(\frac{4\pi \mathrm{v}}{m_h |\Delta_3|^{1/2}}\right)$$

Unless Δ_3 is unobservably small, unitarity loss happens at the scale 4 π v ~ 3 TeV !

Multi-Higgs production

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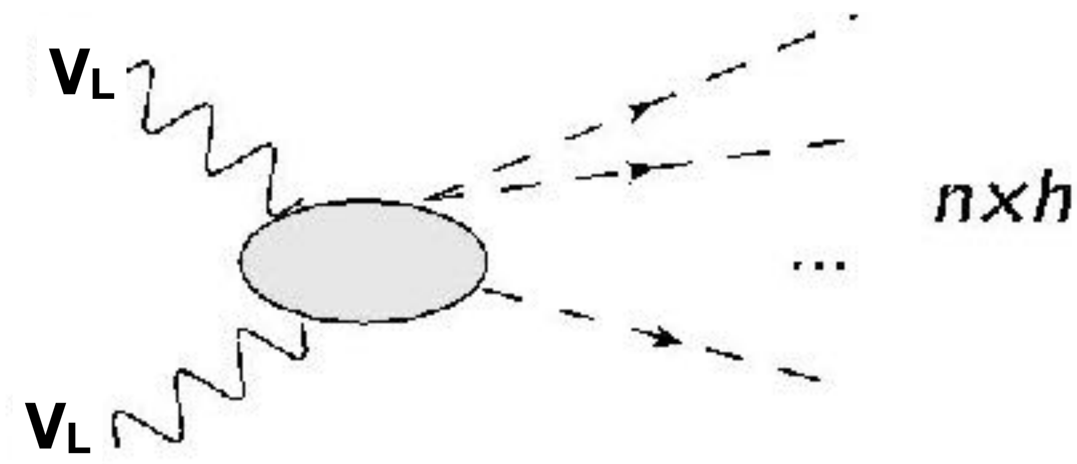
By equivalence theorem, at high energies the same as $GG \to n \times h$



Expanded V contains interactions

$$V \supset = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty} \left(\frac{-h}{\mathrm{v}} \right)^n$$

leading to interaction vertices with arbitrary number of Higgs bosons

s-wave isospin-0 amplitude for GG→h^n is momentum-independent constant proportional to the non-analytic deformation

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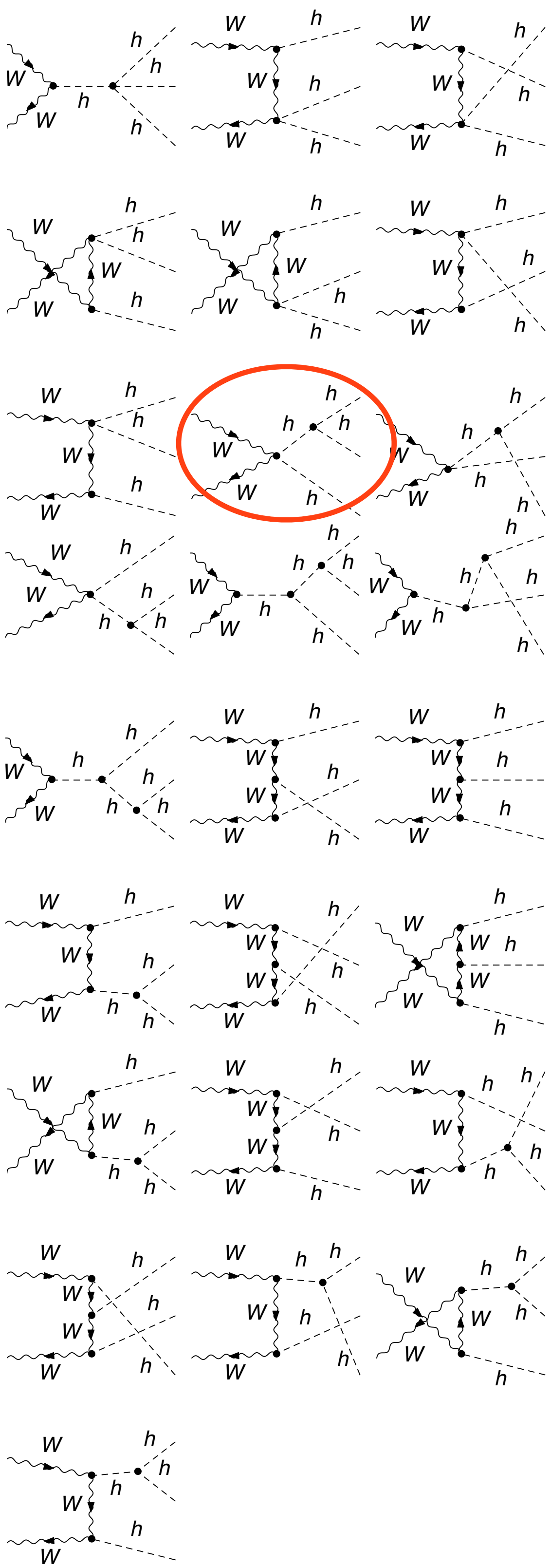
Multi-Higgs production



