The QCD Axion and Unification

Clara Murgui

IFIC, Universitat de València-CSIC

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in collaboration with Pavel Fileviez Pérez and Alexis Plascencia,
CWRU, Cleveland, OH, USA

8 October 2019, University of Zurich

Motivation

- Strong CP problem

$$\mathcal{L}_{\cancel{CP}}(\theta_Y) = \underbrace{-\,\overline{q_L}\,\mathcal{M}\,e^{i\theta_Y}\,q_R + \text{h.c.}}_{\mathcal{L}_Y}$$

$$\left.\begin{aligned} q_L &\to e^{i\alpha/2}\; q_L \\ q_R &\to e^{-i\alpha/2} q_R \end{aligned}\right\}$$

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$$\mathcal{L}_{\cancel{CP}}(\theta_Y, \theta_{\mathrm{QCD}}) = -\underbrace{\overline{q_L}\,\mathcal{M}\,e^{i\theta_Y}\,q_R + \text{h.c.}}_{\mathcal{L}_Y} - \theta_{\mathrm{QCD}}\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

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$$\left.\begin{aligned} q_L &\to e^{i\alpha/2}\,q_L \\ q_R &\to e^{-i\alpha/2}\,q_R \end{aligned}\right\} \quad \cancel{U(1)}_A \quad \left\{\begin{array}{l} \text{Quark masses} \\ \text{Color anomaly} \end{array}\right.$$

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Quark mass matrix ⇄ Topological term

Adler, Bell, Jackiw, 1969
Fujikawa, 1979-1980

$$\mathcal{L}_{CP}(\theta_Y, \theta_{\rm QCD}) \quad \leftrightarrow \quad \mathcal{L}_{CP}(\theta_Y - \alpha, \theta_{\rm QCD} + N_i\alpha)$$

$$N = \sum_{\substack{\text{colored} \\ \text{chiral} \\ \text{fields}}} C_{\Psi^i_L} T_D[R_{\Psi^i_L}] \times \text{mult}[\Psi^i_L]$$

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$$\mathcal{L}_{\cancel{CP}}(\bar{\theta}) = -\frac{g_s^2}{32\pi^2}\underbrace{\left(\arg(\mathrm{Det}\, M) + \theta_{\mathrm{QCD}}\right)}_{\bar{\theta}} G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

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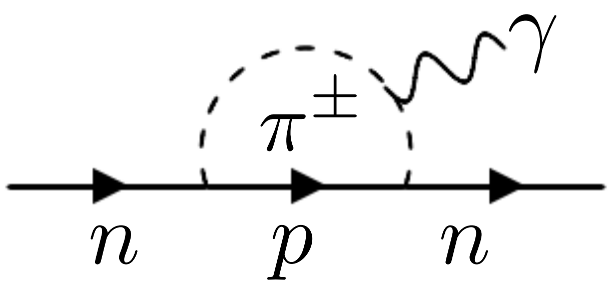
$$\Big\downarrow < \Lambda_{\mathrm{QCD}}$$

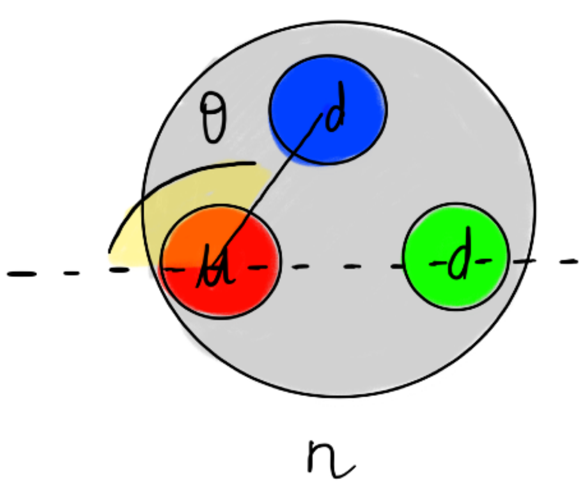
$$\boxed{\chi PT}$$



$$d_n = \mathcal{O}(1)\overline{\theta} \times 10^3 \text{ e fm}$$

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4\, m_u\, m_d}{(m_u + m_d)^2}\sin^2\left(\frac{\bar\theta}{2}\right)}$$

Crewther, Vecchia, Veneziano, Witten, 1979



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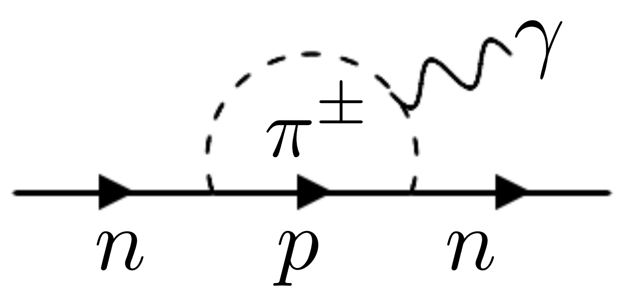

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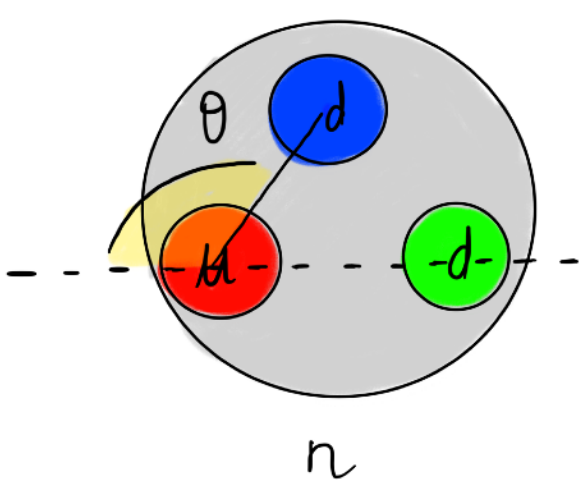
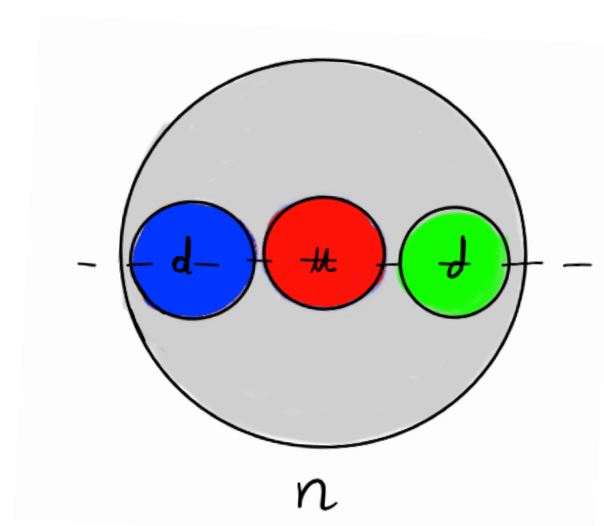
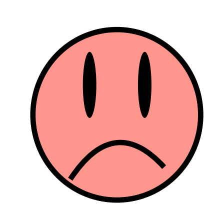


but



$$|d_n|^{\mathrm{exp}} < 3.0 \times 10^{-13} \text{ e fm}$$

$$\Rightarrow \bar\theta < 10^{-10}$$



Motivation

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$$\cancel{U(1)_A} \equiv U(1)_{PQ} \qquad \mathcal{L}_Y = \bar{q_L}\mathcal{M}e^{i\frac{a}{f_a}}q_R + \text{h.c.}$$

$$\mathcal{L}_{\cancel{CP}}\left(\bar{\theta} + \frac{a}{f_a}\right) = -\frac{g_s^2}{32\pi^2}\left(\bar{\theta} + \frac{a}{f_a}\right) G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a + \partial_\mu a \text{ couplings}$$

Peccei-Quinn, 1977

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Peccei-Quinn, 1977

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1-\frac{4\,m_u\,m_d}{(m_u+m_d)^2}\sin^2\left[\frac{1}{2}\left(\bar{\theta}+\frac{a}{f_a}\right)\right]}$$

$$\langle a\rangle = -\bar{\theta} f_a \qquad d_n \propto \frac{a}{f_a}+\bar{\theta} = 0$$

Strong CP problem solved dynamically!

Shift: $a \to a - \bar{\theta} f_a$

Wilczek, 1978
Weinberg, 1978

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$$\langle a \rangle = -\bar\theta f_a \quad \Rightarrow \quad d_n \propto \frac{a}{f_a} + \bar\theta = 0 \quad \Rightarrow$$

Strong CP problem solved dynamically!

