Top-quark mass effects in HJ and HH production via gluon fusion at NLO QCD

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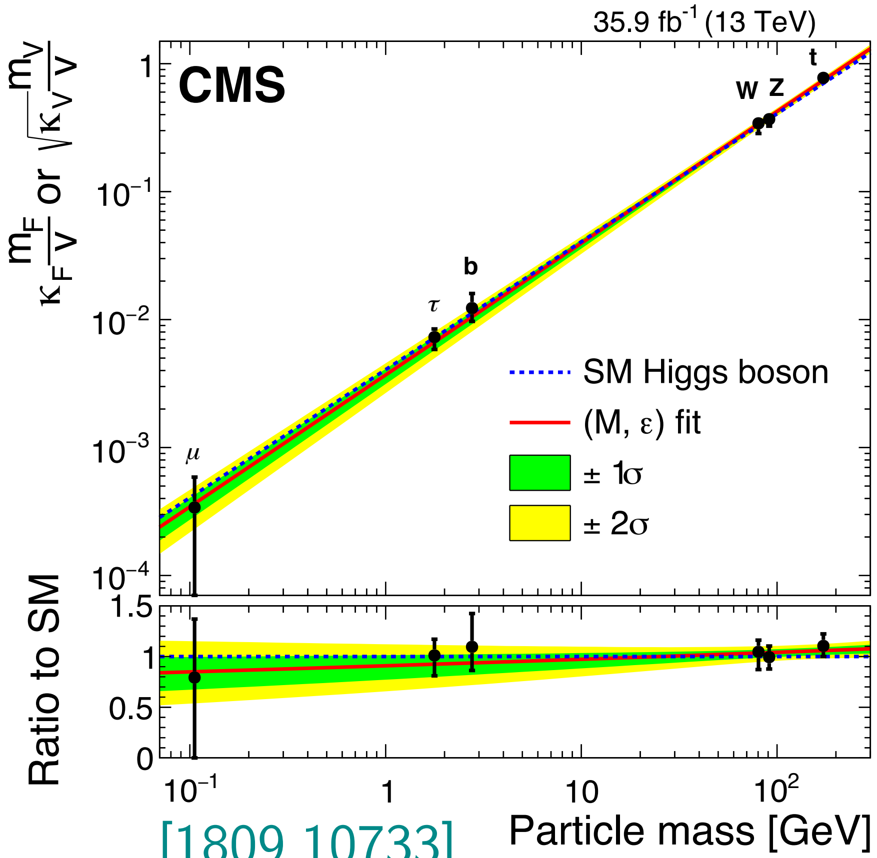
University Zürich

13. November 2018

based on:
Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke arXiv:1604.06447, arXiv:1608.04798
Jones, MK, Luisoni. arXiv:1802.00349

Introduction



[1809.10733]

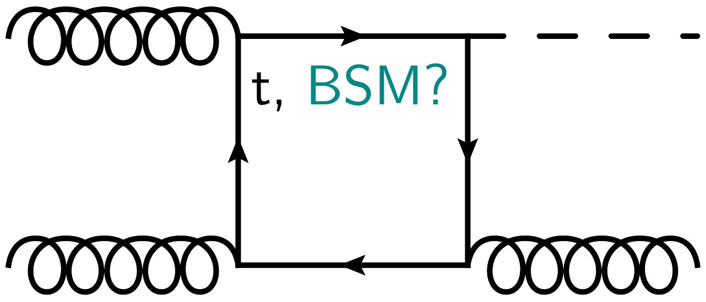
Measurements of Higgs production and decay so far in agreement with Standard Model predictions

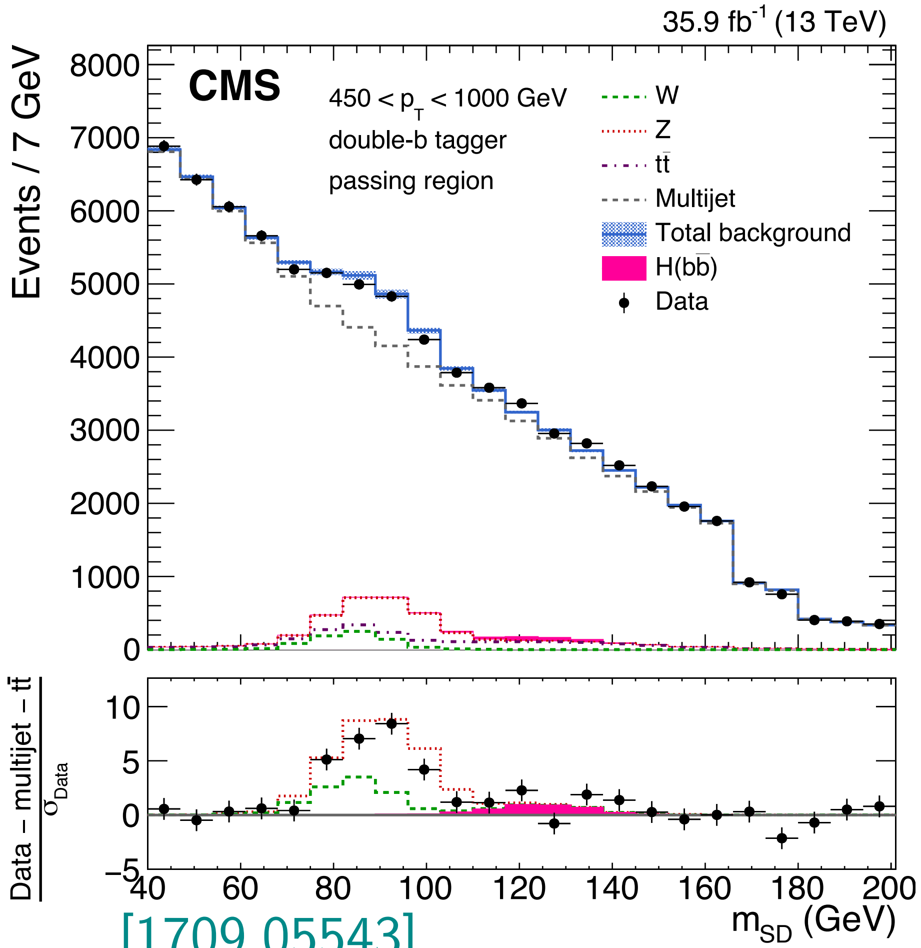
But new physics may hide in distributions

→ CMS: search for boosted Higgs with $p_T > 450\,\mathrm{GeV}$

→ Higgs + Jet(s) production via gluon-fusion

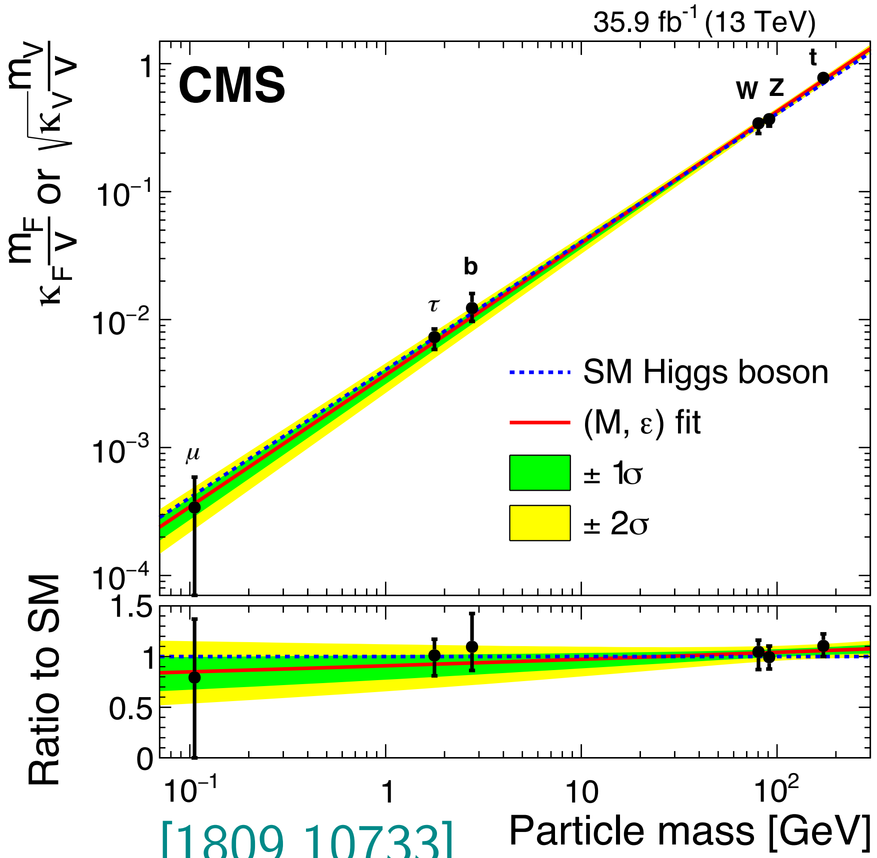
→ boosted production sensitive to particle in loop





[1709.05543]

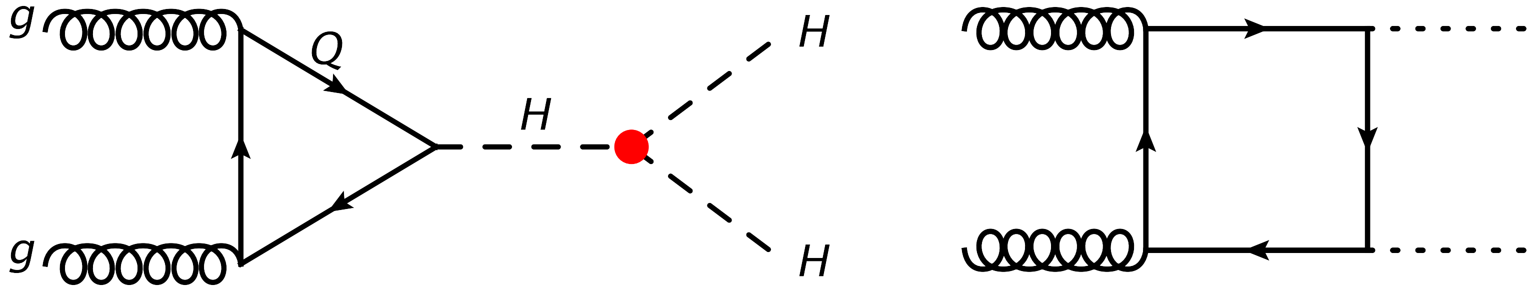
Introduction



[1809.10733]

Higgs self-interaction not yet established

→ Higgs pair production



triple-Higgs coupling

$$V(\Phi) = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4$$

$$\downarrow$$ EW symmetry breaking

$$\frac{m_H^2}{2}H^2 + \boxed{\frac{m_H^2}{2v}H^3} + \frac{m_H^2}{8v^2}H^4$$

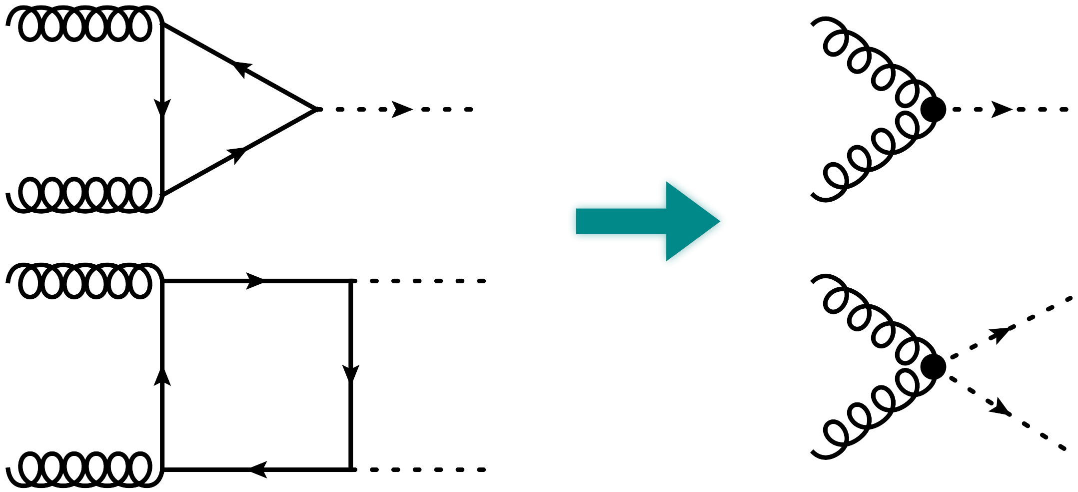
Test of Higgs potential &
EW symmetry breaking

Heavy top limit

Higgs production via gluon-fusion usually calculated in the limit

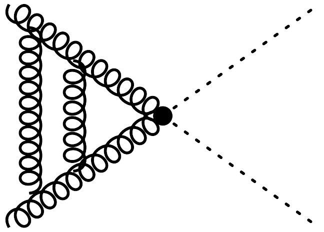
$m_T \to \infty$ limit (Higgs EFT)

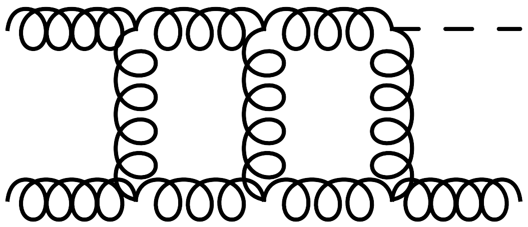
(valid for $\sqrt{s} \ll 2m_T$)



In this limit HJ and HH production are known to NNLO

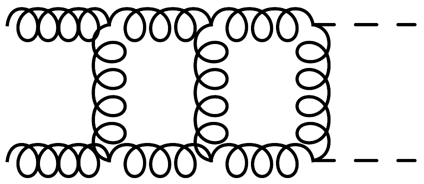
	HJ	HH
NLO:	de Florian, Grazzini, Kunszt 99 Ravindran, Smith, van Neerven 02 Glosser, Schmidt 02	Dawson, Dittmaier, Spira 98
NNLO:	Boughezal, Caola, Melnikov, Petriello, Schulze 13, 14 Chen, (Martinez,) Gehrmann, Glover, Jaquier 14, 16 Boughezal, Focke, Giele, Liu, Petriello 15	de Florian, Mazzitelli 13 Grigo, Melnikov, Steinhauser 14 de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16





contribution to N^3LO

Banerjee, Borowka, Dhani, Gehrmann, Ravindran 18



Heavy top limit

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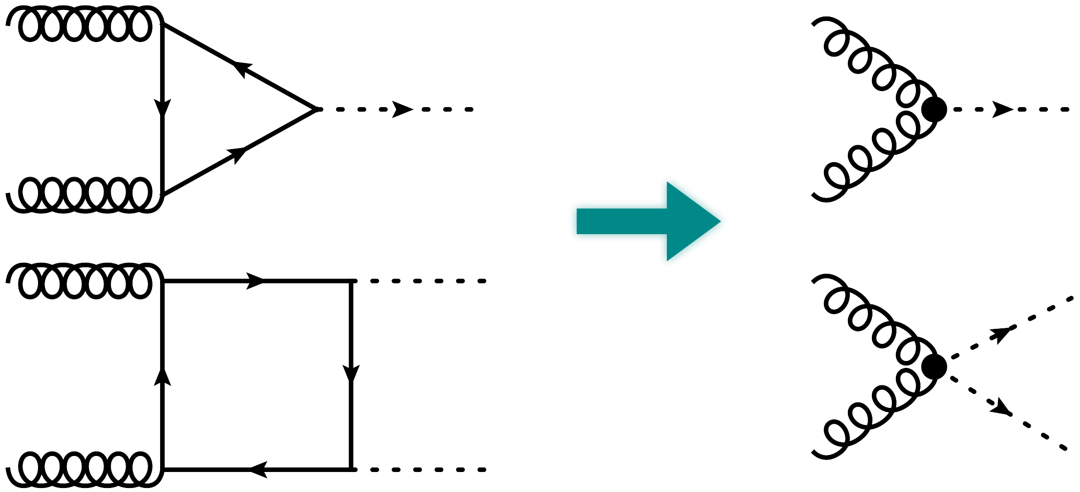
usually calculated in the limit



$m_T \to \infty$ limit (Higgs EFT)
(valid for $\sqrt{s} \ll 2m_T$)

full top-mass dependence required for

- HJ production at large p_T
- HH production ($2m_H < \sqrt{s}$)

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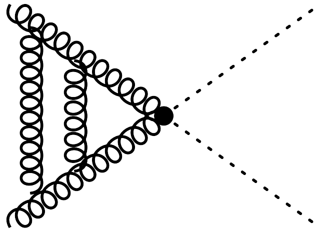