Same calculation can be performed (much more painfully) without resorting to equivalence theorem

Consider $V_L V_L \to hhh$ which depends on triple and other Higgs couplings.

Diagrams with one triple Higgs vertex contribute

$$\mathcal{M}(W_L W_L \to hhh) \sim \frac{m_W^2}{v^2} \frac{m_h^2}{v} (1+\Delta_3) \left(\frac{\sqrt{s}}{m_W}\right)^2 \frac{1}{s - m_h^2}$$

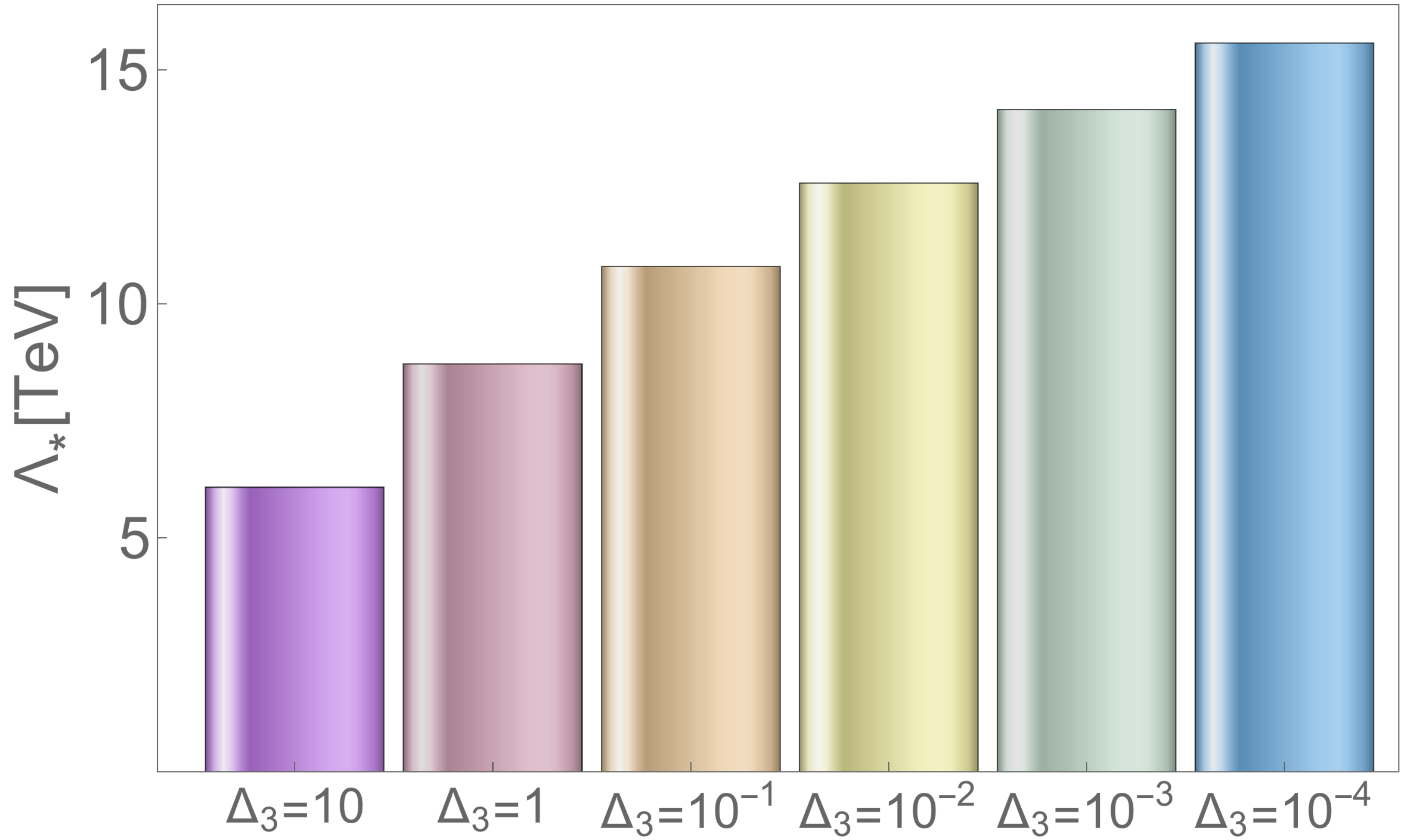
hhWW vertex | **Triple Higgs vertex** | **Longitudinal polarization** | **Higgs Propagator**