Pseudo-Nambu-Goldstone boson: AXION

Wilczek, 1978
Weinberg, 1978

Axions

- Hypothetical pseudoscalar particle:

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}a\, g_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a}(c_q\bar{q}\gamma^\mu\gamma_5 q)$$

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- Axion mass:

$$V(a) \Rightarrow m_a = 5.70(6)(4)\,\mu\text{eV}\left(\frac{10^{12}\text{ GeV}}{f_a}\right)$$

Grilli di Cortona et al., 2016

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- Axion origin:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes \underbrace{U(1)_A}_{\text{global}}$$

Peccei-Quinn (PQ) symmetry

Axion: Pseudo-Nambu-Goldstone boson

Original PQ Axion

Peccei, Quinn, Weinberg, Wilczek

- Relevant Lagrangian:

$$\mathcal{L}_Y \supset Y_u \overline{Q_L}\, \tilde{H}_u\, u_R + Y_d\, \overline{Q_L}\, H_d\, d_R + \text{h.c.}$$

- PQ symmetry:

$$H_u \to e^{iX_u} H_u \qquad u_R \to e^{iX_u} u_R$$
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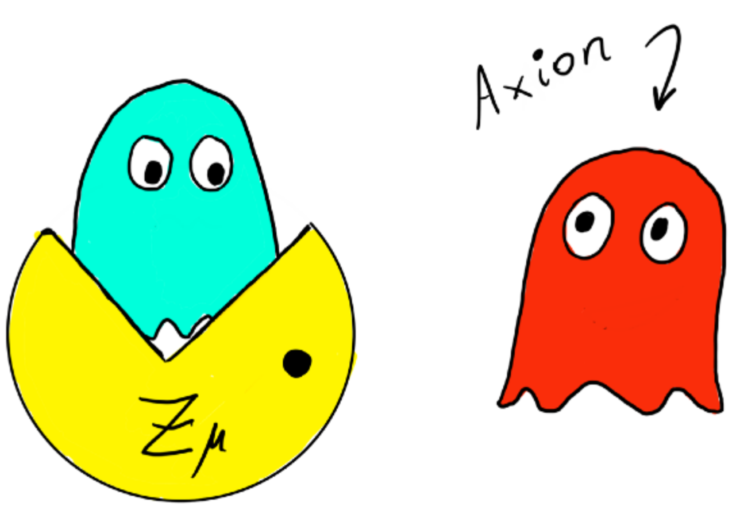
- Scalar potential:

$$V \overset{\text{PQ}}{=} V(H_u^\dagger H_u, H_d^\dagger H_d)$$

$$2\ \text{Goldstones} \begin{cases} 1\ \text{eaten by } Z_\mu \\ 1\ \text{axion} \end{cases}$$



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$$V \overset{\text{SSB}}{\rightarrow} \quad H_u \propto e^{i \frac{v_d}{v_u} \frac{a}{v}}, \quad H_d \propto e^{-i \frac{v_u}{v_d} \frac{a}{v}}$$

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- PQ rotation on fermions:

$$\mathcal{L} \supset \frac{a}{v}\frac{g_s^2}{32\pi^2} N G\tilde{G}, \quad N = \frac{1}{2}\left(\frac{v_d}{v_u} + \frac{v_u}{v_d}\right)$$

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- Experimental bounds:

$$\overbrace{3 \times 10^{-5}\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right)^2}^{\text{Br}(K^+ \to \pi^+ + a)} \gg \overset{\text{KEK bound}}{\overbrace{3.8 \times 10^{-8}}^{\text{Br}(K^+ \to \pi^+ + \text{nothing})}}$$

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$f_a \gg v$ → Invisible axion models

Invisible Axions: DFSZ

Dine, Fischler, Srednicki, 1981

Zhitnitsky, 1980

"Simple generalisation of the PQ scheme with a harmless axion"

$$\mathcal{L}_Y \supset Y_u \overline{Q_L}\, \tilde{H}_u\, u_R + Y_d\, \overline{Q_L}\, H_d\, d_R + \text{h.c.}$$

$S \sim (1,1,0)$

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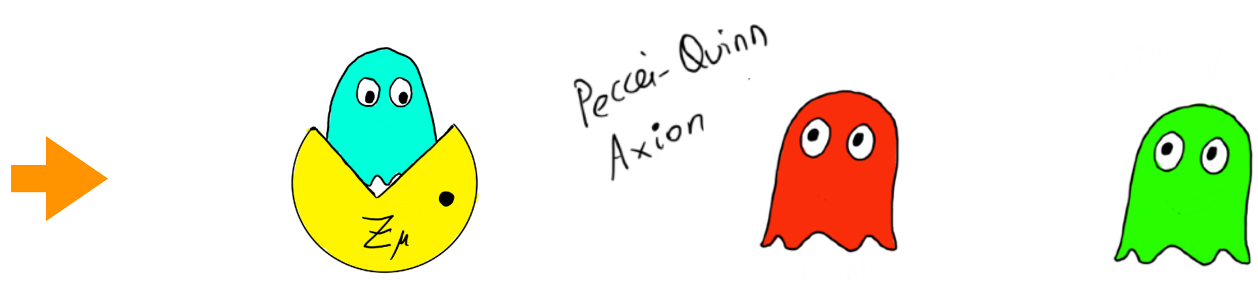

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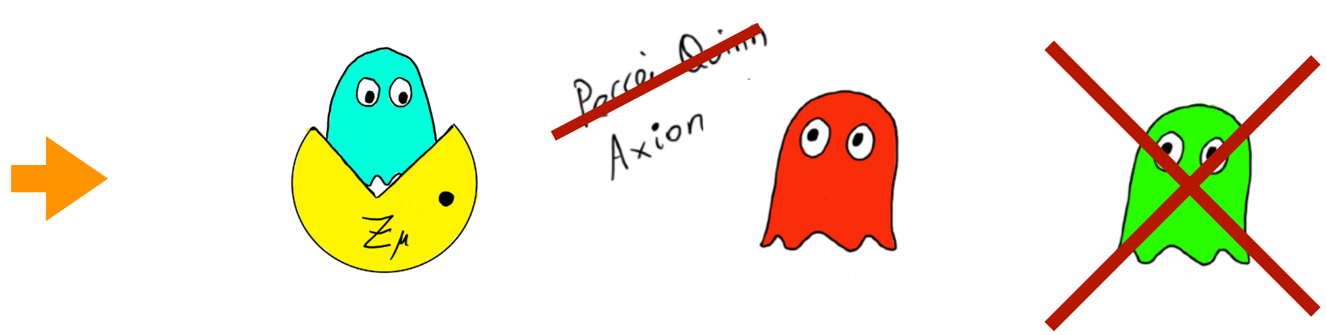

$$V \supset \lambda H_d^\dagger H_u S^2 + \text{h.c.} \quad \Rightarrow X_u + X_d + 2X_S = 0$$

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$$f_a \equiv v_S \gg v$$

Invisible Axions: KSVZ

Kim, 1979

Shifman, Vainshtein, Zakharov, 1980

- "The $U(1)_A$ is spontaneously broken and is realized by the existence of the axion a that does not couple to ordinary quarks at tree level."

$$\mathcal{L}_Y \supset \mathcal{L}_Y^{\text{SM}} + Y_q\,\overline{q_L}\,q_R\,S + M_q\,\overline{q_L}\,q_R + \text{h.c.}$$