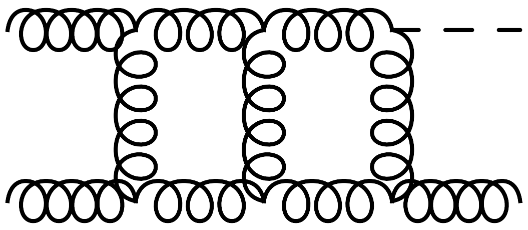

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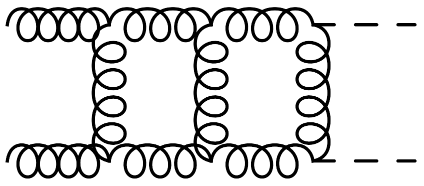
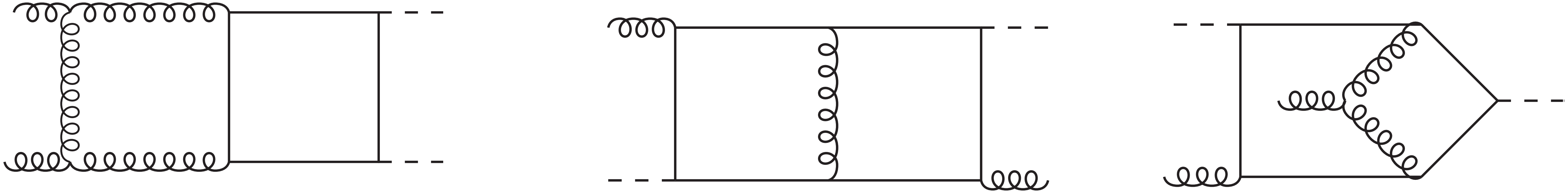


Two Loop Diagrams

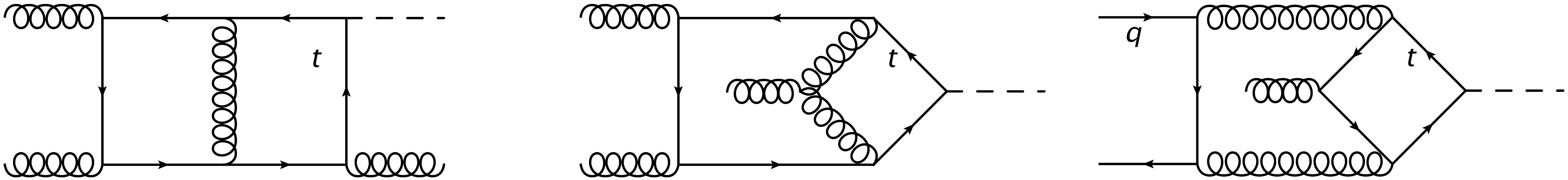
virtual corrections

- 7-propagator 2-loop diagrams
- 4 mass scales $s,\ t,\ m_t,\ m_H$

HH



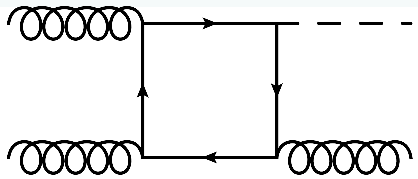
HJ

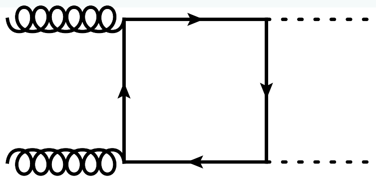


many integrals not known analytically
→ approximations or numerical calculation required

First results with top-mass effects

LO (1-loop) with full m_T dependence





Ellis, Hinchliffe, Soldate, van der Bij 87
Baur, Glover 89

Glover, van der Bij 88

NLO corrections in HEFT can be rescaled with full LO

Dawson, Dittmaier, Spira 98

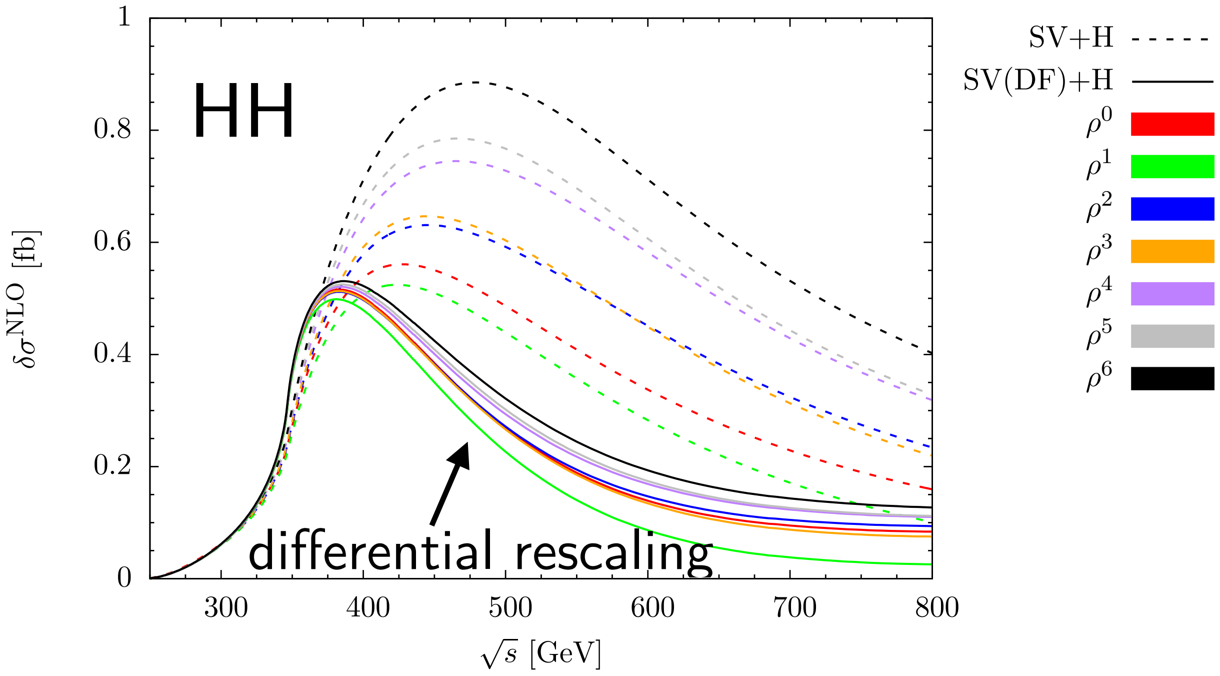
$$d\sigma_{NLO} \approx d\sigma_{NLO}^{HEFT} = \frac{d\sigma_{NLO}(m_t \to \infty)}{d\sigma_{LO}(m_t \to \infty)} d\sigma_{LO}(m_t)$$

... or improved by expansion in $1/m_T$

Harlander, Neumann, Ozeren, Wiesemann 12
Neumann, Wiesemann 14

Grigo, Hoff, (Melnikov,) Steinhauser 13, 15
Degrassi, Giardino, Gröber 16

method extended to NNLO



Grigo, Hoff, Steinhauser 15

expansion in $\rho = m_H^2/m_t^2$

- only works well for $\sqrt{s} < 2m_t$
- NLO corrections to total cross section:
  + 10% when rescaling with total cross section
  - 10% when rescaling differentially in s

First results with top-mass effects

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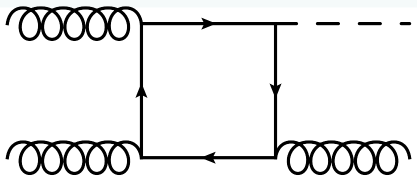
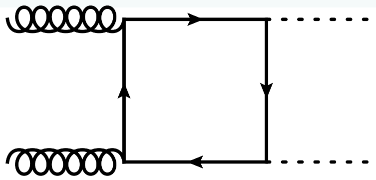


HJ



HH

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method extended to NNLO

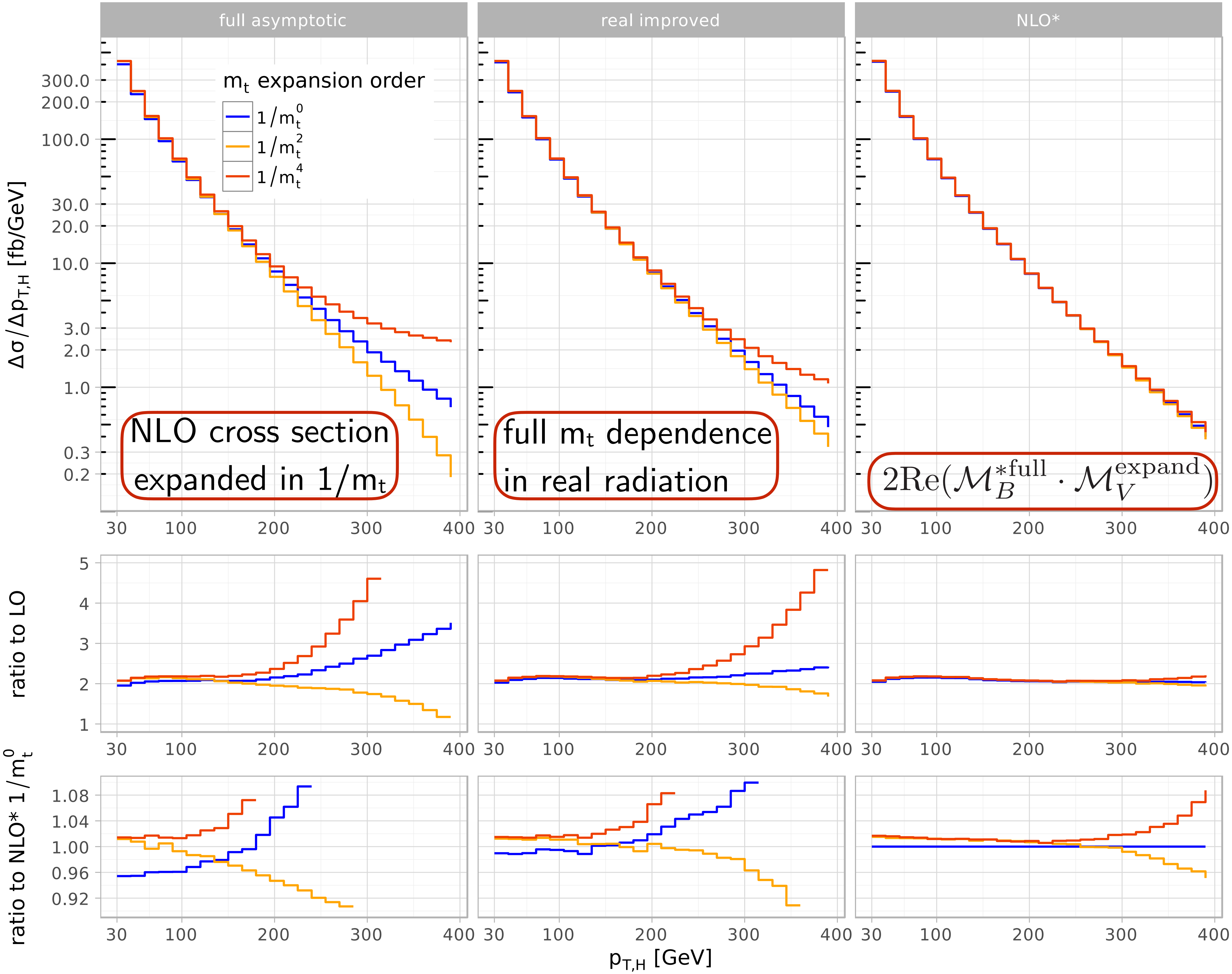
… or combined with full m_T-dependence of real radiation

Buschmann, Goncalves, Kuttimalai, Schonherr, Krauss, Plehn 14
Frederix, Frixione, Vryonidou, Wiesemann 16
Caola, Forte, Marzani, Muselliand, Vita 16
Neumann, Williams 16

Maltoni, Vryonidou, Zaro 14

Approximated results — HJ production

[Neumann, Williams 16]



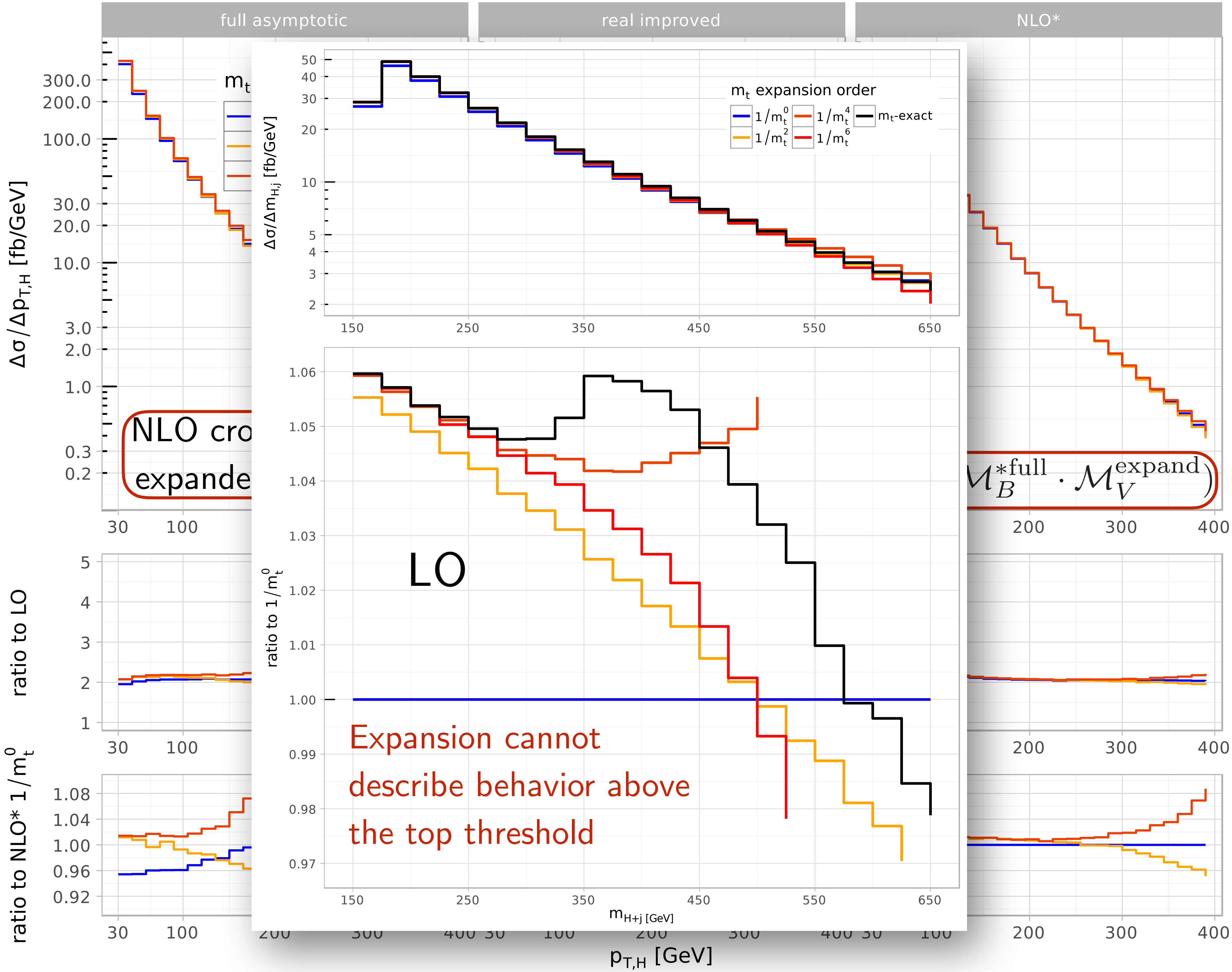
NLO cross section expanded in $1/m_t$

full m_t dependence in real radiation

$2\mathrm{Re}(\mathcal{M}_B^{*\mathrm{full}} \cdot \mathcal{M}_V^{\mathrm{expand}})$

Approximated results — HJ production

[Neumann, Williams 16]



NLO cro

expande

$\mathcal{M}_B^{*\text{full}} \cdot \mathcal{M}_V^{\text{expand}})$

LO

Expansion cannot describe behavior above the top threshold

HJ and HH production at NLO QCD

Overview

- Introduction
- Details of calculation
- Results
  - HJ @ NLO
  - HH @ NLO

  ... and beyond

Analytic results

AA / jj - production via top-quark loop [Becchetti, Bonciani 17]

only planar integrals calculated:

- alphabet containing square roots
- mostly GPLs
- up to 2-fold integrals at weights 3,4

HJ production [Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16]

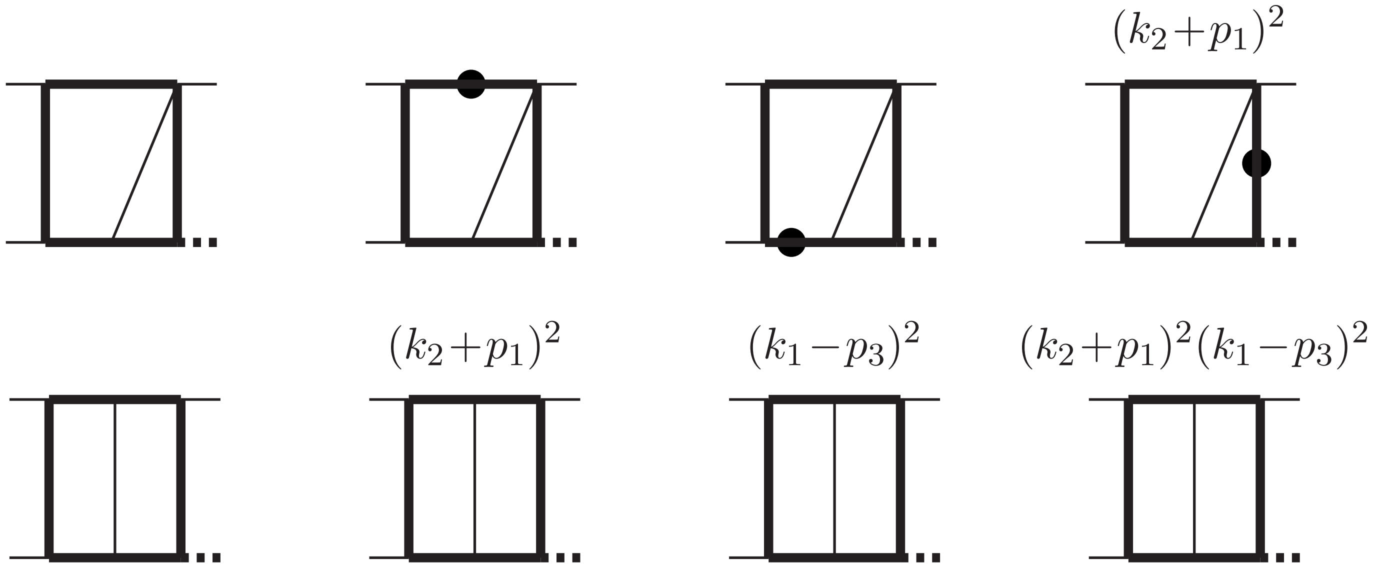
see also
[Primo, Tancredi 16]

most planar integrals can be expressed in terms of

- alphabet with 3 variables,
  49 letters, many square roots
- log, Li2 up to weight 2
- 1-fold integrals at weights 3,4