In SM, various contributions that go like E^0 cancel against each other so that full amplitude behaves as 1/E at high energy, consistently with perturbative unitarity

However, as soon as $\Delta_3 \neq 0$, cancellation is no longer happening, and then tree level $V_L V_L \to hhh$ cross section explodes at high energies

Unitarity constraints

Maximum new physics scale for different Δ_3

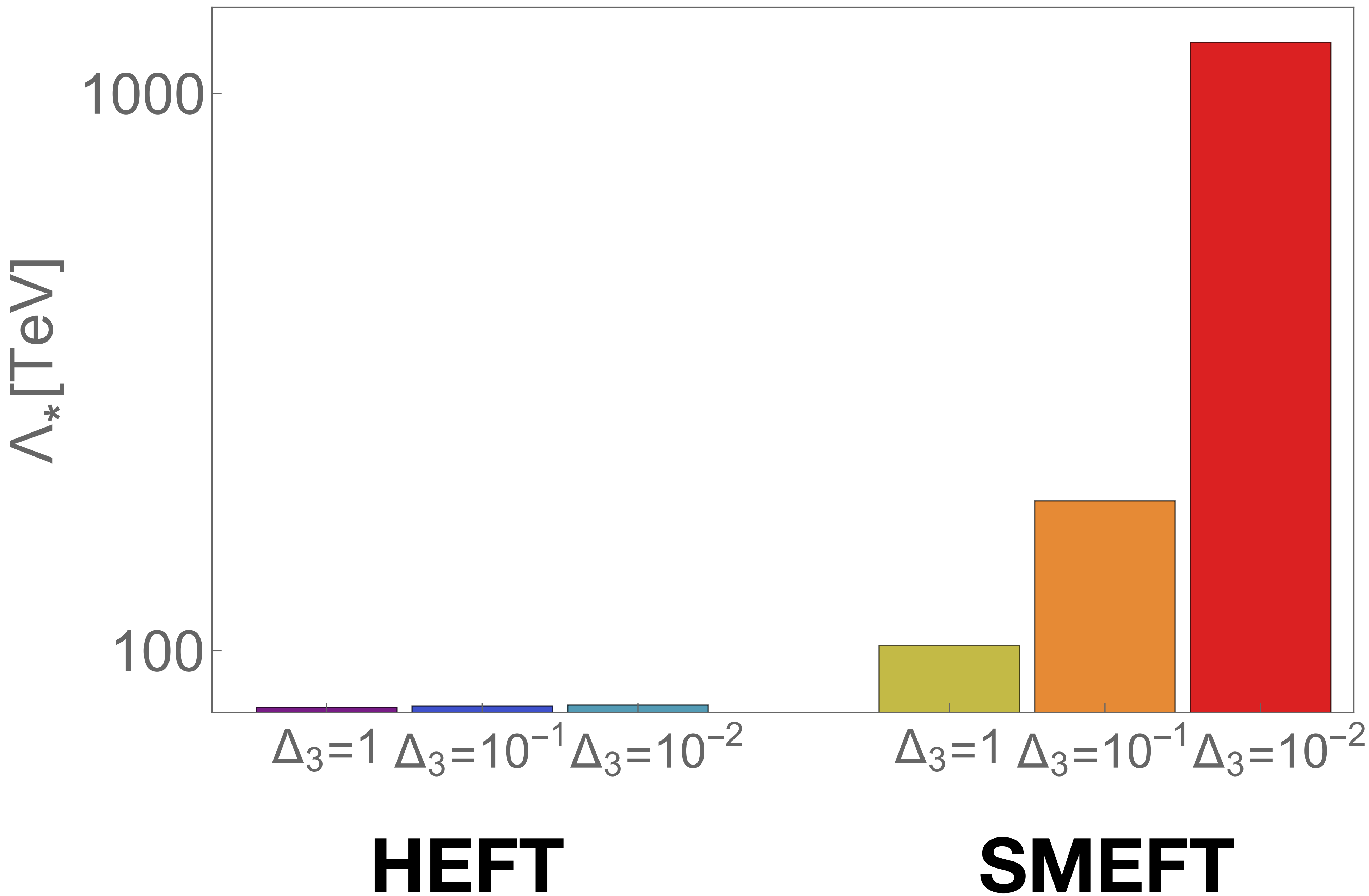


For observable deformations of Higgs cubic, new degrees of freedom must appear below a few TeV

This conclusion does not change much even if cubic deformations are so small so as to be unobservable in practice