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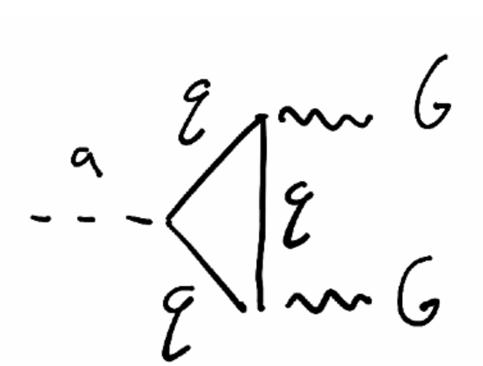
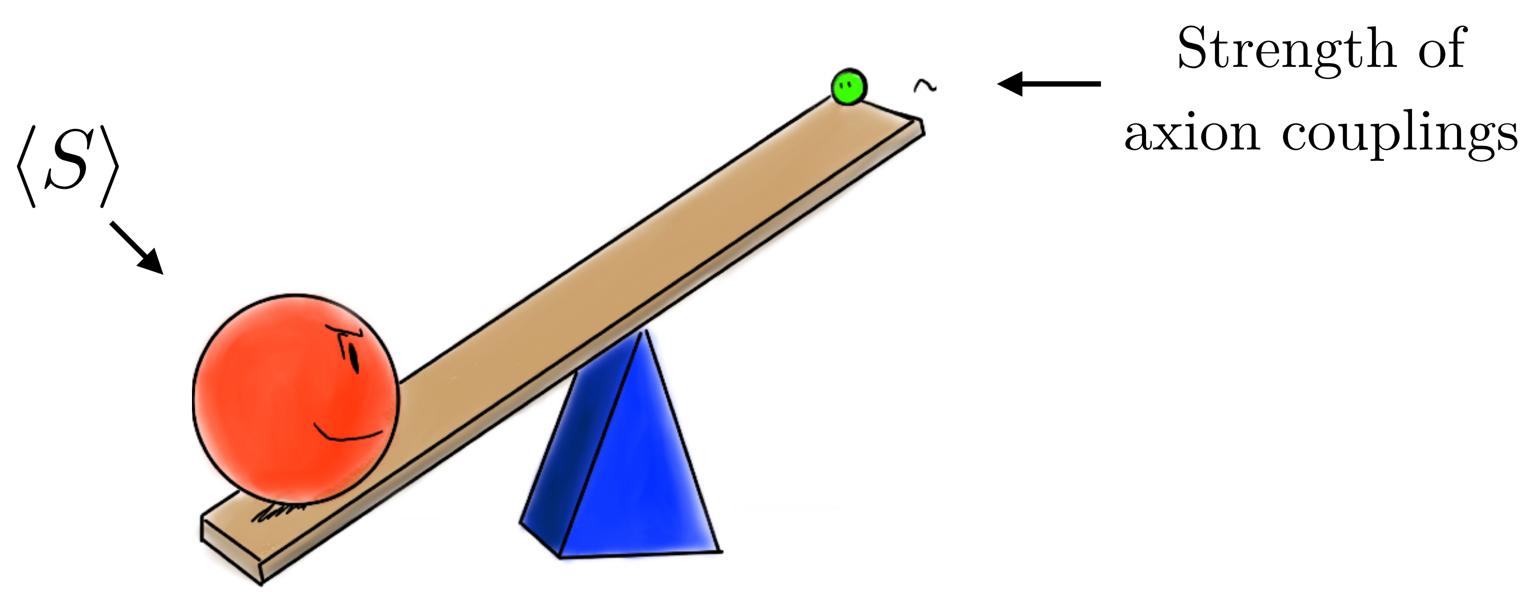


$$\Rightarrow \frac{a}{v_S}\, \underbrace{2\, T_D[q]}_{N}\, \frac{g_S^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu},$$

$$f_a \equiv v_S \gg v$$

Motivation

"The only requirement is the existence of an anomalous U(1) symmetry broken at a large energy scale"



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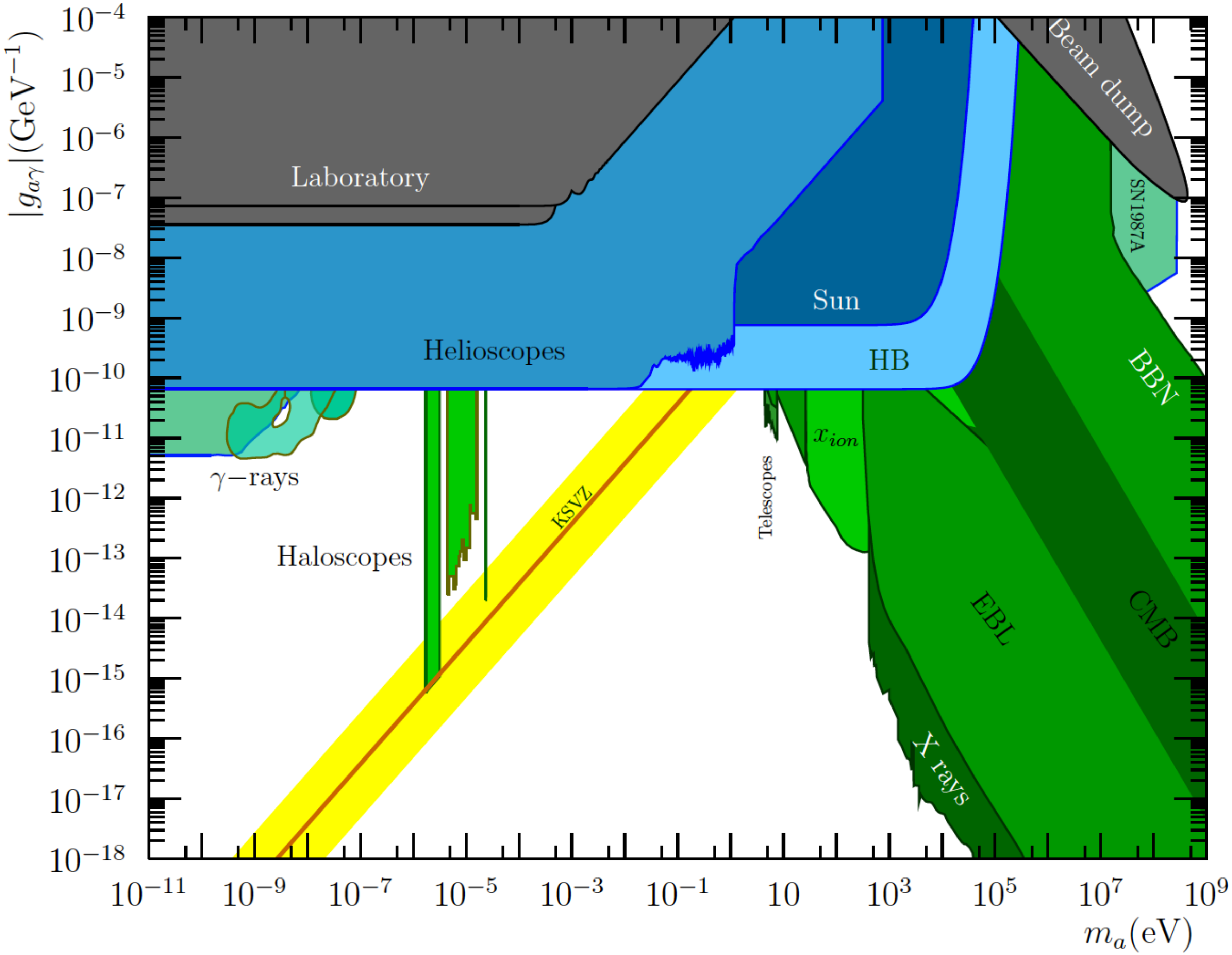