2 sectors contain elliptic functions

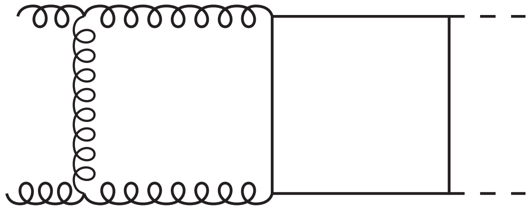
can be expressed as 2- and 3-fold iterated integrals with elliptic kernel



so far no non-planar 4-point integrals

Computational Method

Method for calculating virtual amplitude:

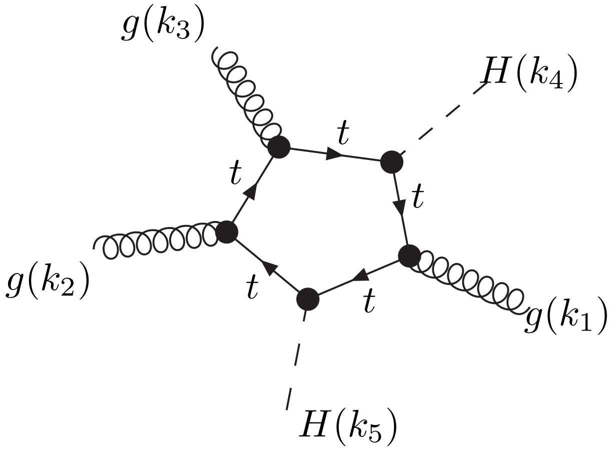


1. Form factor decomposition
2. Integral reduction
3. Sector decomposition
4. Numerical integration of loop integrals
   using Quasi Monte Carlo algorithm
5. Generate histograms of virtual contribution
   using unweighted LO events for phase-space sampling

Combine with real radiation at histogram level

→ 1-loop 5-point amplitudes

generated with GoSam Cullen et.al.



Integral Reduction

form-factor decomposition of virtual amplitude

- obtained by applying projectors
- linear combinations of many scalar integrals

HJ: 3767 integrals

up to 3 inverse propagators for 7-propagator integrals

up to 4 inverse propagators for factorizing 6-propagator integrals

HH: 1601 integrals

up to 4 inverse propagators

$$\int \mathrm{d}^d p_i \frac{\partial}{\partial p_i^\mu} \left[q^\mu \, \mathbf{I}'(p_1, \ldots, p_l; k_1, \ldots, k_m) \right] = 0$$

integral reduction

apply integration-by-parts identities

Tkachov 81; Chetyrkin 81; Laporta 01

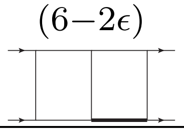
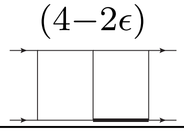
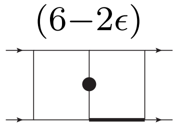
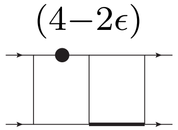
→ minimal set of linearly independent master integrals:

→ can be chosen to be finite:

von Manteuffel, Panzer, Schabinger 14

- divergences only in coefficients
- simplifies numerical evaluation of integrals
- integrals in shifted dimensions Tarasov 96; Lee 10

(quasi-)finite integrals

				run time	rel. error
 (s,t)	280 s	1.00×10^{-3}	 (s,t)	214135 s	8.29×10^{-3}
 (s,t)	294 s	1.21×10^{-3}	 (s,t)	3484378 s	30.9

[von Manteuffel, Schabinger 17]

results obtained using SecDec 3

Integral Reduction

IBP reduction obtained using Reduze 2 [von Manteuffel, Studerus 12]

but reduction with 4 independent scales $(s,\ t,\ m_t^2,\ m_H^2)$ challenging

→ modifications to Reduze code:

- specify list of required integrals
  → consider only equations containing these integrals
- change order of solving the system of equations, sorting the equations by number of unreduced integrals

useful additional simplification: fix m_t and m_H

HH reduction with fixed masses:
$m_t = 173\,\mathrm{GeV},\ m_H = 125\,\mathrm{GeV}$
but we did not manage to obtain reduction of non-planar integrals!

→ rewrite inverse propagators as scalar products to reduce rank

→ directly calculate them numerically

full HJ reduction obtained twice:

- with $m_H^2/m_t^2 = 12/23$
- with full m_t and m_H dependence

$$\int \mathrm{d}^d p_1 \mathrm{d}^d p_2 \frac{(p_1+k_1)^2}{p_1^2} f(p_i,k_i) = \int \mathrm{d}^d p_1 \mathrm{d}^d p_2 \left(1 + \frac{k_1^2}{p_1^2} + \frac{2\,p_1\cdot k_1}{p_1^2}\right) f(p_i,k_i)$$

rank-2

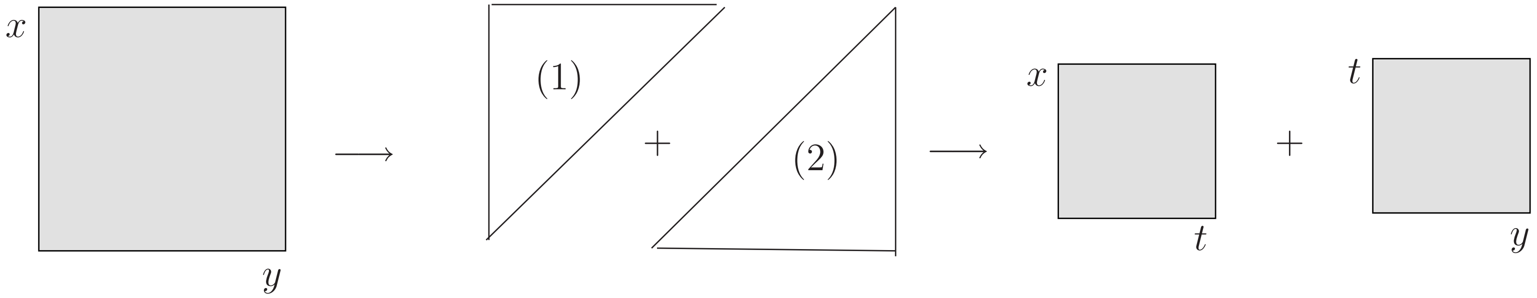
rank-1

up to 4 inverse propagators → up to rank-4 tensors

Sector Decomposition

- sector decomposition

  Binoth, Heinrich 00



$$\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \, \frac{1}{(x_1+x_2)^{2+\varepsilon}}$$

$\leftarrow$ overlapping singularities

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \, \frac{1}{(x_1+x_2)^{2+\varepsilon}} \left[\theta(x_1-x_2)+\theta(x_2-x_1)\right]$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^{x_1} \mathrm{d}x_2 \, \frac{1}{(x_1+x_2)^{2+\varepsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^{x_2} \mathrm{d}x_1 \, \frac{1}{(x_1+x_2)^{2+\varepsilon}}$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}t \, \frac{x_1}{(x_1+tx_1)^{2+\varepsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}t \, \frac{1}{x_2^{1+\varepsilon}(1+t)^{2+\varepsilon}}$$

$\leftarrow$ singularities factorized

- subtraction of poles

$$\int_0^1 \mathrm{d}x \, x^{-1-\varepsilon} g(x,\varepsilon) = -\frac{1}{\varepsilon} g(0,\varepsilon) + \int_0^1 \underline{\mathrm{d}x \, x^{-1-\varepsilon} \left(g(x,\varepsilon)-g(0,\varepsilon)\right)}$$

$\rightarrow$ finite

- expansion in ε

→ finite integrals for each order in ε → numeric integration possible

Loop Integrals

- (scalar) loop integral after Feynman parametrization:

$$G = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j - 1} \delta(1 - \sum_{i=1}^{N} x_i) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu - LD/2}(\vec{x}, s_{ij})}$$

with Symanzik polynomials $\mathcal{U}$, $\mathcal{F}$:

$$\mathcal{U}(\vec{x}) \;=\; \sum_{T \in \mathcal{T}_1} \Big[\prod_{j \in \mathcal{C}(T)} x_j \Big] \qquad \mathcal{F}(\vec{x}) \;=\; \mathcal{F}_0(\vec{x}) + \mathcal{U}(\vec{x}) \sum_{j=1}^{N} x_j m_j^2 \qquad \mathcal{F}_0(\vec{x}) \;=\; \sum_{\hat{T} \in \mathcal{T}_2} \Big[\prod_{j \in \mathcal{C}(\hat{T})} x_j \Big] (-s_{\hat{T}}) - i0$$

- sector decomposition leads to

$$G = \sum_i G_i, \quad G_i = c_i(\varepsilon) \int_0^1 \prod_j \mathrm{d}x_j x_j^{\nu_j - 1} \frac{\mathcal{U}_i(\vec{x})^{e_U(\varepsilon)}}{\mathcal{F}_i(\vec{x}, s_{ij})^{e_F(\varepsilon)}}$$

$$\mathcal{U}_i = 1 + u_i(\vec{x}),$$
$$\mathcal{F}_i = -f_{0,i} + f_i(\vec{x})$$

- contour deformation (required if some $s_{ij} > 0$)

Soper 00; Binoth, et al. 05; Nagy, Soper 06; Borowka et al. 12

$$\vec{x} \to \vec{z}(\vec{x}) = \vec{x} - i\vec{\tau}(\vec{x}) \qquad\qquad \tau_k(\vec{x}) = \lambda \, x_k (1 - x_k) \frac{\partial \mathcal{F}(\vec{x})}{\partial x_k}$$

generates correct imaginary part at thresholds $\mathcal{F}(\vec{x}) = 0$

$$\mathcal{F}(\vec{x}) \to \mathcal{F}(\vec{z}(\vec{x})) = \mathcal{F}(\vec{x}) - i \sum_k \tau_k(\vec{x}) \left(\frac{\partial \mathcal{F}}{\partial x_k} \right)^2 + \mathcal{O}(\tau_k(\vec{x})^2)$$

pySecDec

pySecDec Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke

a toolbox for the calculation of dimensionally regulated parameter integrals using sector decomposition

publicly available: secdec.hepforge.org

improvements compared to SecDec 3

- implementation using python and Form Vermaseren, Ruijl, Ueda
- modular structure

  can act on (almost) any polynomial

  any number of regulators is possible
- generates libraries

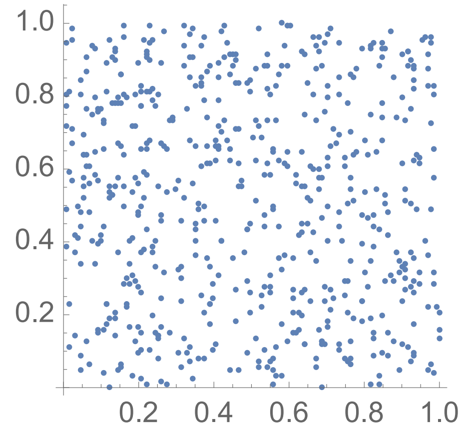
  can be directly linked to amplitude code
- handling of non-logarithmic poles improved
- improved symmetry finder
- …
- coming soon: QMC integration

SecDec 3 used for calculation of HJ and HH production

Numerical Integration Methods

We need to (numerically) integrate

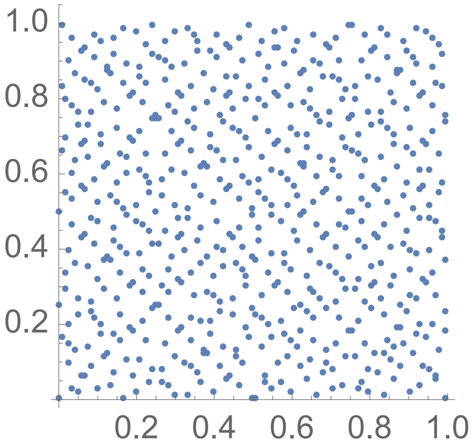
$$I[f] = \int_{[0,1]^d} \mathrm{d}\mathbf{x}\, f(\mathbf{x}) \quad \approx \quad Q[f] = \frac{1}{N} \cdot \sum_{i=1}^{N} w_i \cdot f(\mathrm{x}_i)$$



- Monte Carlo

  randomly select N sampling points

  integration error: $\varepsilon \approx \mathrm{Var}[f]/\sqrt{N}$



- Quasi-Monte Carlo

  using points with low discrepancy D_N

  integration error: $\varepsilon \leq D_N \cdot \mathrm{V}[f]$ $\quad \varepsilon = \mathcal{O}\left(\log^{\overset{\uparrow}{d}}(N)/N\right)$

→ Does not work well for large d

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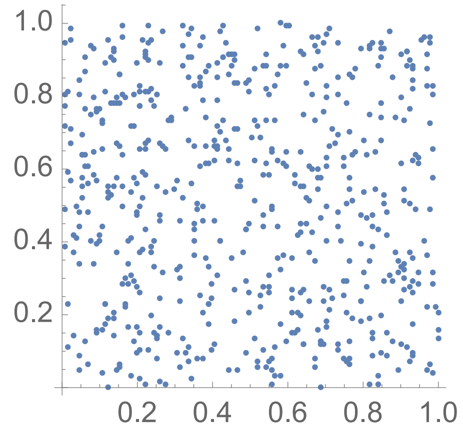

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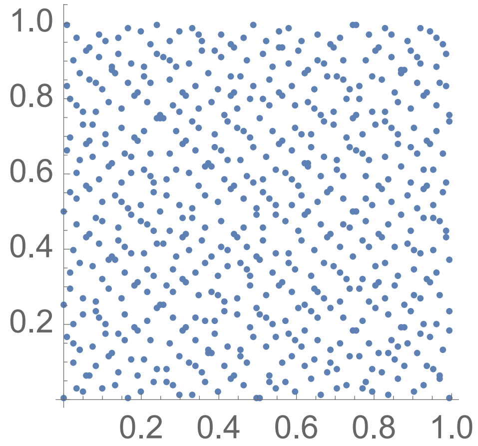


- Quasi-Monte Carlo

  using points with low discrepancy D_N

  integration error: $\varepsilon \leq D_N \cdot \mathrm{V}[f]$ $\qquad \varepsilon = \mathcal{O}\left(\log^{d}(N)/N\right)$

  → Does not work well for large d

But we can do better:

(if norm of f in weighted function space is finite)

→ Quasi-Monte Carlo in weighted function space

integration error: $\varepsilon \leq \epsilon_\gamma \cdot ||f||_\gamma$

Quasi-Monte Carlo in weighted function space

assign weights $\gamma_{\mathfrak{u}}$ to each subset of dimensions $\mathfrak{u} \subseteq \{1, \ldots, d\}$

Review: Dick, Kuo, Sloan `13

weighted function spaces, typically considered in literature:

Sobolev space

norm $$||f||_\gamma^2 = \sum_{\mathfrak{u} \subseteq \{1,\ldots,d\}} \frac{1}{\gamma_{\mathfrak{u}}} \int_{[0,1]^{|\mathfrak{u}|}} \left(\int_{[0,1]^{d-|\mathfrak{u}|}} \frac{\partial^{|\mathfrak{u}|} f(\mathbf{x})}{\partial \mathbf{x}_{\mathfrak{u}}} \mathrm{d}\mathbf{x}_{-\mathfrak{u}} \right)^2 \mathrm{d}\mathbf{x}_{\mathfrak{u}}$$

Korobov space

$$||f||_\gamma^2 = \sum_{\mathbf{h} \in \mathbb{Z}^d} \frac{\prod_{j \in \mathfrak{u}(\mathbf{h})} |h_j|^{2\alpha}}{\gamma_{\mathfrak{u}(\mathbf{h})}} |\hat{f}(\mathbf{h})|^2$$

Fourier coefficient

use rank-1 lattice rule $$I[f] \approx I_k = \frac{1}{N} \cdot \sum_{i=1}^{N} f(\mathbf{x}_{i,k}), \quad \mathbf{x}_{i,k} = \left\{ \frac{i \cdot \mathbf{z}}{N} + \mathbf{\Delta}_k \right\}$$