HEFT vs SMEFT

Maximum new physics scale for different Δ_3

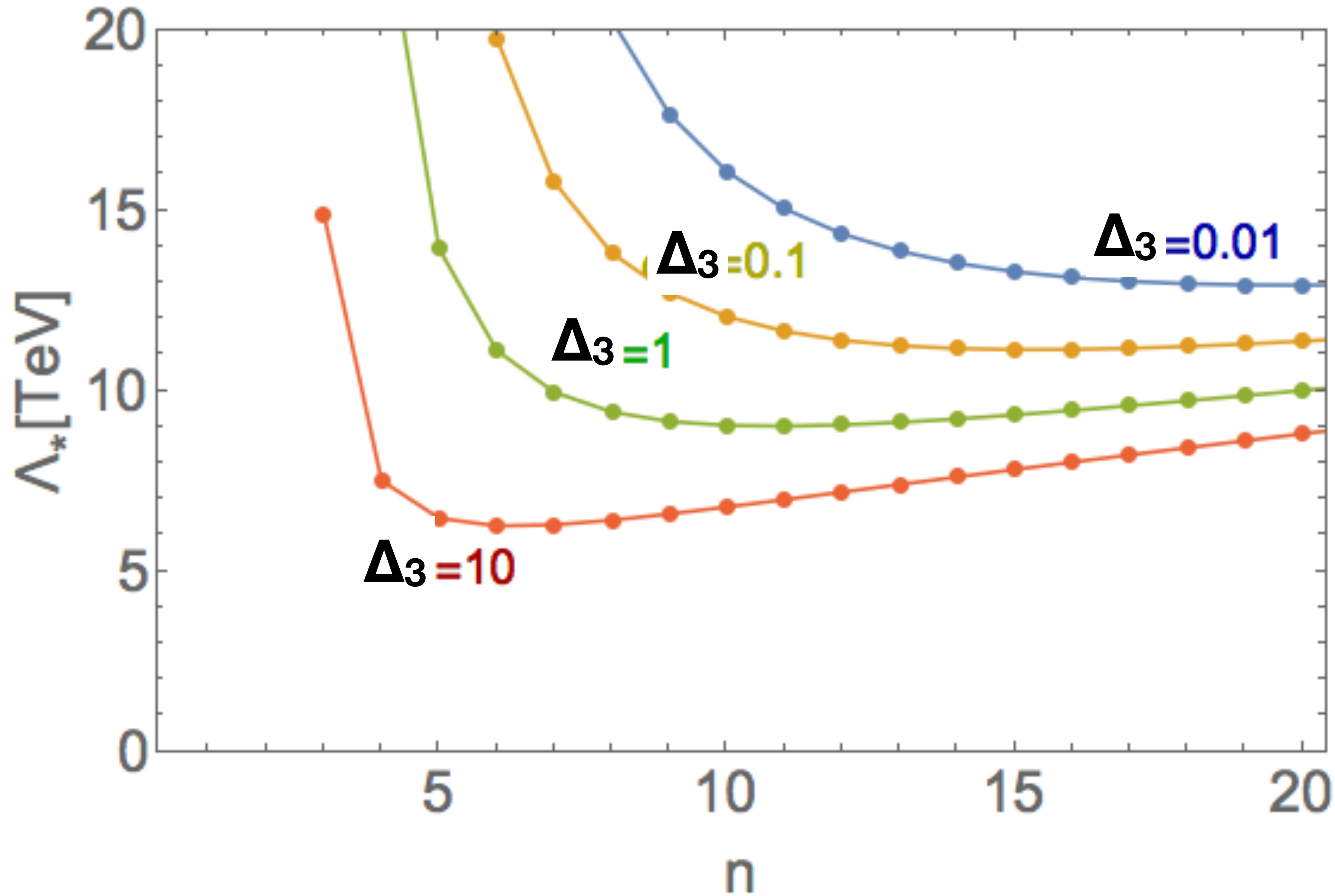


For SMEFT maximum new physics scale increases as $(\Delta_3)^{-1/2}$

$$\Delta_3 \sim \frac{c}{\Lambda^2} \quad \Rightarrow \quad \Lambda_* = \frac{4\pi}{\sqrt{|\Delta_3|}}$$

Unitarity constraints

Unitarity bounds separately for each n



The smaller Δ_3, the larger multiplicity n which dominates unitarity bounds. But even for tiny Δ_3, dominant n is order 10, so neglecting Higgs masses in phase space is justified a posteriori

Summary of unitarity constraints

- SM with deformed cubic loses perturbative unitarity at the scale of order 4 π v, and has to be UV completed around that scale

- This is true even if the deformation is tiny and unobservable in practice

- The same lesson applies to <u>any</u> HEFT theory that is not part of the SMEFT parameter space, even when it is a continuous and small deformation of the SM Lagrangian!

Unitarity arguments extended to other Higgs couplings in **Abu-Ajamieh et al 2009.11293**

Perspective on HEFT

- In effective theories, non-analytic terms in Lagrangian appear due to integrating out light or massless degrees of freedom

- More precisely, non-analyticity at $H \to 0$ signals that particle whose mass vanishes as $H \to 0$ has been integrated out

- Thus, HEFT is effective theory for UV models containing particles who get their masses from EW symmetry breaking. This explains why the cutoff cannot be parametrically raised above $4\pi v$.