Irastorza, Redondo, 2018

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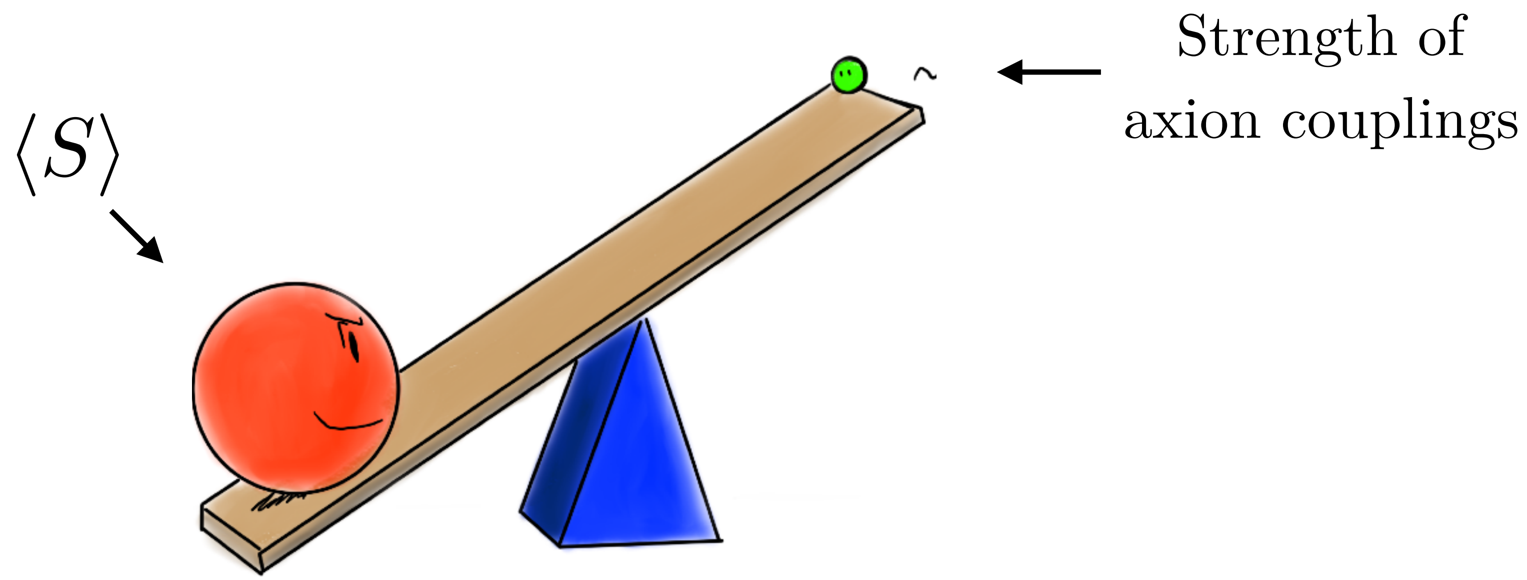
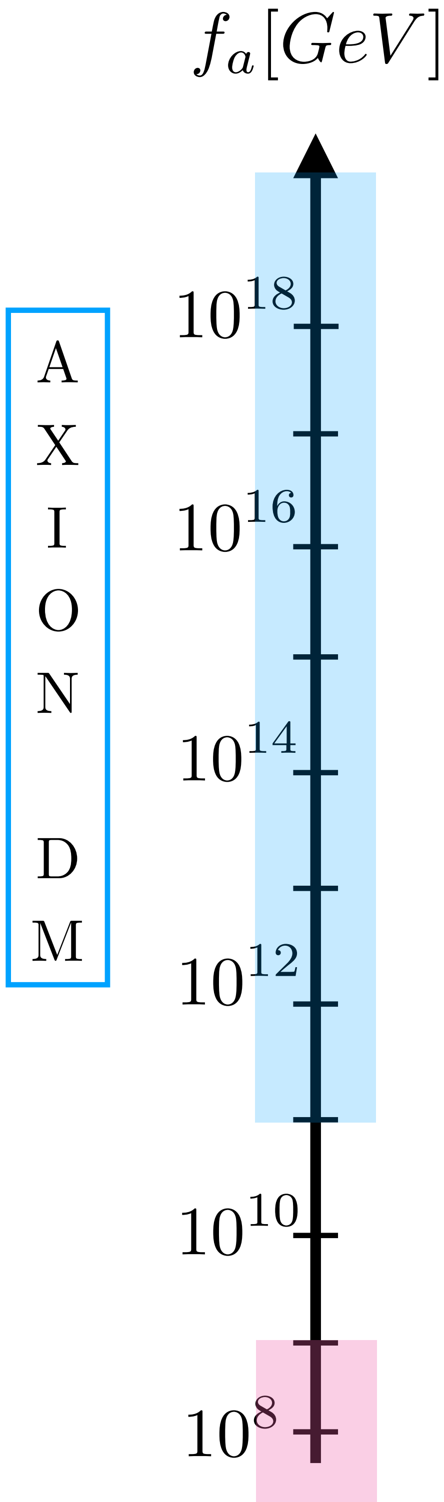


- Astrophysical constraints: $f_a \gtrsim 10^7$ GeV

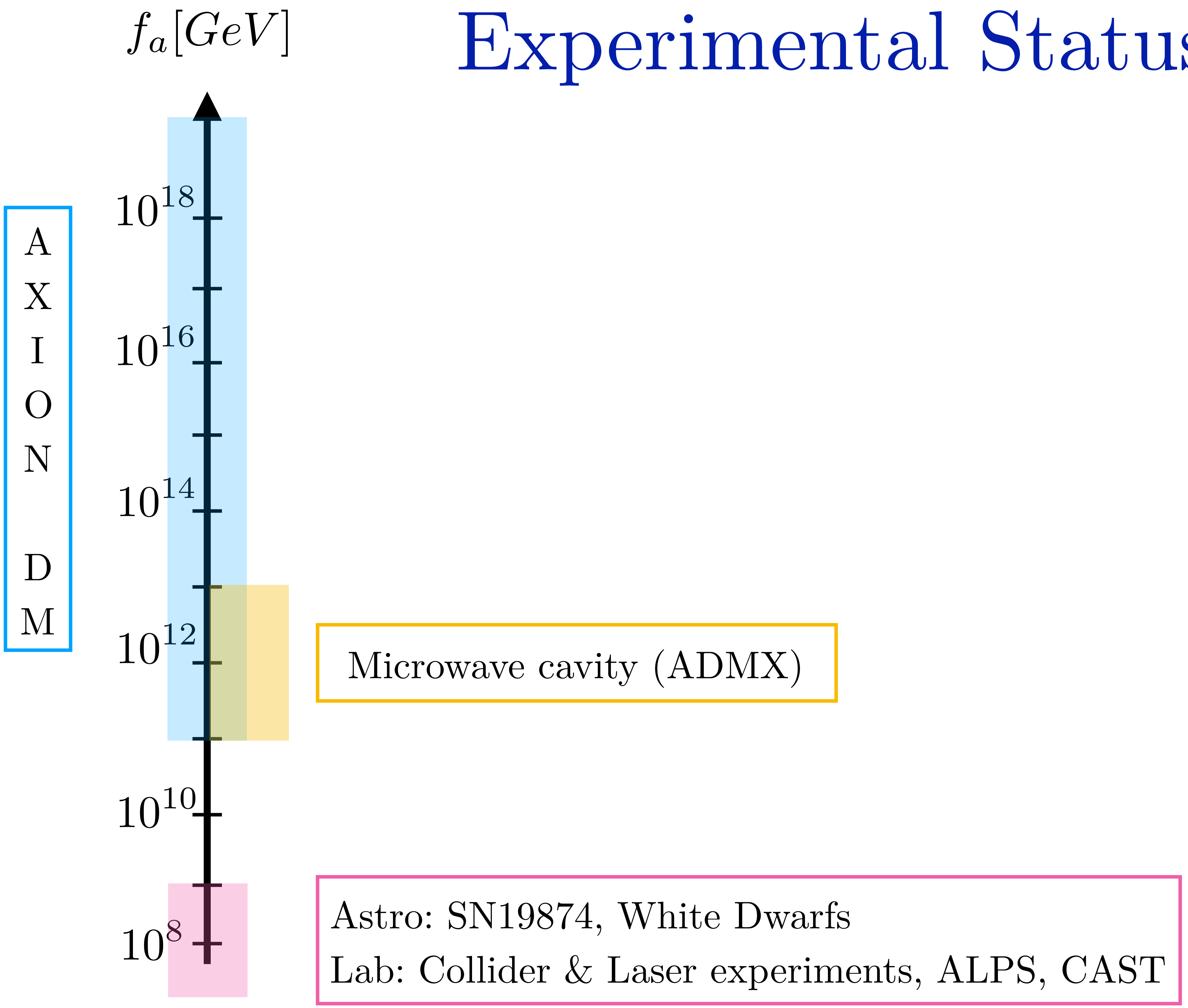
Experimental Status



$$f_a^{\mathrm{DM}} \gtrsim 10^{11} - 10^{12}\ \mathrm{GeV}$$

Astro: SN19874, White Dwarfs
Lab: Collider & Laser experiments, ALPS, CAST

Experimental Status



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- Axion halosopes: exploit coherent effects to detect axion DM

Axions highly non-relativistic ➡ monochromatic photons

$$\frac{\Delta\omega}{\omega} \sim \sigma_v^2 \sim 10^{-6}$$

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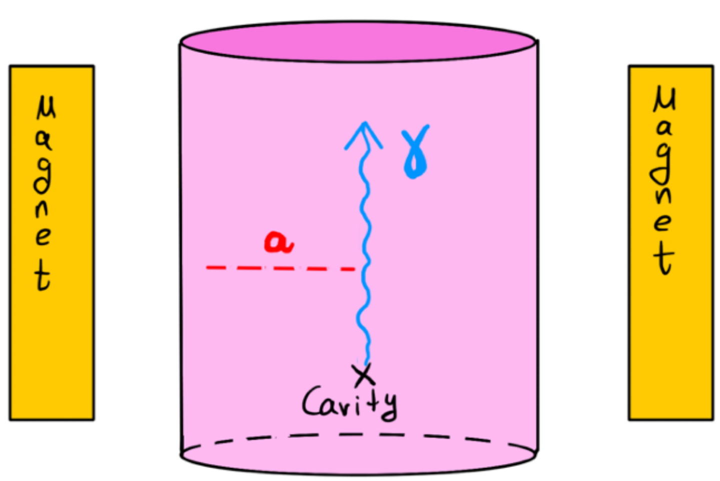
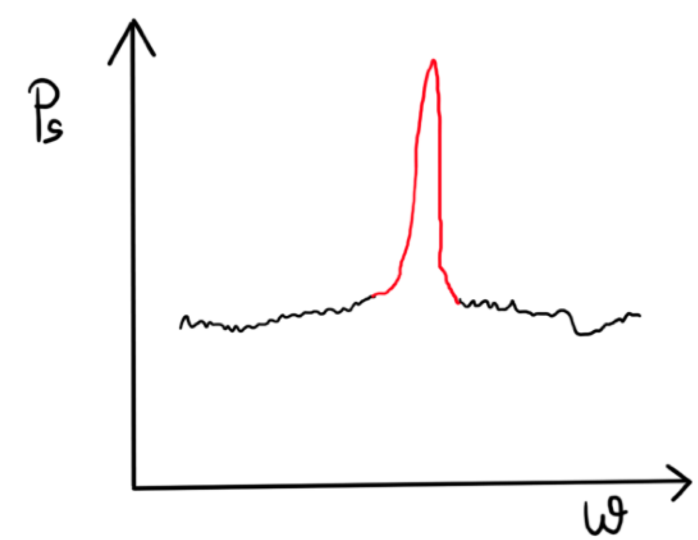
High Q microwave cavities