$\{\ldots\} =$ fractional part ($\rightarrow x \in [0;1[$)

$\mathbf{\Delta}_k =$ randomized shifts

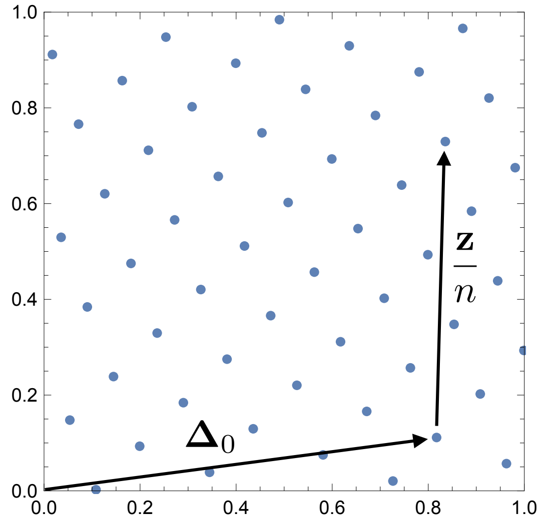
$\rightarrow$ m different estimates of Integral: $I_1, \ldots, I_m$

$\rightarrow$ error estimate of result

$\mathbf{z} =$ generating vector

constructed component-by-component Nuyens `07

minimizing worst-case error ϵ_γ



worst-case error

$$\epsilon_\gamma^2 \leq \left(\frac{1}{\psi(N)} \sum_{\emptyset \neq \mathfrak{u} \subseteq \{1,\ldots,d\}} \gamma_{\mathfrak{u}}^\lambda \left(\frac{2\zeta(2\lambda)}{(2\pi^2)^\lambda} \right)^{|\mathfrak{u}|} \right)^{\frac{1}{\lambda}}$$

$\lambda \in]1/2, 1]$

$\epsilon_\gamma = \mathcal{O}\left(N^{-1}\right)$

$$\epsilon_\gamma^2 \leq \left(\frac{1}{\psi(N)} \sum_{\emptyset \neq \mathfrak{u} \subseteq \{1,\ldots,d\}} \gamma_{\mathfrak{u}}^\lambda \left(2\zeta(2\alpha\lambda)\right)^{|\mathfrak{u}|} \right)^{\frac{1}{\lambda}}$$

$\lambda \in]1/(2\alpha), 1]$

smoothness α

$\epsilon_\gamma = \mathcal{O}\left(N^{-\alpha}\right)$

first application to sector-decomposed loop integrals: Li, Wang, Yan, Zhao 15

implementation in public library coming soon! arXiv:1811.????? Borowka, Heinrich, Jahn, Jones, MK, Schlenk

can be used on CPUs and GPUs

Evaluation of amplitude

after sector decomposition and expansion in ε:

→ amplitude written in terms of $\mathcal{O}(10\,\mathrm{k})$ finite integrals

Some optimizations required to reduce run time:

- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral}\, i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \qquad\qquad \sigma_i = c_i \cdot t_i^{-e}$$

$$\sigma_i = \text{error estimate (including coefficients in amplitude)}$$

$$\lambda = \text{Lagrange multiplier} \qquad\qquad \sigma = \text{precision goal}$$

- parallelization on gpu
- avoid reevaluation of integrals for different orders in ε and form factors

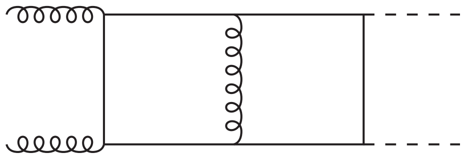
compute only once

$$F^a = \sum_i \left[\left(\sum_j C^a_{i,j} \varepsilon^j \right) \cdot \left(\sum_k I_{i,k} \varepsilon^k \right) \right] = \frac{C^a_{1,-2} I_{1,0} + C^a_{1,-1} I_{1,-1} + \dots}{\varepsilon^2} + \frac{C^a_{1,-1} I_{1,0} + \dots}{\varepsilon^1} + \dots$$

HH Amplitude Evaluation — Example

$$\sqrt{s} = 327.25\,\text{GeV},\ \sqrt{-t} = 170.05\,\text{GeV},\ M^2 = s/4$$

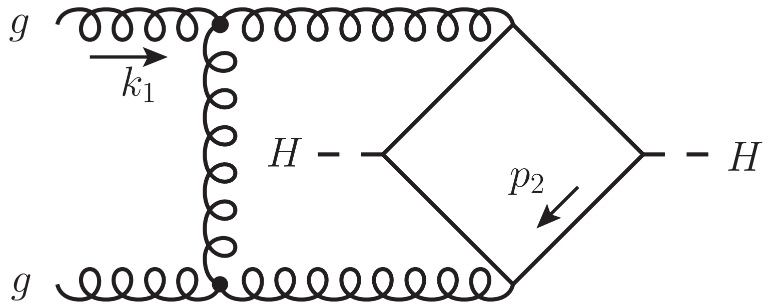
contributing integrals:



integral	value	error	time [s]
. . .			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
. . .			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342

≈ 700 integrals

$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon}\Gamma(3+2\epsilon)M^{-4}\left(\frac{A_{-2}}{\epsilon^2}+\frac{A_{-1}}{\epsilon^1}+A_0+\mathcal{O}(\epsilon)\right)$$



sector decomposition

sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
. . .				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

HH Amplitude Evaluation — Example

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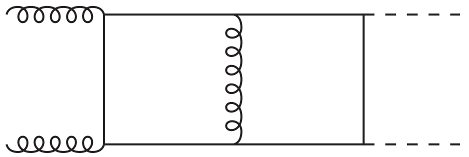
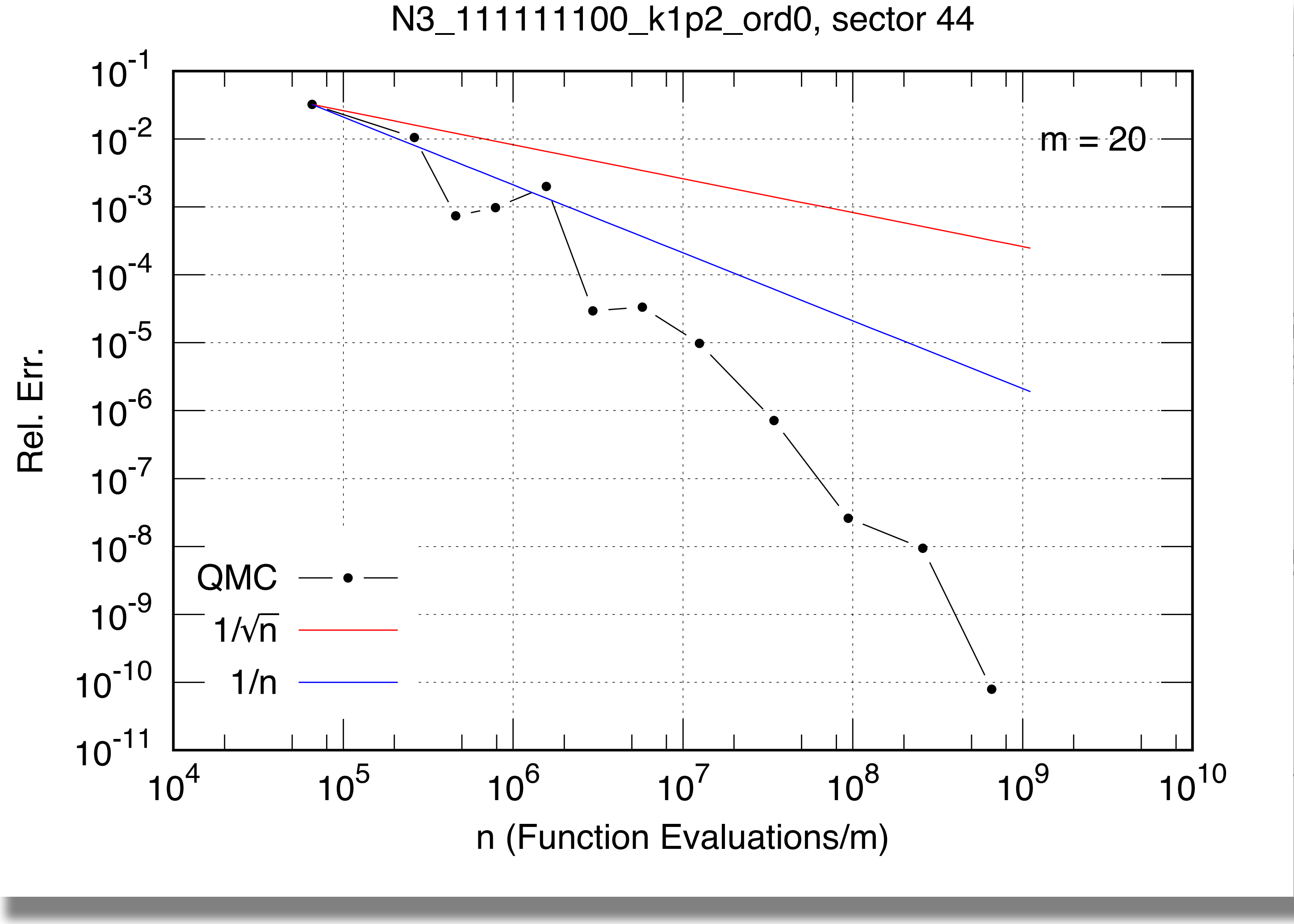
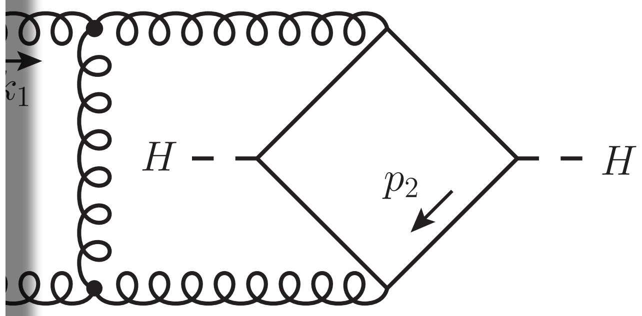


integral	value	error	time [s]
. . .			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
. . .			
N3_111111100_k1p2			412
N3_111111100_1_or			896
N3_111111100_k1p2			794
N3_111111100_k1p2			342

≈ 700 integrals

$I(s,t,m_t^2,m_h^2) = -($





sector	in			
5	(-1.34e-0			
6	(-1.58e-03			
. . .				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
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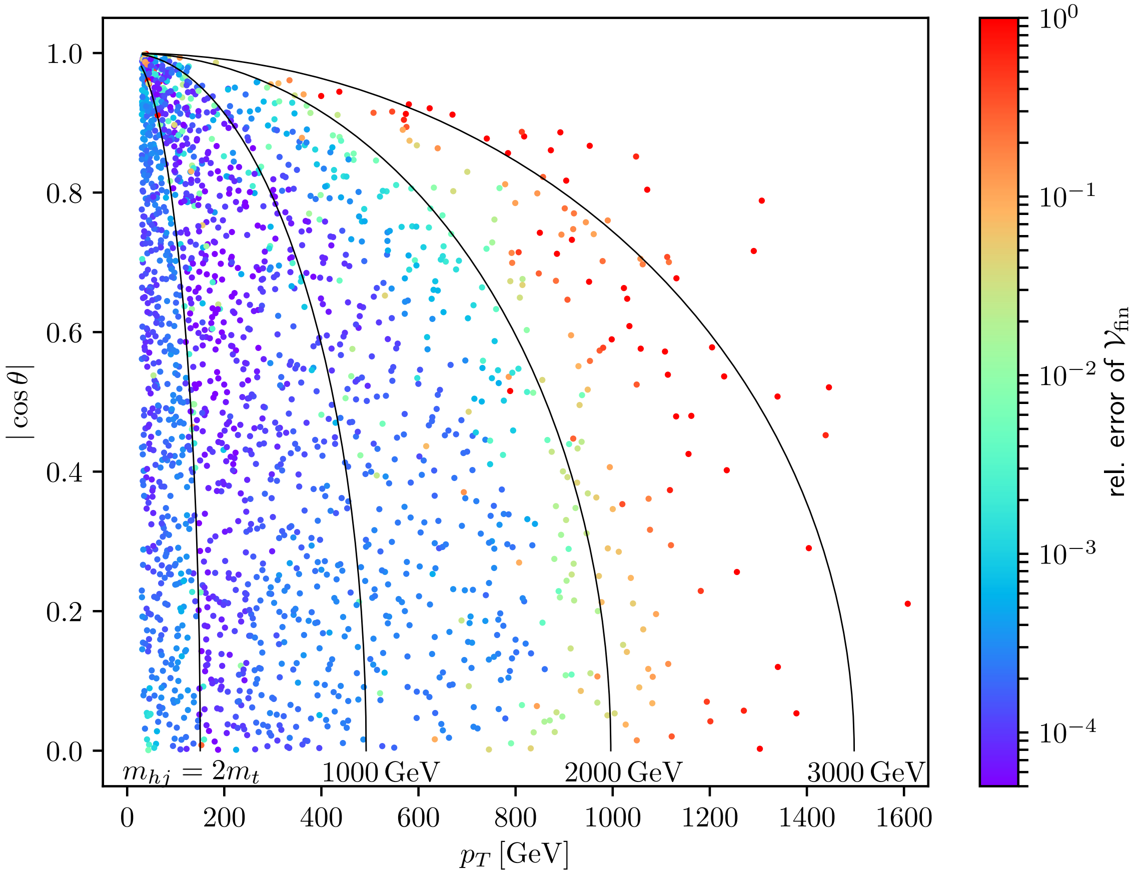
HJ Numerical Stability & Run Time

numerical evaluation of virtual amplitude:

- precision goal: 0.5% for each form factor
- wall-clock limit: 2d GPU-time (Tesla K20X GPUs)

accuracy reached for $|\mathcal{M}|^2$:

- better than per-mill
  for most points below $m_{hj} = 1.5\,\mathrm{TeV}$
- region $m_{hj} \gtrsim 2\,\mathrm{TeV}$ numerically challenging



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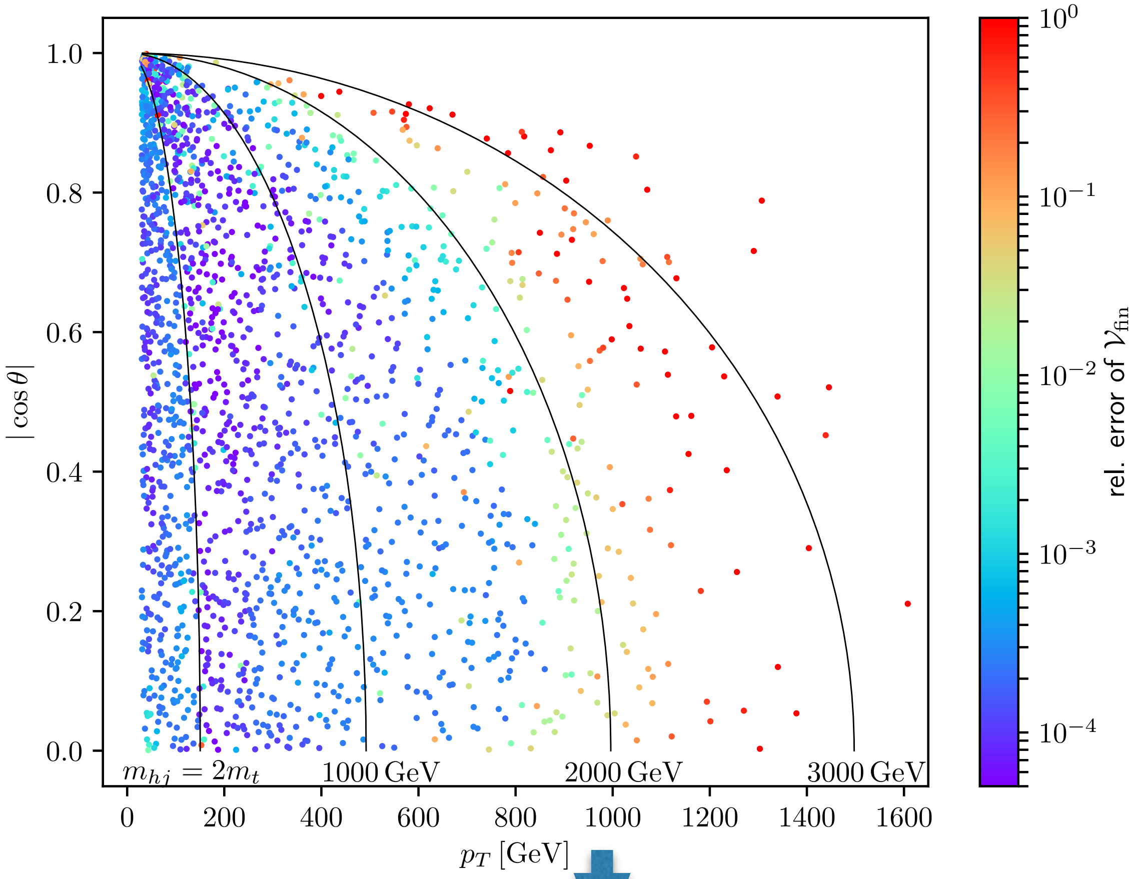