- In contrast, SMEFT is the effective theory for UV models where new particles can be decoupled in the limit $v \to 0$, that is they have mass independent of EW symmetry breaking

Perspective on HEFT

Example of UV model leading to non-analytic terms in low-energy effective theory

Consider a funny version of the 2-Higgs-double model:

$$\mathscr{L}_{\mathrm{UV}} = \mathscr{L}_{\mathrm{SM}} - \frac{\kappa}{2}|\Phi|^4 + \mu^2(\Phi^\dagger H + \mathrm{h.c.})$$

Eqs of motion: $\Phi = \left(\frac{\mu^2}{\kappa H^\dagger H}\right)^{1/3} H$

Effective Lagrangian: $\mathscr{L}_{\mathrm{EFT}} \approx \mathscr{L}_{\mathrm{SM}} + \frac{3\mu^{8/3}}{2\kappa^{1/3}} \left(H^\dagger H\right)^{2/3}$

Non-analyticity appears because of integrating out a particle that would be massless in the absence of EW symmetry breaking

More familiar example is integrating out 4th chiral generation at one loop, which produces Log$|H|^2$ terms in the Coleman-Weinberg potential

Another example is integrating out tachyonic electroweak doublets or triplets

Cohen et al 2008.08597

HEFT vs SMEFT

$$\mathscr{L}_{\text{HEFT}} = \frac{1}{2} f_h(h)(\partial_\mu h)^2 - V(h) + \frac{\mathrm{v}^2}{4} f_1(h)\mathrm{Tr}[\partial_\mu U^\dagger \partial_\mu U] + \mathrm{v}^2 f_2(h)\left(\mathrm{Tr}[U^\dagger \partial_\mu U \sigma_3]\right)^2 + \ldots$$

$$U = \exp\left(\frac{i\pi^a\sigma^a}{\mathrm{v}}\right) \quad .$$

One can always re-express non-linear Lagrangian in linear language by replacing:

$$U \to \frac{\left(\tilde{H}, H\right)}{\sqrt{H^\dagger H}}$$

$$h \to \sqrt{2H^\dagger H} - v$$

After this substitution, Lagrangian has linearly realized electroweak symmetry but, for a generic point in parameter space, it contains terms that are non-analytic (that is, not continuously differentiable) at H=0

A HEFT Lagrangian belongs to the SMEFT class, if, after this substitution, non-analytic terms cancel up to equations of motion and field redefinitions

HEFT vs SMEFT

$$\mathscr{L}_{\text{HEFT}} = \frac{1}{2} f_h(h)(\partial_\mu h)^2 - V(h) + \frac{\text{v}^2}{4} f_1(h) \text{Tr}[\partial_\mu U^\dagger \partial_\mu U] + \text{v}^2 f_2(h) \left(\text{Tr}[U^\dagger \partial_\mu U \sigma_3] \right)^2 + \dots$$