If $m_a \subset \omega_c \pm \frac{\omega_c}{Q}$

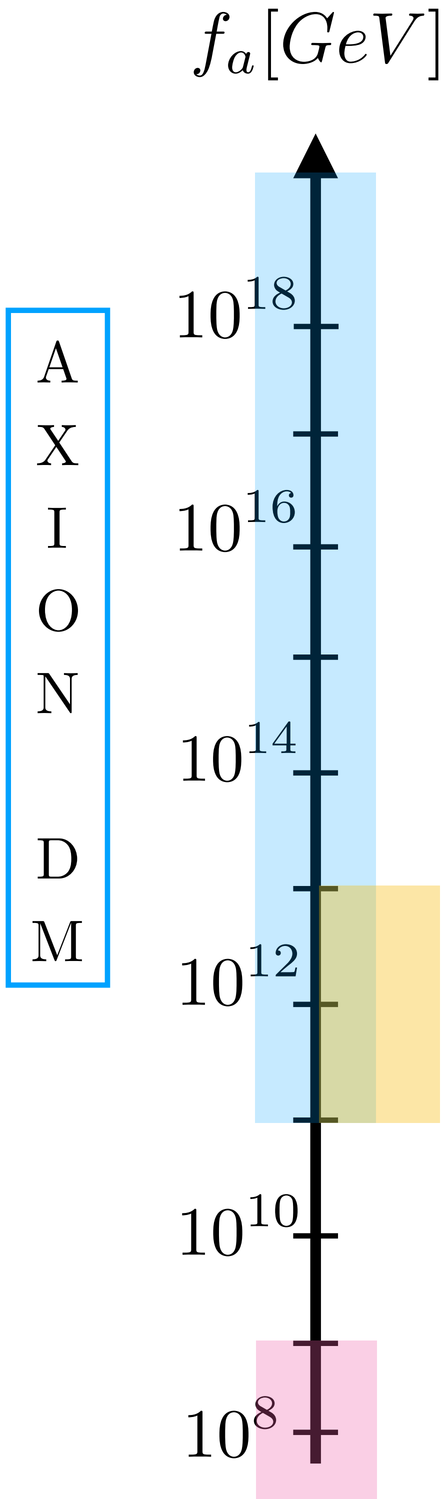


Resonantly enhanced conversion

$a \to \gamma$

$$P_s \propto \frac{Q}{m_a} g_{a\gamma\gamma}^2 B_e |G_m|^2 V \rho_a$$

Experimental Status



Microwave cavity (ADMX)

Astro: SN19874, White Dwarfs

Lab: Collider & Laser experiments, ALPS, CAST

And what if we decouple the detector V from the axion ω_a?

ABRACADABRA

Kahn, Said, Thaler, 2016

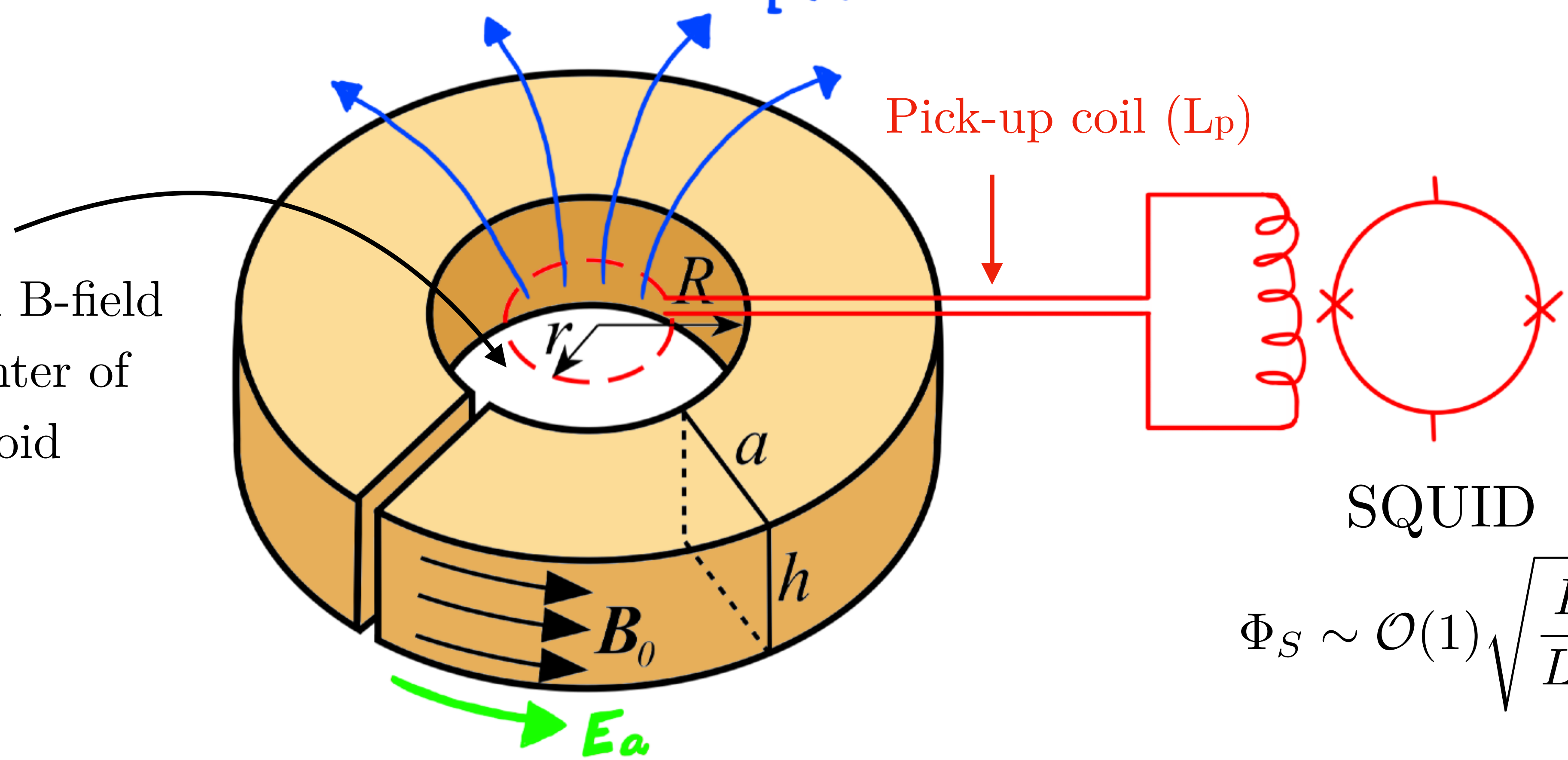
A Broadband/Resonant Approach to Cosmic Axion Detection with an Amplifying B-field Ring Apparatus

And what if we decouple the detector V from the axion ω_a?