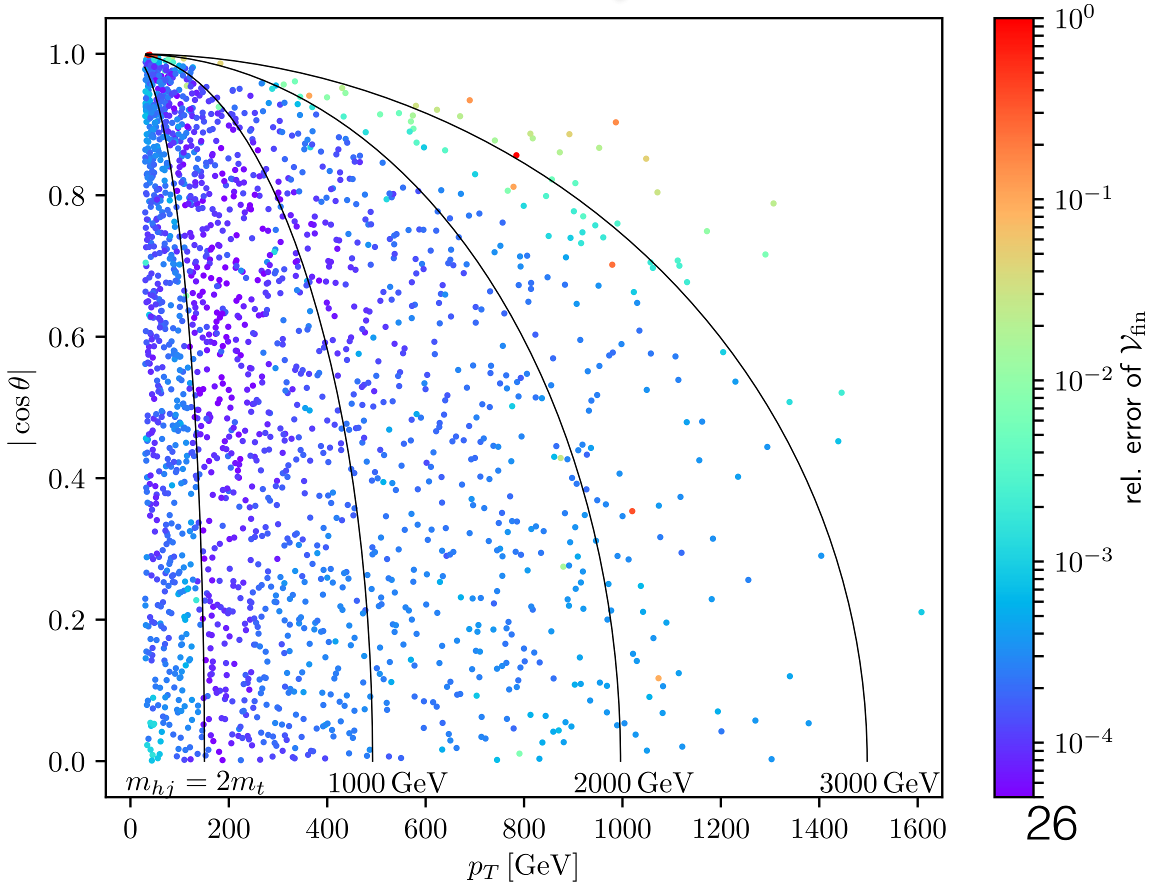
improved basis choice

- use finite integrals with $\mathrm{exponent}(\mathcal{F}) = -1$
  $\to$ possibly better convergence
- avoid poles in sectors with large #prop
- prefer basis with simple, factorizing denom.

➡ reduced median runtime 15h → <2h

➡ reduced size of code for coefficients

➡ avoid spurious poles & cancellations



Phase-Space Integration

Evaluation of virtual amplitude very slow
→ good sampling of phase space required

Phase-space integration of virtual corrections:

- generate unweighted events based on differential LO cross section
  → nearly perfect importance sampling for evaluating total cross section
- for HJ: include additional p_T-dependent reweighing factor
  enhances number of events in tail of distribution, reducing their weight

Only $\mathcal{O}(1\,k)$ virtual amplitude results required

HJ and HH production at NLO QCD

Overview

- Introduction
- Details of calculation
- Results
  - HJ @ NLO
  - HH @ NLO

  … and beyond

HJ Results — Total cross section

- LHC @ 13 TeV
- $p_{T,j} > 30\,\mathrm{GeV}, \; R = 0.4$, anti-k$_\mathrm{T}$
- scale: $\frac{H_T}{2} = \frac{1}{2}\left(\sqrt{m_\mathrm{H}^2 + p_{t,\mathrm{H}}^2} + \sum_i |p_{t,i}|\right)$
- PDF4LHC15
- $m_H = 125\,\mathrm{GeV}$

  $m_t = \sqrt{23/12}\, m_H \approx 173.05\,\mathrm{GeV}$

FT$_\mathrm{approx}$:

$$\mathrm{d}\sigma_\mathrm{NLO}^{\mathrm{FT_{approx}}} = \int \mathrm{d}PS_2 \left(\mathrm{d}\sigma_\mathrm{B}^\mathrm{Full} + \frac{\mathrm{d}\sigma_\mathrm{B}^\mathrm{Full}}{\mathrm{d}\sigma_\mathrm{B}^\mathrm{HEFT}}\, \mathrm{d}\sigma_\mathrm{V}^\mathrm{HEFT} \right) + \int \mathrm{d}PS_3\, \mathrm{d}\sigma_\mathrm{R}^\mathrm{Full}$$

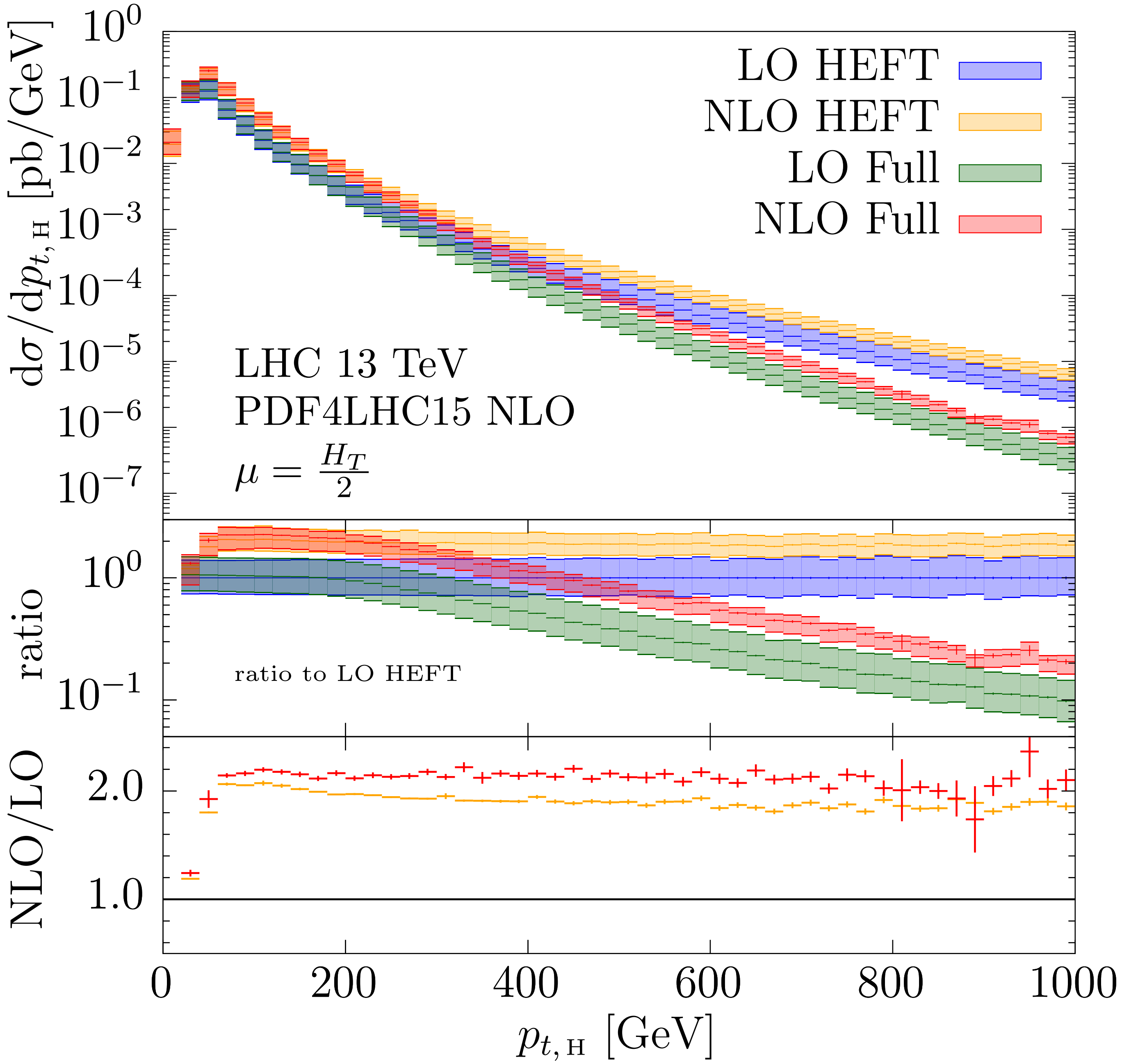
THEORY	LO [pb]	NLO [pb]
HEFT:	$\sigma_\mathrm{LO} = 8.22^{+3.17}_{-2.15}$	$\sigma_\mathrm{NLO} = 14.63^{+3.30}_{-2.54}$
FT$_\mathrm{approx}$:	$\sigma_\mathrm{LO} = 8.57^{+3.31}_{-2.24}$	$\sigma_\mathrm{NLO} = 15.07^{+2.89}_{-2.54}$
Full:	$\sigma_\mathrm{LO} = 8.57^{+3.31}_{-2.24}$	$\sigma_\mathrm{NLO} = 16.01^{+1.59}_{-3.73}$

+9%

+6%

HJ Results — p_T of Higgs boson

mass effects compared to HEFT



HEFT and full theory predict different scaling of $d\sigma/dp_T^2$

$\sim p_T^{-2}$ in HEFT

$\sim p_T^{-4}$ in full theory

[Caola, Forte, Marzani, Muselli, Vita, 15,16]

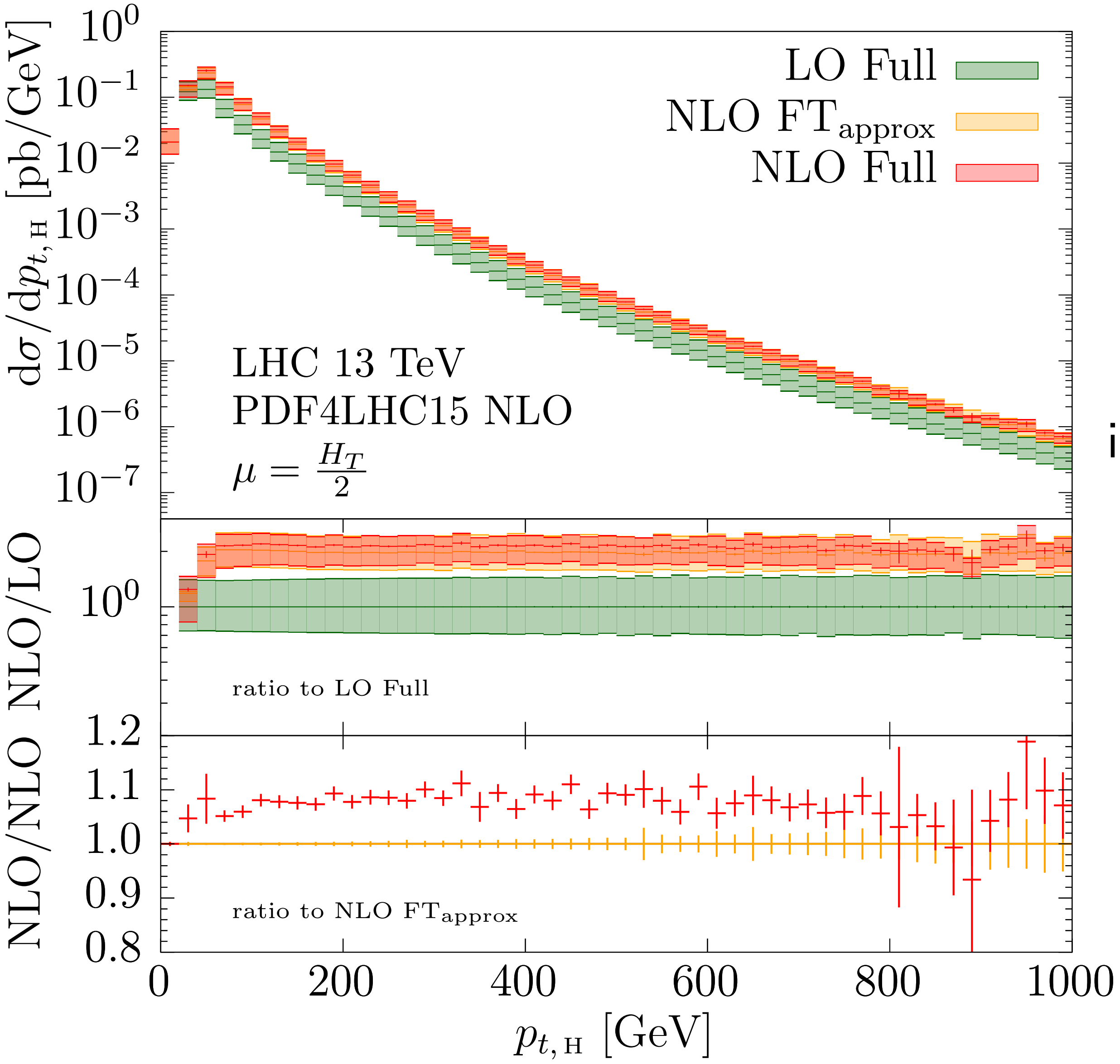
confirmed at NLO

nearly constant K-factor in full theory

HJ Results — p_T of Higgs boson

mass effects compared to FT_{approx}

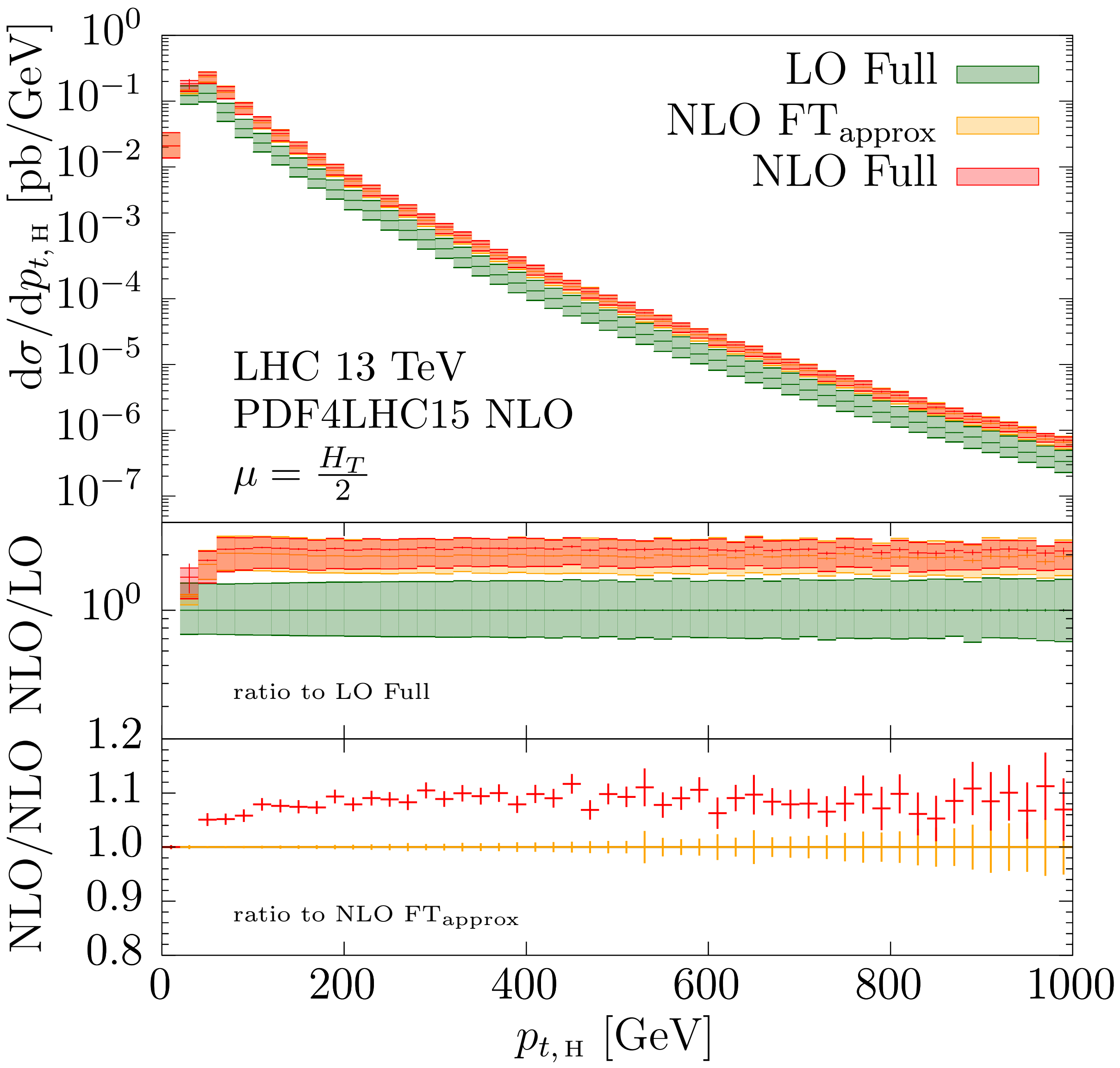
- full mt dependence in real radiation
- virtual correction in HEFT, rescaled by $\mathrm{B}(m_t)/\mathrm{B}(m_t \to \infty)$



improved

→

basis



- FT_{approx} and full theory predict same shape of p_T distribution
- nearly constant increase of ~8% due to top mass in virtual contribution