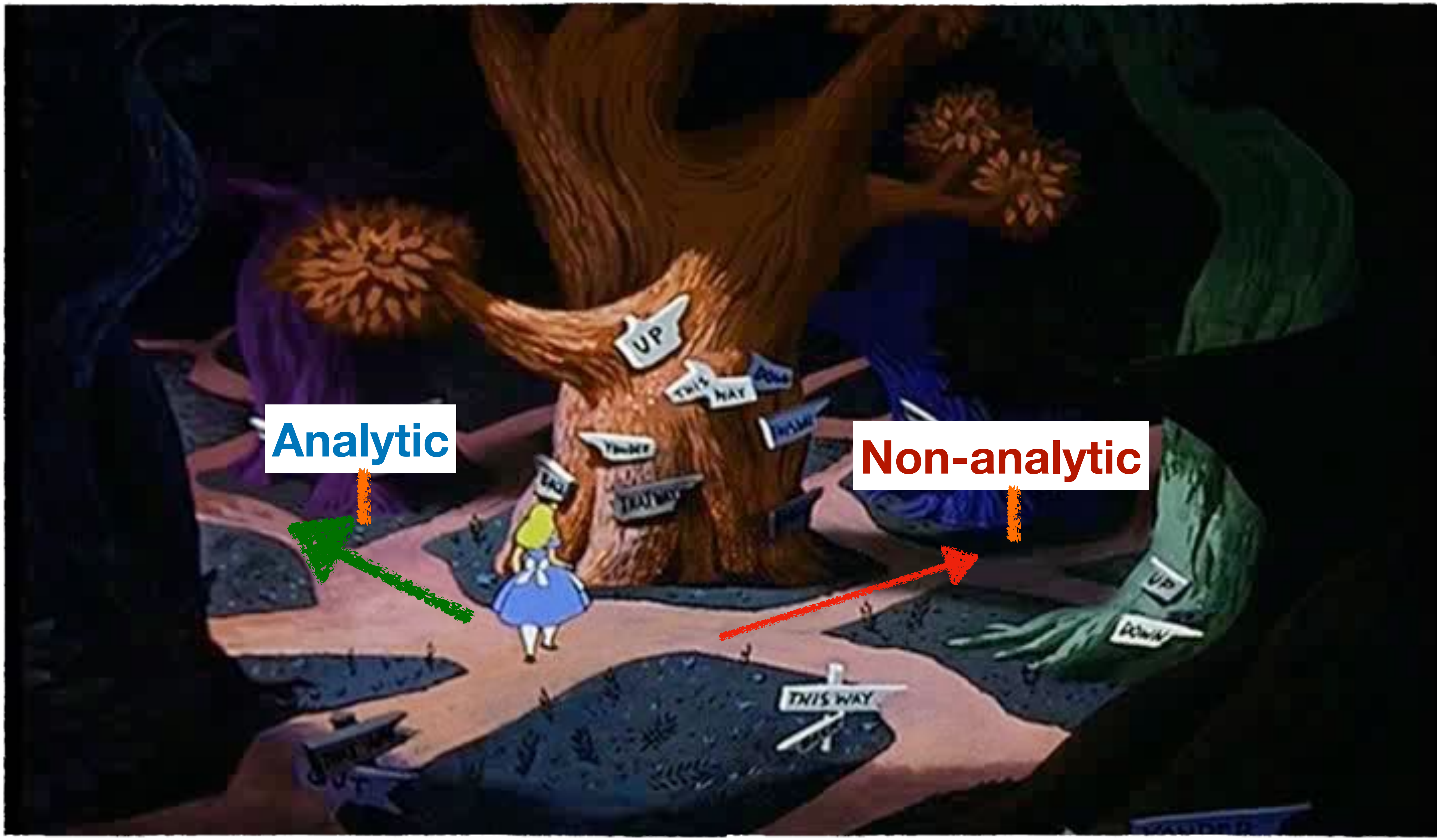
Geometric criterion to distinguish HEFT from SMEFT introduced in **Alonso et al 1511.00724**