$\vec{E}_a$ sources a $\vec{B}_a \perp \vec{B}_0$ ➔ oscillating magnetic flux

$$\phi(t) = g_{a\gamma\gamma} B_0^{\max} \sqrt{2\rho_{\rm DM}} \cos(m_a t) V_B$$

No external B-field in the center of the toroid

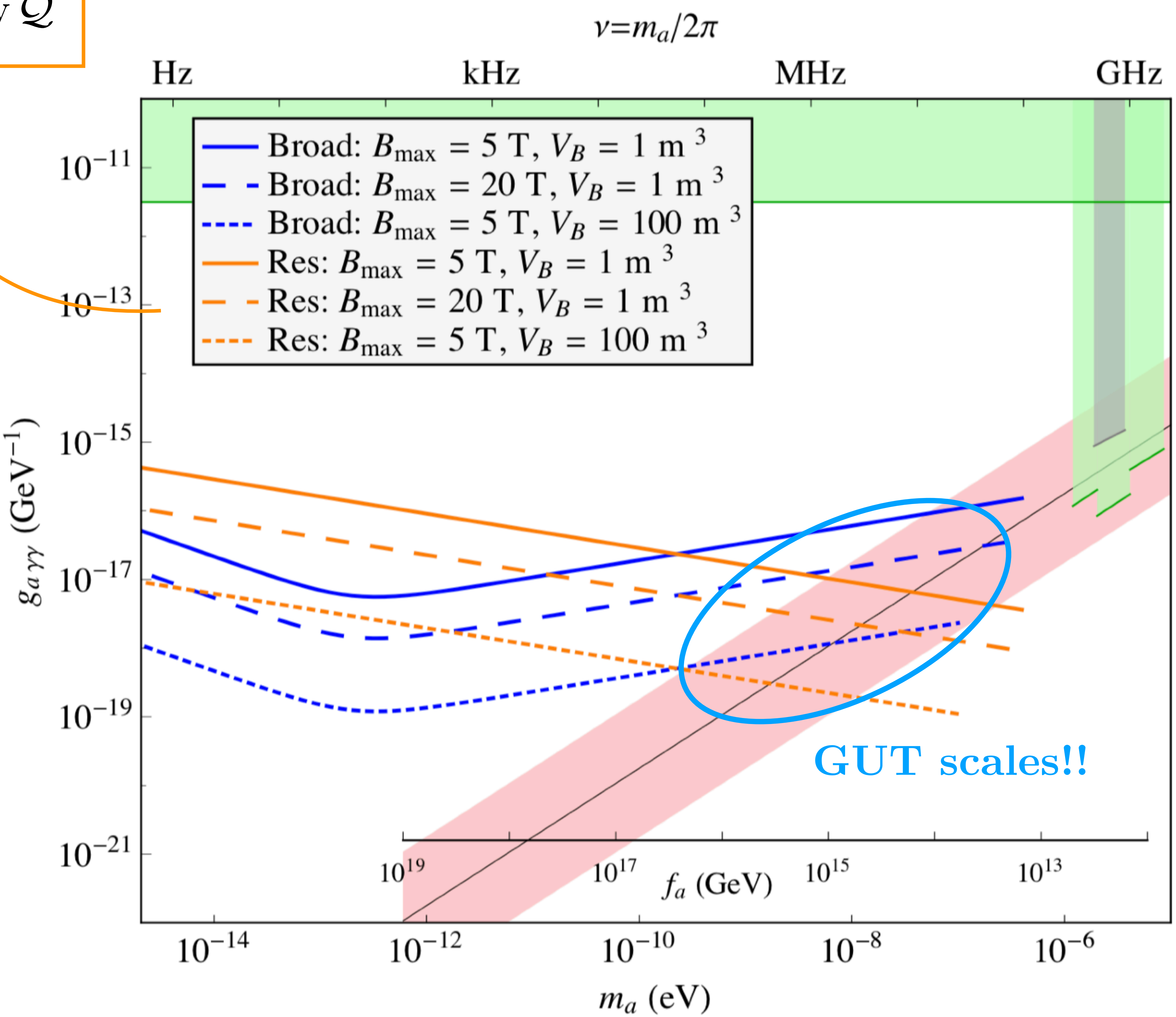


$$\Phi_S \sim \mathcal{O}(1)\sqrt{\frac{L}{L_p}}\,\Phi$$

$\vec{B}_0$ generates an $\vec{E}_a$ through Ampère's circuit law

ABRACADABRA

Enhance the signal by $\mathcal{Q}$



CASPEr

Graham, Rajendran, 2013

Cosmic Axion Precession Experiment

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \boxed{\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}} + \frac{1}{4}a\, g_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + \boxed{\frac{\partial_\mu a}{2f_a}(c_q\bar{q}\gamma^\mu\gamma_5 q)}$$

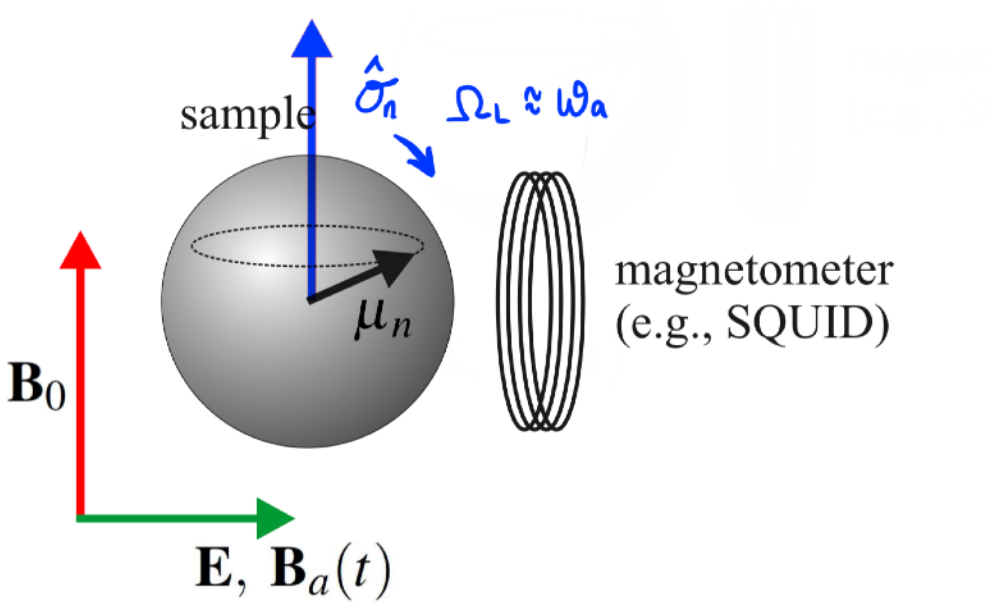
$E < \Lambda_{\rm QCD}$

$$\underbrace{-\frac{i}{2}\,g_d\, a\overline{\Psi_n}\sigma_{\mu\nu}\gamma_5\Psi_n F^{\mu\nu}}_{\mathcal{L}_{\rm EDM}} + \underbrace{g_{aNN}\,(\partial_\mu a)\overline{\Psi_n}\gamma^\mu\gamma_5\Psi_n}_{\mathcal{L}_{\rm spin}}$$

CASPEr ELECTRIC

$$\vec{d_n}(t) = g_d\,\frac{\sqrt{2\rho_a}}{m_a}\,\cos(\omega_a t)\hat{\sigma_n}$$

$$H_{\rm EDM} = -\vec{d_n}(t)\cdot\vec{E}$$



Detecting an oscillating nuclear-spin-dependent energy shift using **NMR techniques**

CASPEr

Cosmic Axion Precession Experiment

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \boxed{\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}} + \frac{1}{4}a\, g_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + \boxed{\frac{\partial_\mu a}{2f_a}(c_q\bar{q}\gamma^\mu\gamma_5 q)}$$

$$\Big\downarrow \; E < \Lambda_{\mathrm{QCD}}$$

$$\underbrace{-\frac{i}{2}\, g_d\, a\overline{\Psi_n}\sigma_{\mu\nu}\gamma_5\Psi_n F^{\mu\nu}}_{\mathcal{L}_{\mathrm{EDM}}} + \underbrace{g_{aNN}\,(\partial_\mu a)\overline{\Psi_n}\gamma^\mu\gamma_5\Psi_n}_{\mathcal{L}_{\mathrm{spin}}}$$