HJ Results — p_T of Higgs boson

comparison to small m_t expansion Kudashkin, Melnikov, Wever 17
Lindert, Kudashkin, Melnikov, Wever 18

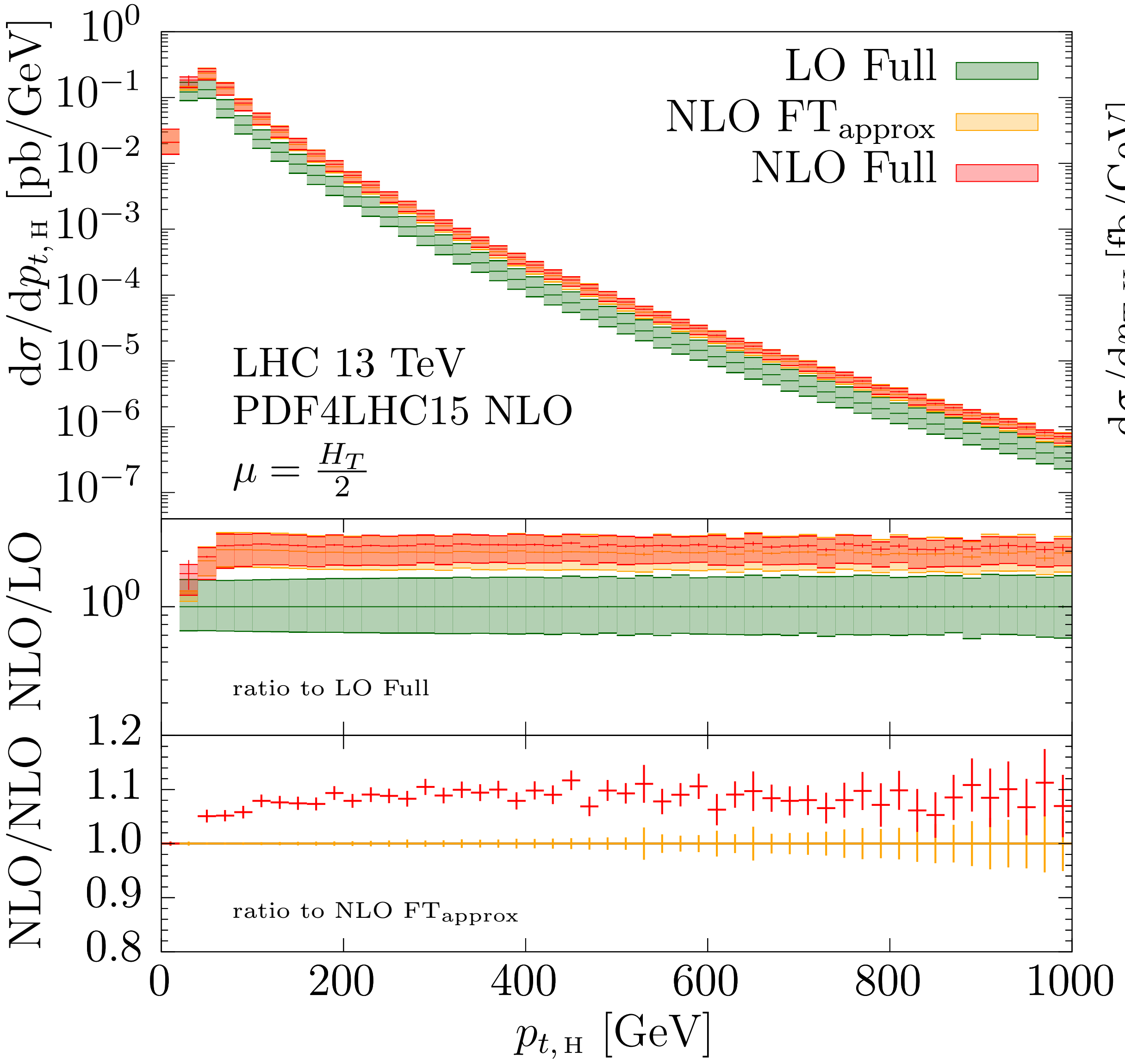
see also
Neumann 18

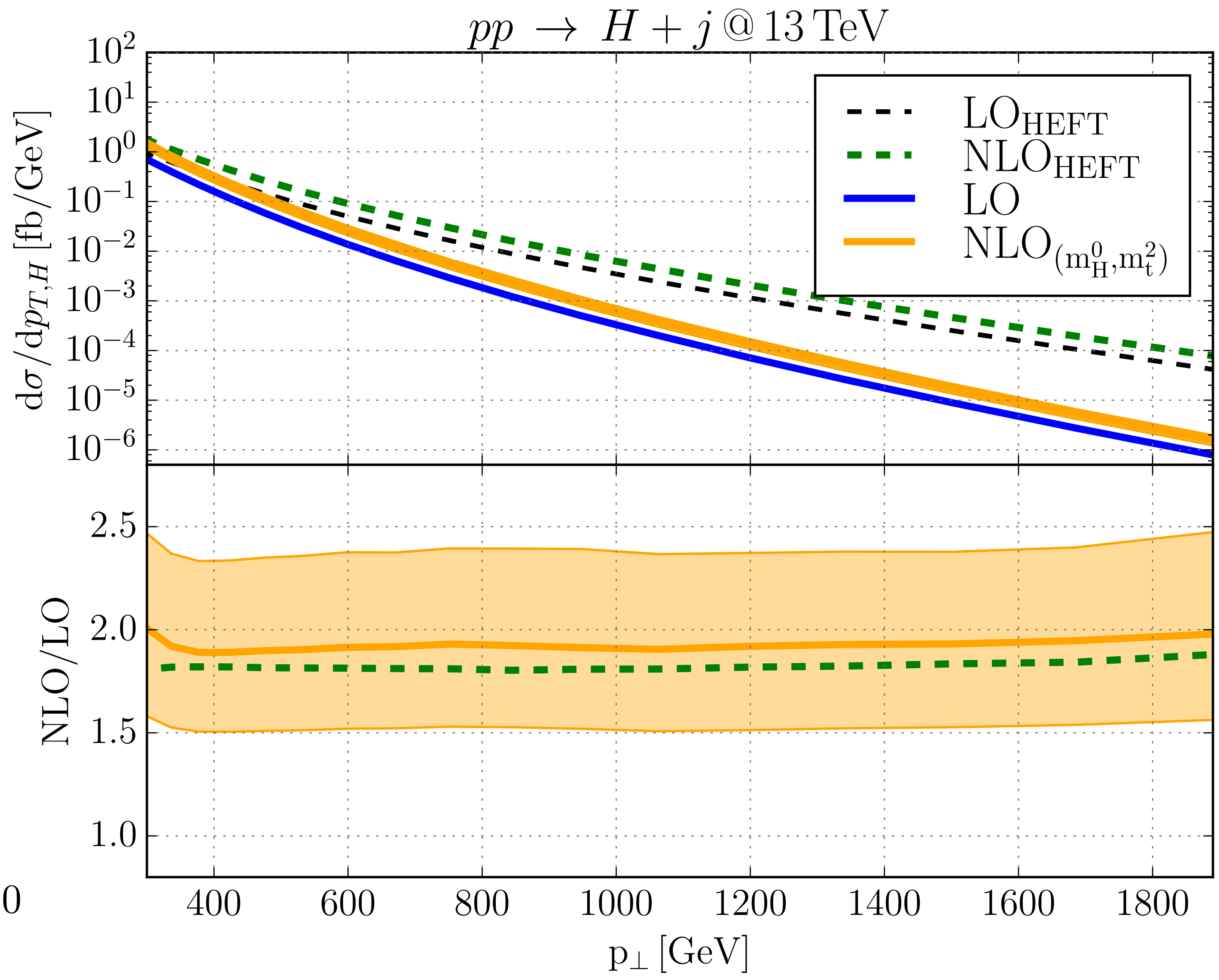
expansion in $\eta = -\frac{m_h^2}{4m_t^2}\,, \quad \kappa = -\frac{m_t^2}{s}$ to $\mathcal{O}(\eta^0\kappa^1)$

leads to $\frac{K^{SM}}{K^{FT_{\text{approx}}}} = 1.04\,..\,1.06$

minor difference to full result

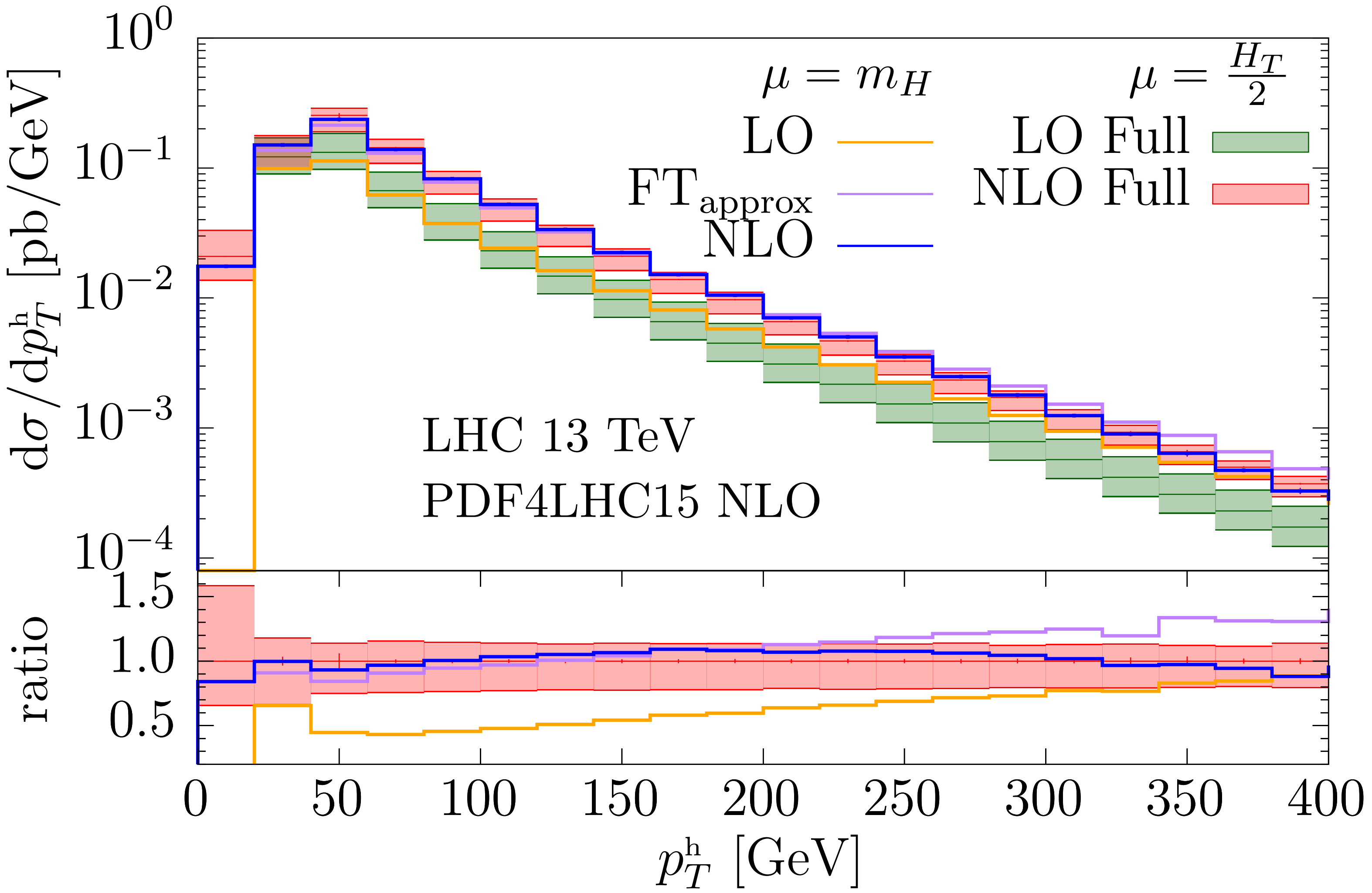
possibly due to missing $\mathcal{O}(\eta^1)$ terms





HJ Results — p_T of Higgs boson

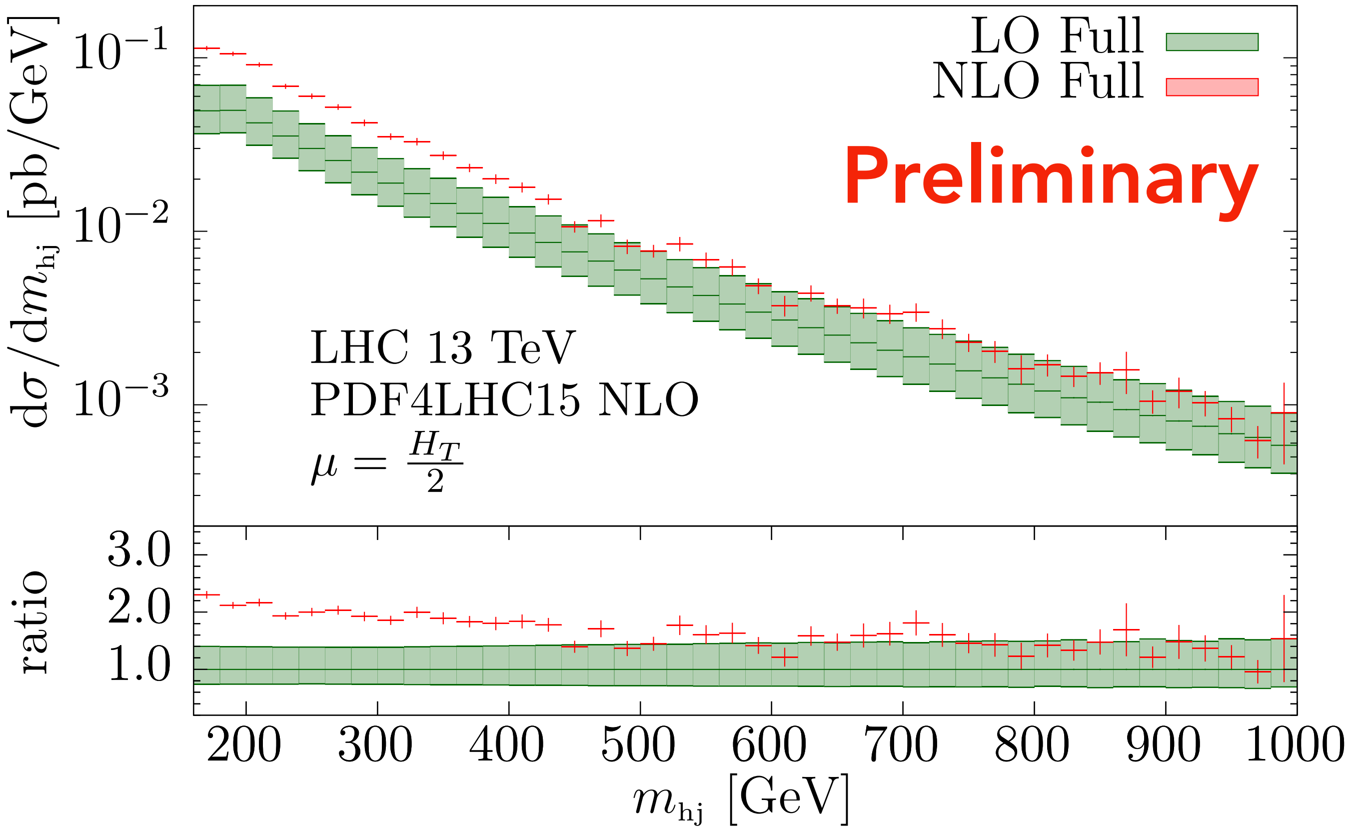
Results obtained using a fixed scale $\mu = m_H$



- NLO results in good agreement
- different shape of LO result → phase-space dependent K-factor
- $\mathrm{FT}_{\mathrm{approx}}$ overestimates cross section in tail

HJ Results

invariant mass m_{Hj} of Higgs jet system



- K-factor decreases for large invariant masses
- large cancellations of real and virtual corrections in tail
  → obtaining stable results challenging