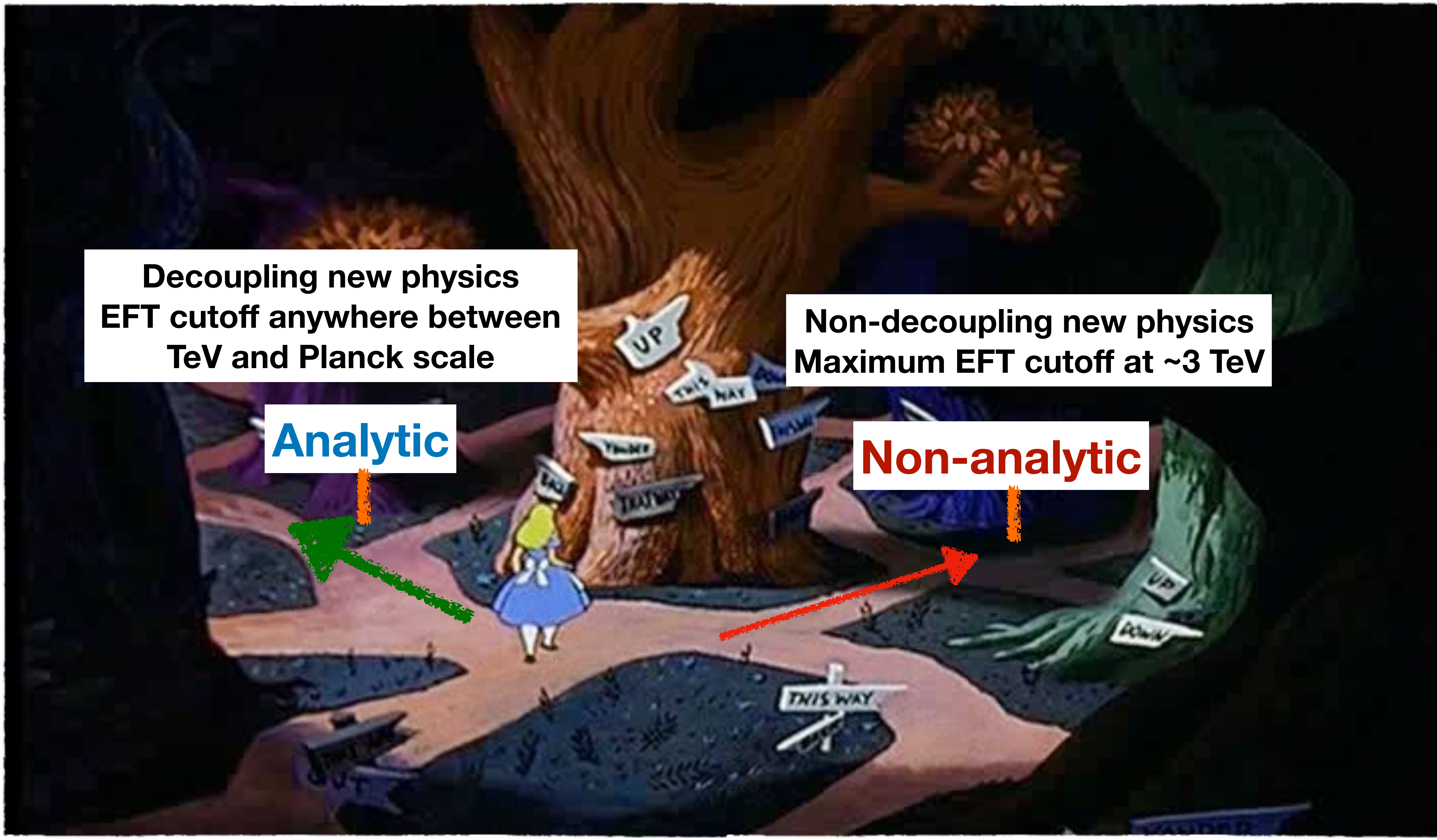
For $f_h(h)=1$ and $f_2(h)=0$, the Lagrangian belongs to the SMEFT class if the scalar manifold has an O(4) fixed point, that is if exists h_f such that $f_1(h_f)=0$

As it stands, this is not equivalent to the analyticity condition advertised in the previous slide

Geometric criterion recently clarified in **Cohen et al 2008.08597**

For the candidate O(4) fixed point h_f such that $f_1(h_f)=0$,
the potential V(h) has to be analytic at h_f (it has convergent Taylor expansion at h_f),
and the metric of the scalar manifold has to be analytic at h_f ,
in particular the curvature and its covariant derivatives have to be finite at h_f


Analytic
Non-analytic
UP
THIS WAY
DOWN
THIS WAY


Decoupling new physics
EFT cutoff anywhere between
TeV and Planck scale
Analytic
Non-decoupling new physics
Maximum EFT cutoff at ~3 TeV
Non-analytic
THIS WAY

Summary

- HEFT = SMEFT + non-analytic interactions
- Non-analytic term → infinite series of interactions suppressed by v^n → cut-off near $4\pi v$
- Manifested as n>2-body Higgs production violating perturbative unitarity bounds around that scale
- Non-analytic terms can be understood as effective description of BSM models where new particles get their masses only from the Higgs VEV