CASPEr ELECTRIC

$$\vec{d_n}(t) = g_d\,\frac{\sqrt{2\rho_a}}{m_a}\,\cos(\omega_a t)\hat{\sigma_n}$$

$$H_{\mathrm{EDM}} = -\vec{d_n}(t)\cdot\vec{E}$$

CASPEr WIND

$$H_{\mathrm{spin}} = g_{aNN}\,\vec{\nabla}a\cdot\hat{\sigma_n}$$

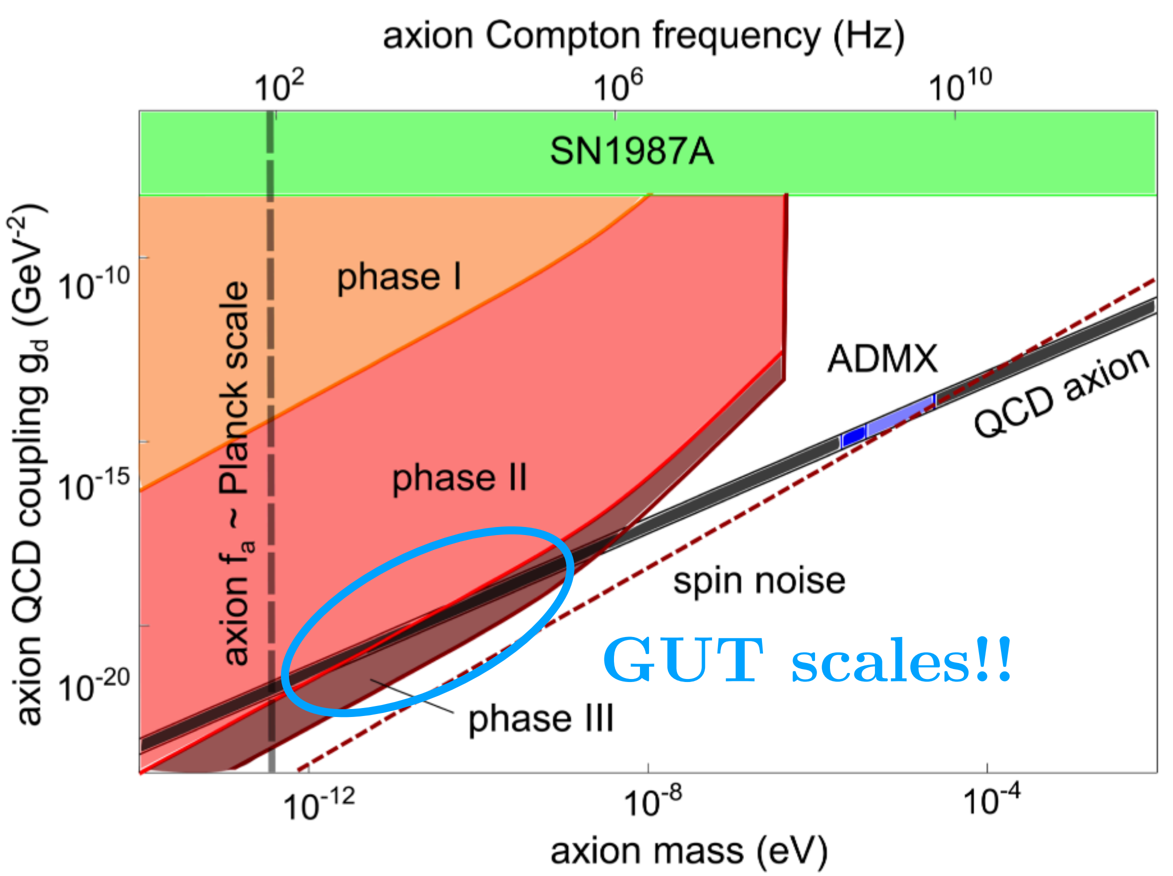
$$H_{\mathrm{wind}} = -\vec{\mu_n}\cdot\vec{B_a}(t)$$

Detecting an oscillating nuclear-spin-dependent energy shift using **NMR techniques**

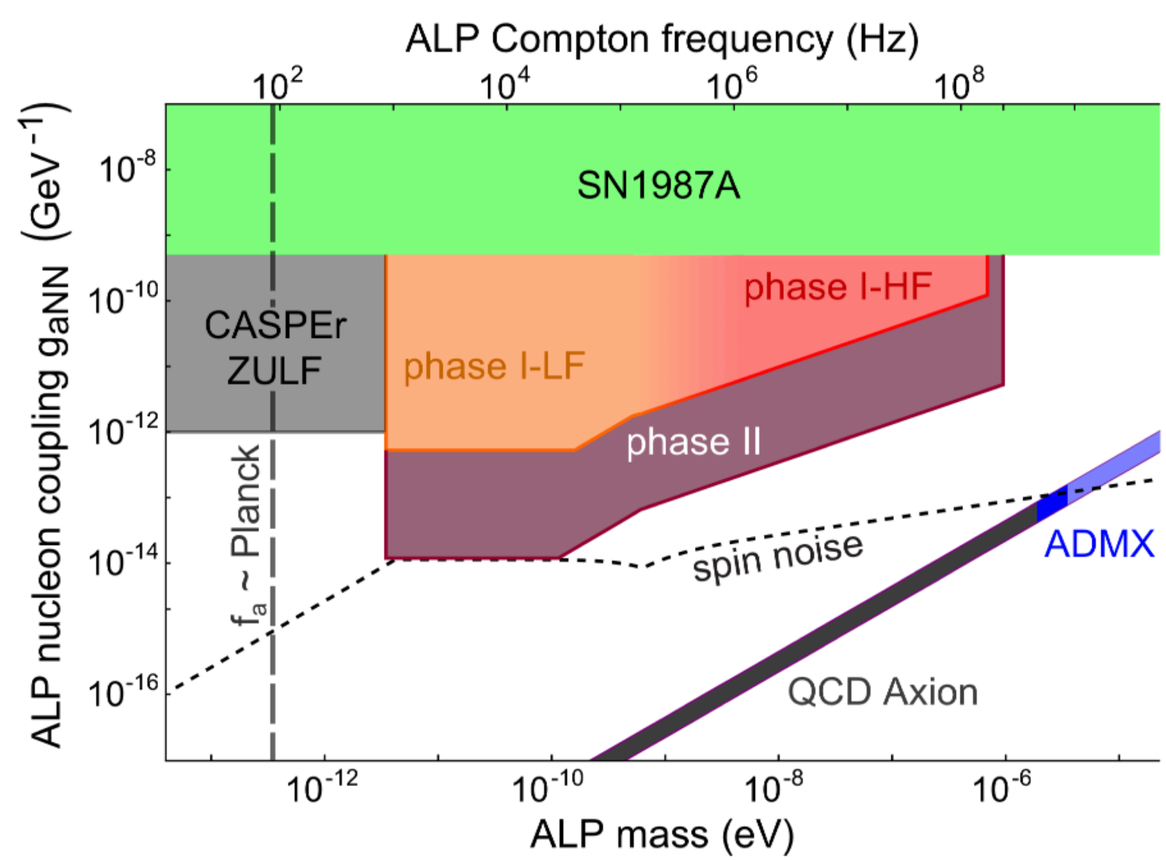
CASPEr

Cosmic Axion Precession Experiment

CASPEr electric



CASPEr wind



Budker, Graham, Bedbetter, Rajendran, Sushkov, 2014

Strongest magnet 30 T $\quad \Rightarrow m_a < 10^{-6}$ eV

Motivation

Wise, Georgi, Glashow, 1981

Grand Unified Theories (GUTs)

$SU(5)$

Georgi, Glashow

- Rank 4

$$r[SU(5)] = r[SU(3)_c \otimes SU(2)_L \otimes U(1)_Y]$$

$$\lambda_3, \lambda_8 \qquad \sigma_z \qquad \mathbb{I}$$

$$T_3 = \frac{1}{2}\begin{pmatrix} & & & 0 & 0 \\ & \lambda_3 & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad T_8 = \frac{1}{2}\begin{pmatrix} & & & 0 & 0 \\ & \lambda_8 & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T_{23} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & & \\ 0 & 0 & 0 & & \end{pmatrix} \qquad T_{24} = \frac{1}{\sqrt{15}}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{pmatrix}$$

Grand Unified Theories (GUTs)

$SU(5)$

- Rank 4
- 15 Weyl d.o.f. fit perfectly in $\bar{5}$ and 10

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \; (u^c)_L, \; (d^c)_L, \; (e^c)_L$$

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ -e \\ \nu \end{pmatrix}_L, \; 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}_L$$

Grand Unified Theories (GUTs)

$SU(5)$

- Rank 4
- 15 Weyl d.o.f. fit perfectly in $\bar{5}$ and 10
- Charge quantisation

$$\mathcal{Q} = T_{23}(\equiv \frac{1}{2}\sigma_z) + \sqrt{\frac{5}{3}}T_{24}$$

Matter content

$$\bar{5}, 10$$

Grand Unified Theories (GUTs)

$SU(5)$

- Rank 4
- 15 Weyl d.o.f. fit perfectly in $\bar{5}$ and 10
- Charge quantisation

Scalar content

$$5_H, 24_H$$

Matter content

$$\bar{5}, 10$$

$$5_H = \begin{pmatrix} T \\ H_1 \end{pmatrix}$$

$$\text{“}SU(5) \stackrel{\langle 24_H \rangle}{\to} SM \stackrel{\langle 5_H \rangle}{\to} SU(3)_C \otimes U(1)_Q\text{”}$$

$$24_H = \begin{pmatrix} \Sigma_8 - \frac{2}{\sqrt{30}}\Sigma_0 & \Sigma_{(\bar{3},2)} \\ \Sigma_{(3,2)} & \Sigma_3 + \frac{3}{\sqrt{30}}\Sigma_0 \end{pmatrix}$$