HJ and HH production at NLO QCD

Overview

- Introduction
- Details of calculation
- Results
  - HJ @ NLO
  - HH @ NLO

  … and beyond

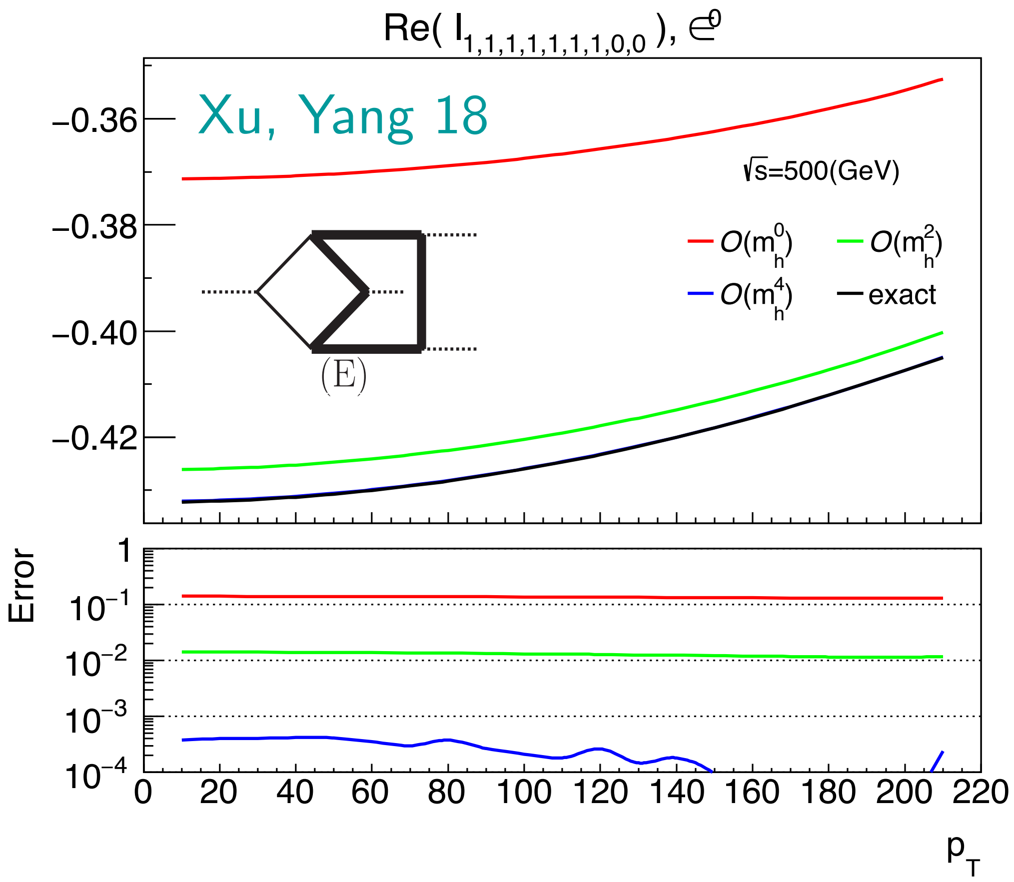
HH production — approximated results

approximated of 2-loop integrals:

- expansion in small m_t Davies, Mishima, Steinhauser 18

  so far only expansion of planar integrals
- expansion in small m_H Xu, Yang 18

  so far only individual integrals



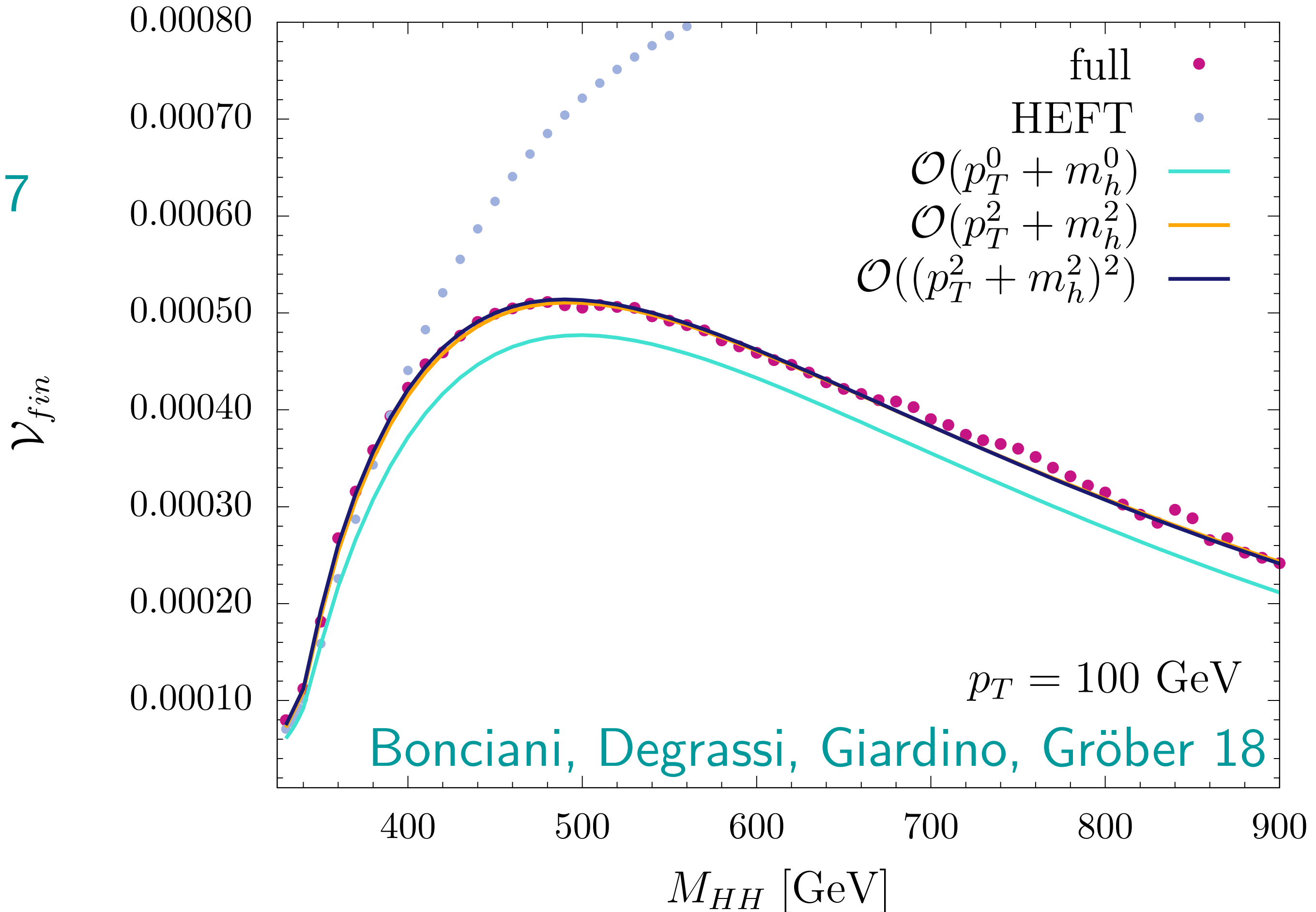
approximated of 2-loop amplitude:

- Padé approximation Gröber, Maier, Rauh 17

  includes heavy top expansion

  & threshold logarithms
- Expansion in $p_T^2 + m_H^2$

  Bonciani, Degrassi, Giardino, Gröber 18

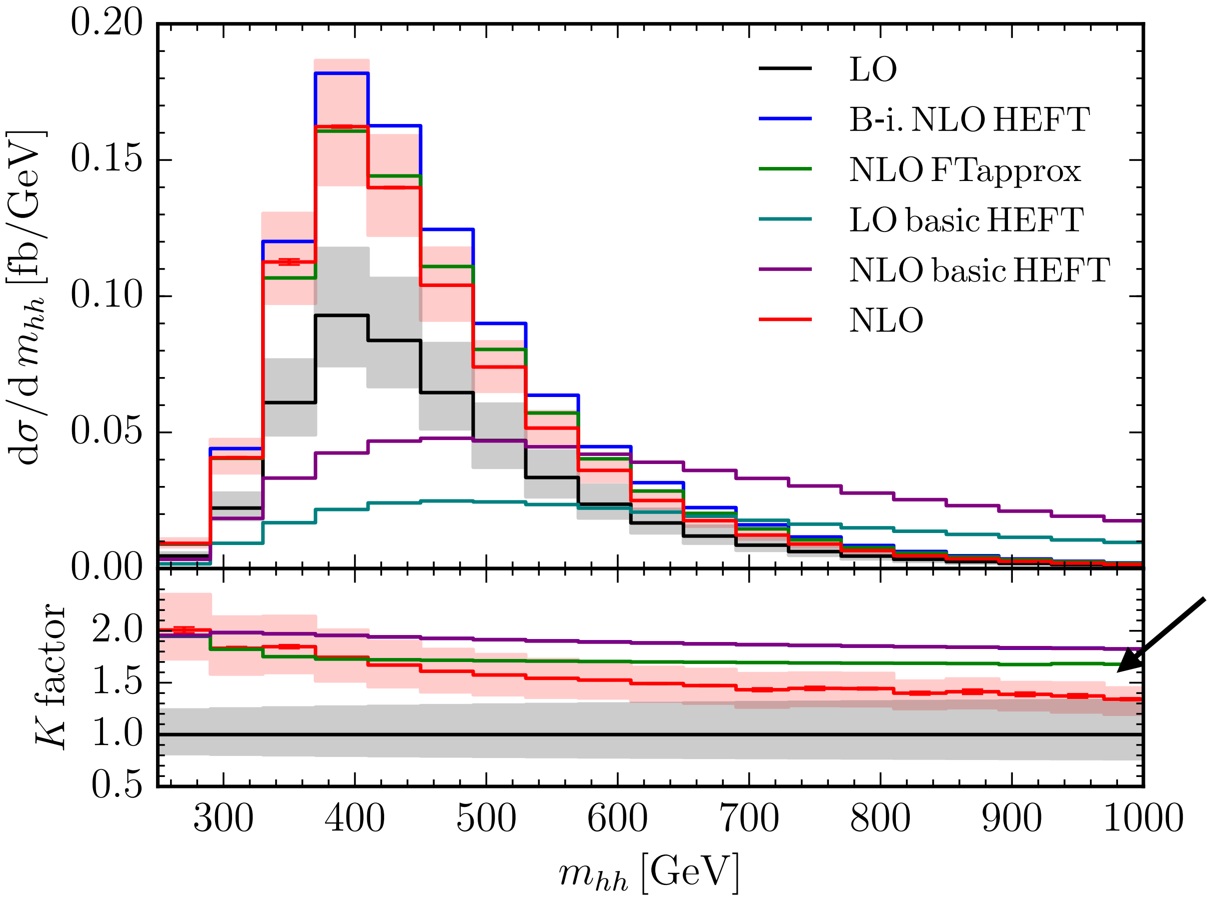


HH results — fixed order NLO

$\sqrt{s}$	LO	B-i. NLO HEFT	NLO FT$_{approx}$	NLO
14 TeV	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$34.26^{+14.7\%}_{-13.2\%}$	$32.91^{+13.6\%}_{-12.6\%}$
100 TeV	$731.3^{+20.9\%}_{-15.9\%}$	$1511^{+16.0\%}_{-13.0\%}$	$1220^{+11.9\%}_{-10.7\%}$	$1149^{+10.8\%}_{-10.0\%}$

-14% wrt. NLO HEFT

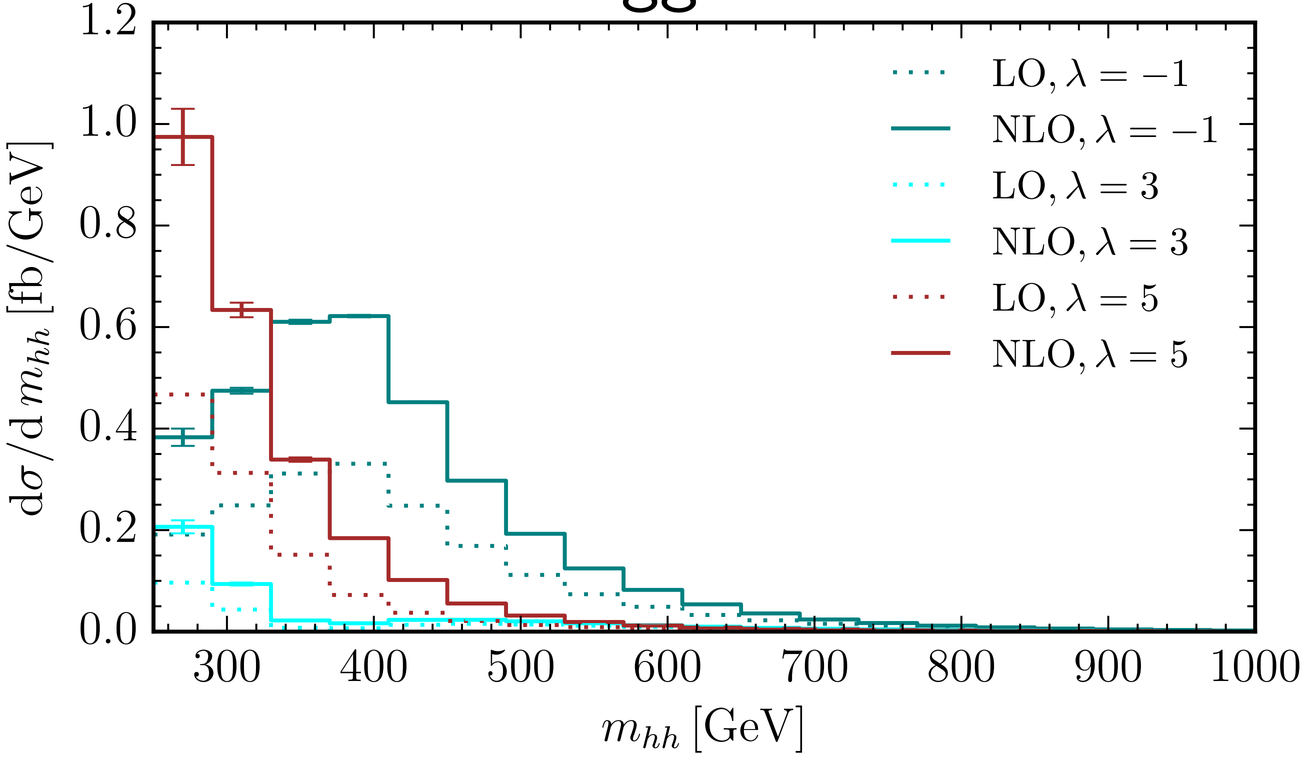
-4% wrt. NLO FT$_{\text{approx}}$



large dependence of K-factor on m_{hh}

large top mass effects in high m_{hh} region

modified Higgs self-interaction



Grid interpolation (so far only HH)

Calculation of fixed order results:

1. generate unweighted LO events
2. evaluate virtual amplitude at these points
3. obtain histogram of virtual contribution
4. add real radiation (at histogram level)

Problems:

- slow (2h GPU time per phase-space point)
- impractical for
  - combining with parton showers, etc.
  - providing results to other groups

→ provide results of virtual amplitude together with grid interpolation framework

- use pre-computed amplitude results as input
- obtain interpolated amplitude result for arbitrary phase-space points
- fast & can be interfaced to other codes

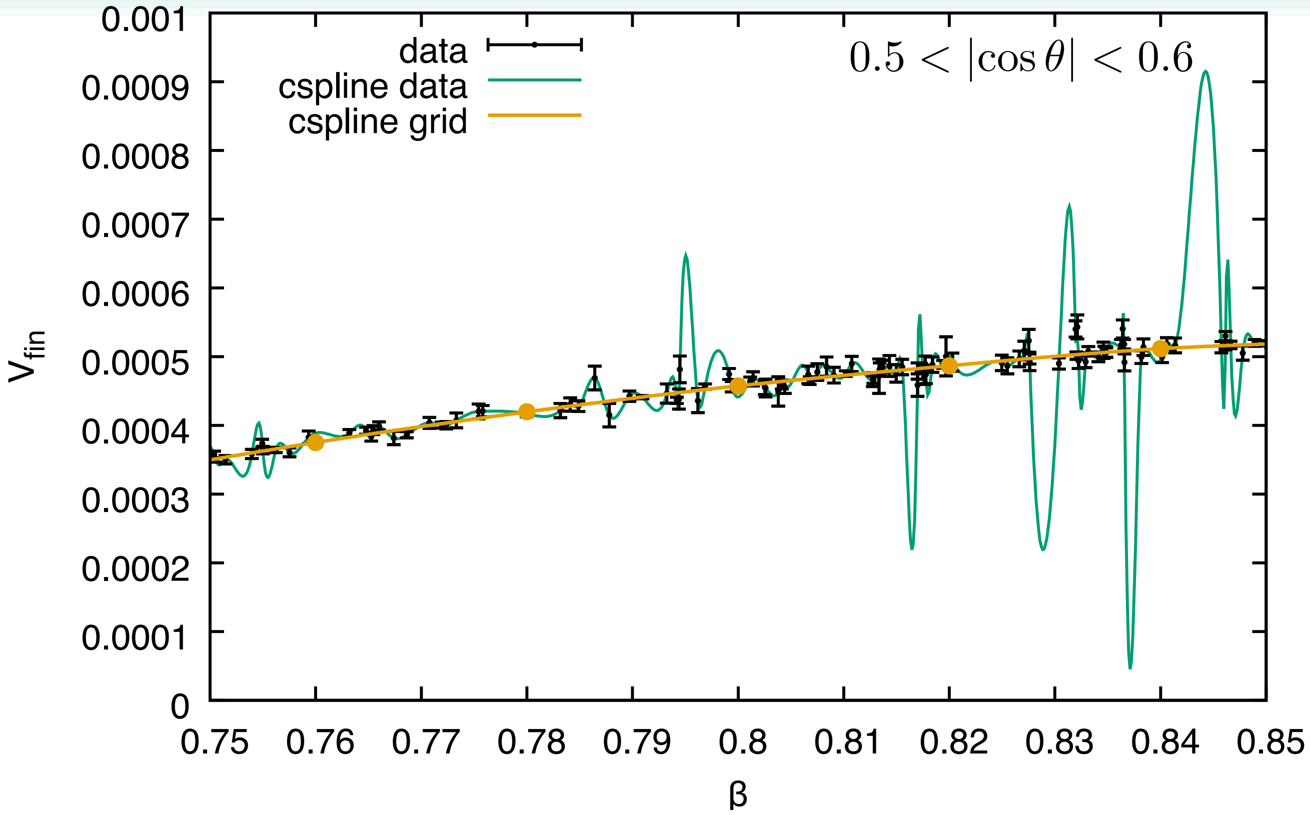
available at github.com/mppmu/hhgrid

Grid interpolation details (so far only HH)

2-dimensional grid interpolation $(\hat{s},\, \hat{t})$

Problems during construction of grid:

- interpolation can enhance numerical uncertainties
- input data not evaluated on equidistant grid points



Details of grid interpolation:

- input parameters $x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left|\frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})}\right|,$ with $\beta = \left(1 - \frac{4m_H^2}{\hat{s}}\right)^{\frac{1}{2}}$

  → nearly uniform distribution of phase space points in $(x, c_\theta) \in [0,1]^2$ if $f(\beta)$ chosen according to cumulative distribution of points in original calculation
- interpolation done in 2 steps:
  1. choose equidistant grid points, estimate result at each grid point with least-square fit to linear function of amplitude results in vicinity
  2. Clough-Tocher interpolation (as implemented in SciPy) to estimate amplitude at arbitrary sampling points

  → reduces sensitivity to uncertainties of input-data points

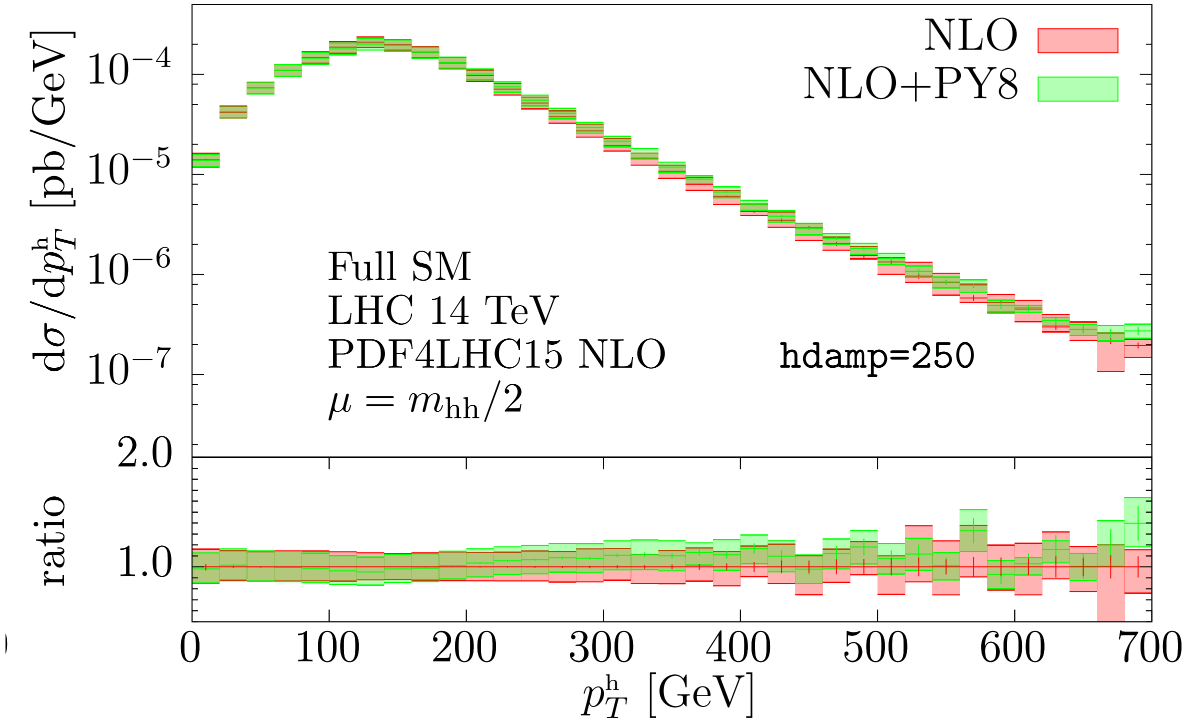
HH Results — Parton Shower

combination with parton shower

$\rightarrow$ publicly available in PowhegBox-V2

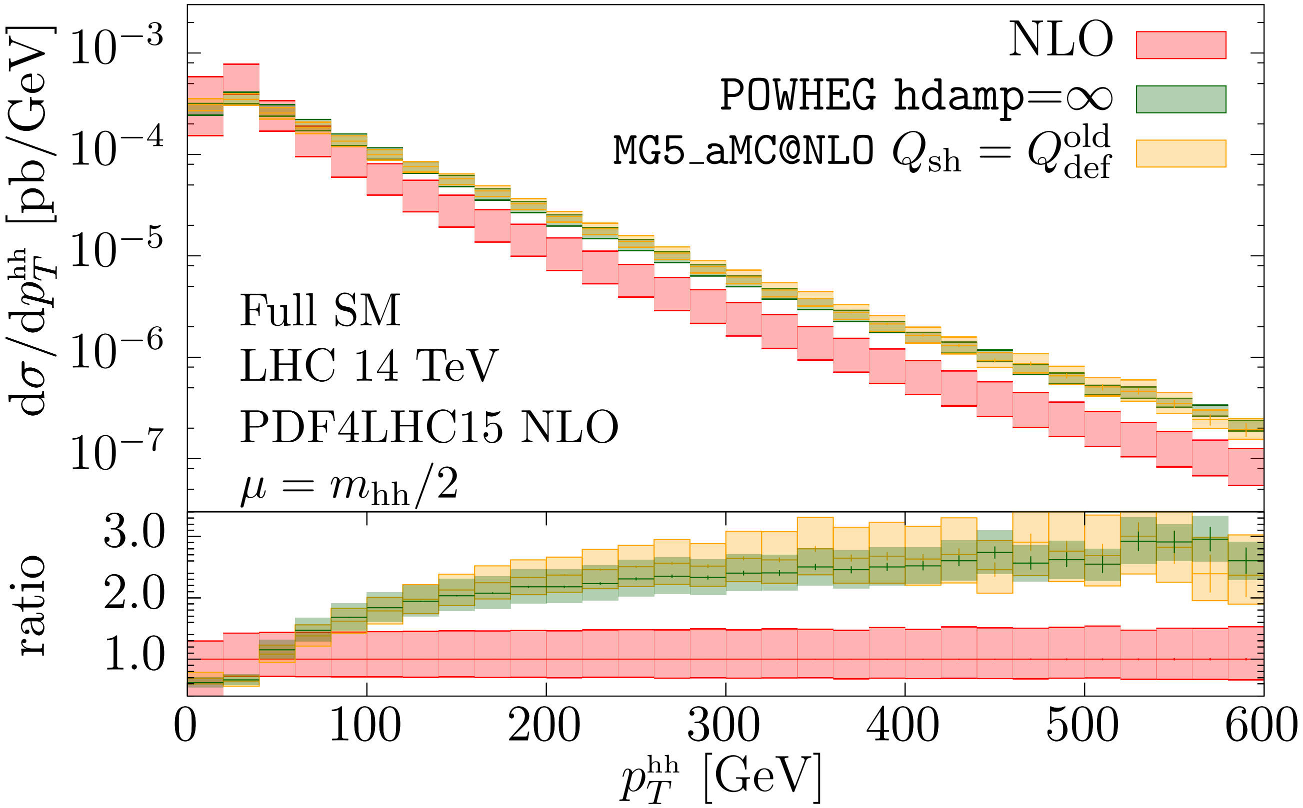
Heinrich, Jones, MK, Luisoni, Vryonidou 17

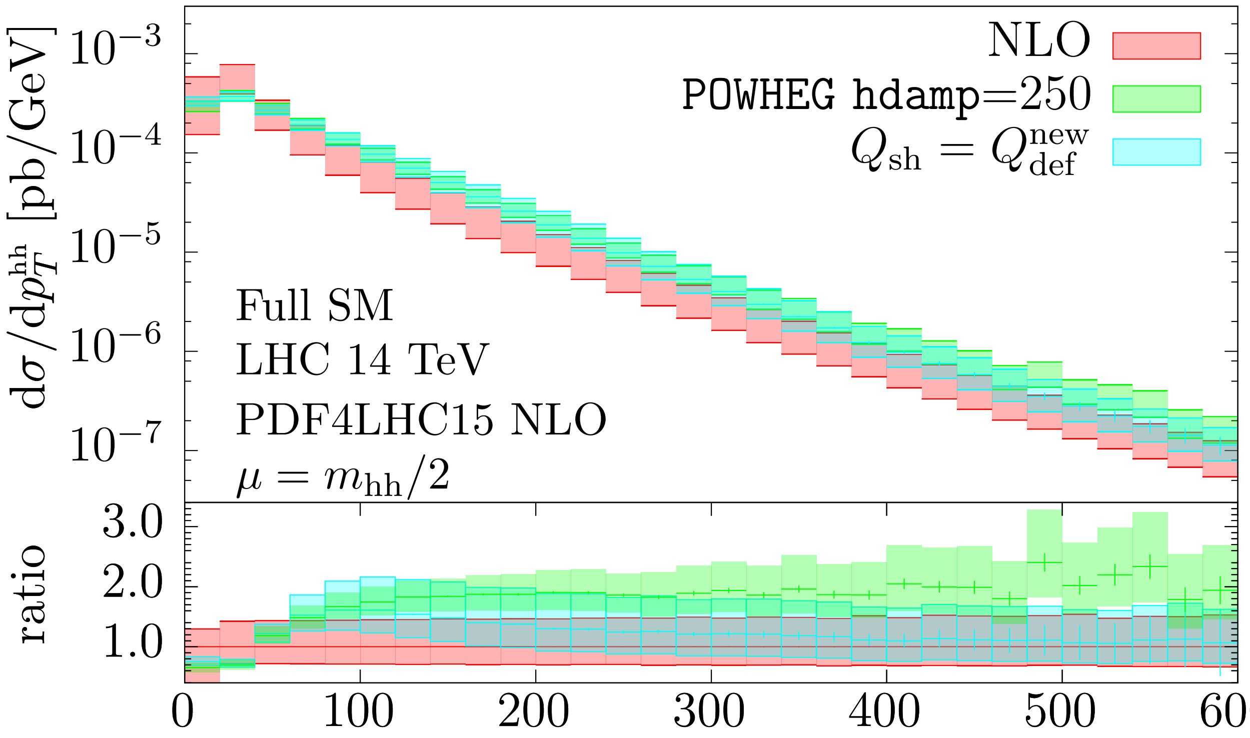
see also Jones, Kuttimalai 17



$\rightarrow$ small effects for NLO accurate observables

large dependence of p_T^{hh} on shower parameters:





HH — NNLO

Grazzini, Heinrich, Jones, Kallweit, MK, Lindert, Mazzitelli 18

combination with NNLO $(m_t \to \infty)$

$\to$ approx. m_t dependence at NNLO

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\mathrm{NLO_{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
$\mathrm{NNLO_{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$\mathrm{NNLO_{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$\mathrm{NNLO_{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. $\mathrm{NNLO_{FTapprox}}$	±2.6%	±2.7%	±3.4%	±4.6%
$\mathrm{NNLO_{FTapprox}}$/NLO	1.118	1.116	1.096	1.067

3 different methods:

1) $\mathrm{NNLO_{NLO\text{-}i}}$

rescale NLO by $K_{\mathrm{NNLO}} = \mathrm{NNLO_{HEFT}}/\mathrm{NLO_{HEFT}}$

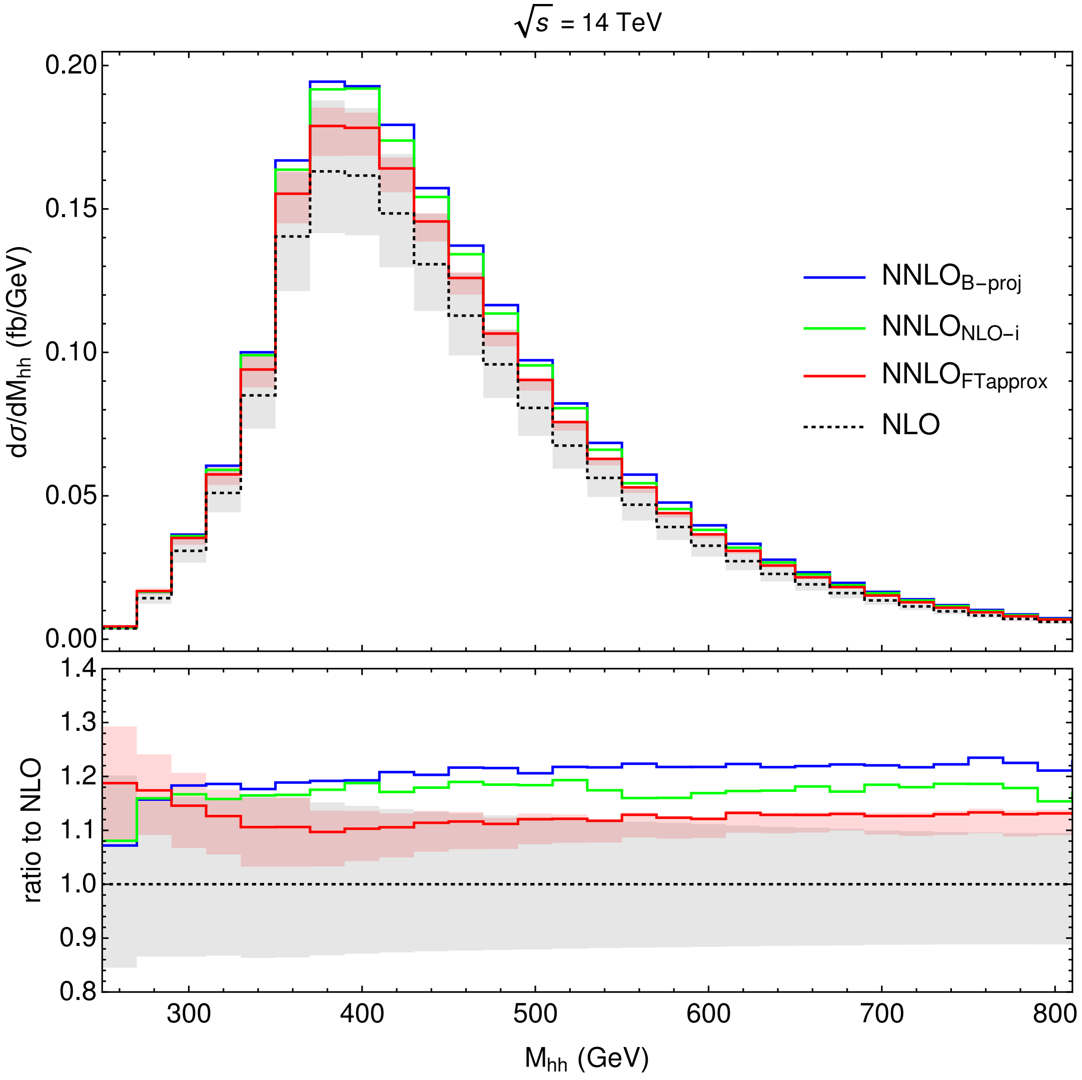
2) $\mathrm{NNLO_{B\text{-}proj}}$

project all real radiation contributions to Born configuration, rescale by $\mathrm{LO}/\mathrm{LO_{HEFT}}$

3) $\mathrm{NNLO_{FTapprox}}$

calculate $\mathrm{NNLO_{HEFT}}$ and for each multiplicity rescale by

$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}^{\mathrm{Born}}_{\mathrm{Full}}(ij \to HH + X)}{\mathcal{A}^{(0)}_{\mathrm{HEFT}}(ij \to HH + X)}$$



Conclusion

HH and HJ production at NLO QCD with full m_t-dependence

- calculation using numerical approach
  - based on sector decomposition
  - viable alternative, if analytic results not available
  - results can be obtained without full reduction
  - numerical integration slow, but grid interpolation can be used for fast evaluation of virtual amplitude
- HH production
  - top mass effects decrease cross section by ~4% compared to $\mathrm{FT}_{\mathrm{approx}}$
  - size of corrections increase for large m_{HH}
  - results beyond fixed order NLO available:
    - interface to parton shower
    - combination with $\mathrm{NNLO}_{\mathrm{HEFT}}$
- HJ production
  - top mass effects increase cross section by ~6% compared to $\mathrm{FT}_{\mathrm{approx}}$
  - work in progress: grid generation

    → combination with parton shower & $\mathrm{NNLO}_{\mathrm{HEFT}}$