Grand Unified Theories (GUTs)

$SU(5)$

$$V_{24} \sim \underbrace{(8,1,0)}_{G_\mu} \oplus \underbrace{(1,3,0)}_{W_\mu} \oplus \underbrace{(3,2,-5/6)}_{V_\mu^c} \oplus \underbrace{(\bar{3},2,5/6)}_{V_\mu} \oplus \underbrace{(1,1,0)}_{\lambda_\mu}$$

$$5_H \sim \underbrace{(1,2,1/2)}_{H_1} \oplus \underbrace{(3,1,-1/3)}_{T}$$

$$24_H \sim \underbrace{(8,1,0)}_{\Sigma_8} \oplus \underbrace{(1,3,0)}_{\Sigma_3} \oplus \underbrace{(3,2,-5/6)}_{\Sigma_{(3,2)}} \oplus \underbrace{(\bar{3},2,5/6)}_{\Sigma_{(\bar{3},2)}} \oplus \underbrace{(1,1,0)}_{\Sigma_{24}}$$

Scalar content	Gauge bosons	Matter content
$5_H, 24_H$	24_V	$\bar{5}, 10$

$$5_H = \begin{pmatrix} T \\ H_1 \end{pmatrix} \qquad A_\mu = \begin{pmatrix} G_\mu - \frac{1}{\sqrt{15}} B_\mu & V_\mu^c \\ V_\mu & W_\mu + \frac{3}{2\sqrt{15}} B_\mu \end{pmatrix}$$

$$24_H = \begin{pmatrix} \Sigma_8 - \frac{2}{\sqrt{30}} \Sigma_0 & \Sigma_{(\bar{3},2)} \\ \Sigma_{(3,2)} & \Sigma_3 + \frac{3}{\sqrt{30}} \Sigma_0 \end{pmatrix}$$

Grand Unified Theories (GUTs)

$SU(5)$

$$V_{24} \sim \underbrace{(8,1,0)}_{G_\mu} \oplus \underbrace{(1,3,0)}_{W_\mu} \oplus \underbrace{(3,2,-5/6)}_{V_\mu^c} \oplus \underbrace{(\bar{3},2,5/6)}_{V_\mu} \oplus \underbrace{(1,1,0)}_{\lambda_\mu}$$

$$5_H \sim \underbrace{(1,2,1/2)}_{H_1} \oplus \underbrace{(3,1,-1/3)}_{T}$$

$$24_H \sim \underbrace{(8,1,0)}_{\Sigma_8} \oplus \underbrace{(1,3,0)}_{\Sigma_3} \oplus \underbrace{(3,2,-5/6)}_{\Sigma_{(3,2)}} \oplus \underbrace{(\bar{3},2,5/6)}_{\Sigma_{(\bar{3},2)}} \oplus \underbrace{(1,1,0)}_{\Sigma_{24}}$$

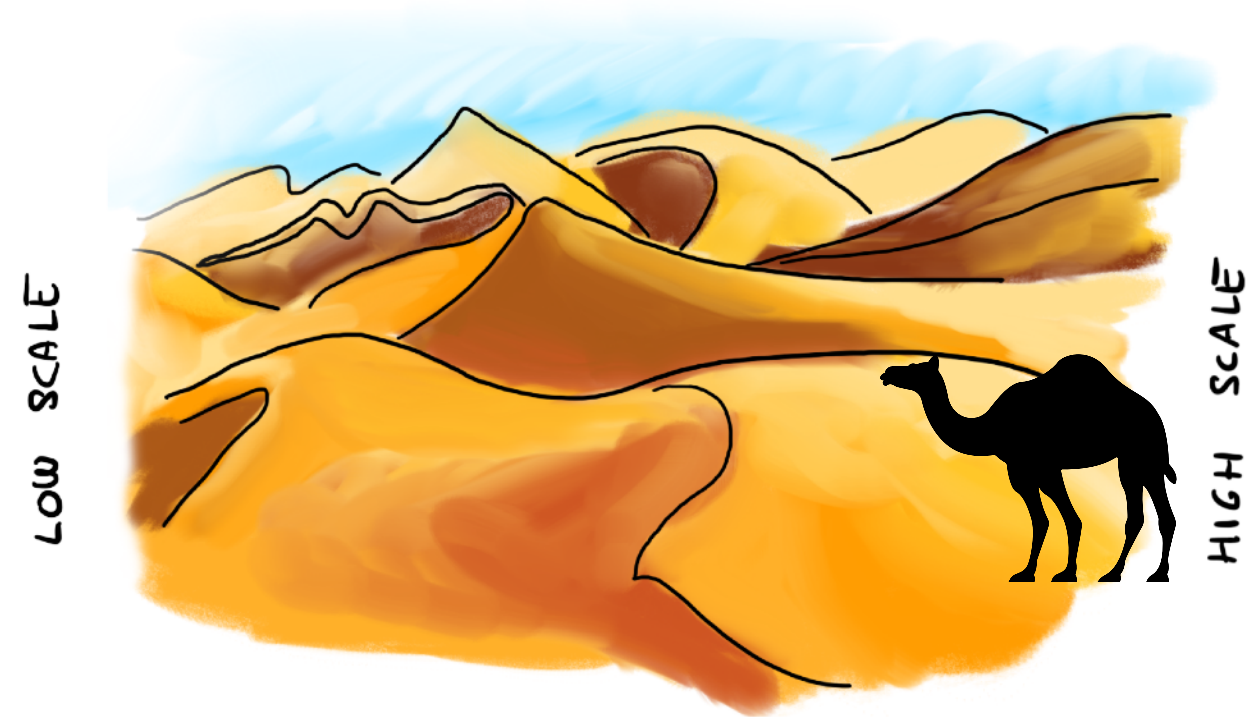
Scalar content	Gauge bosons	Matter content
$5_H, 24_H$	24_V	$\bar{5}, 10$

$$5_H = \begin{pmatrix} T \\ H_1 \end{pmatrix} \qquad A_\mu = \begin{pmatrix} G_\mu - \frac{1}{\sqrt{15}} B_\mu & V_\mu^c \\ V_\mu & W_\mu + \frac{3}{2\sqrt{15}} B_\mu \end{pmatrix}$$

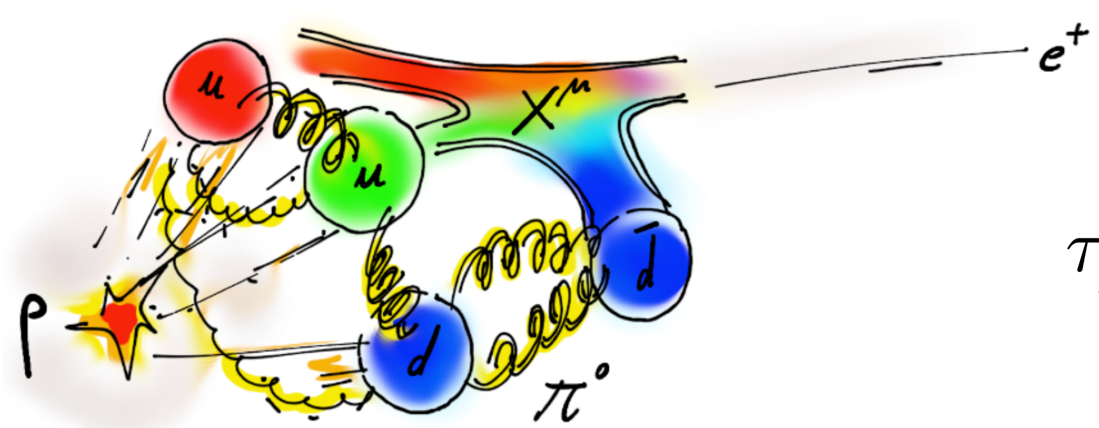
$$24_H = \begin{pmatrix} \Sigma_8 - \frac{2}{\sqrt{30}} \Sigma_0 & \Sigma_{(\bar{3},2)} \\ \Sigma_{(3,2)} & \Sigma_3 + \frac{3}{\sqrt{30}} \Sigma_0 \end{pmatrix}$$

Grand Unified Theories (GUTs)

$SU(5)$



Scalar content	Gauge bosons	Matter content
$5_H, 24_H$	24_V	$\bar{5}, 10$



$$\tau_p \gtrsim 10^{34} \text{ yr} \Rightarrow M_{\text{GUT}} \sim 10^{15} \text{ GeV}$$

Grand Unified Theories (GUTs)

$SU(5)_{GG}$

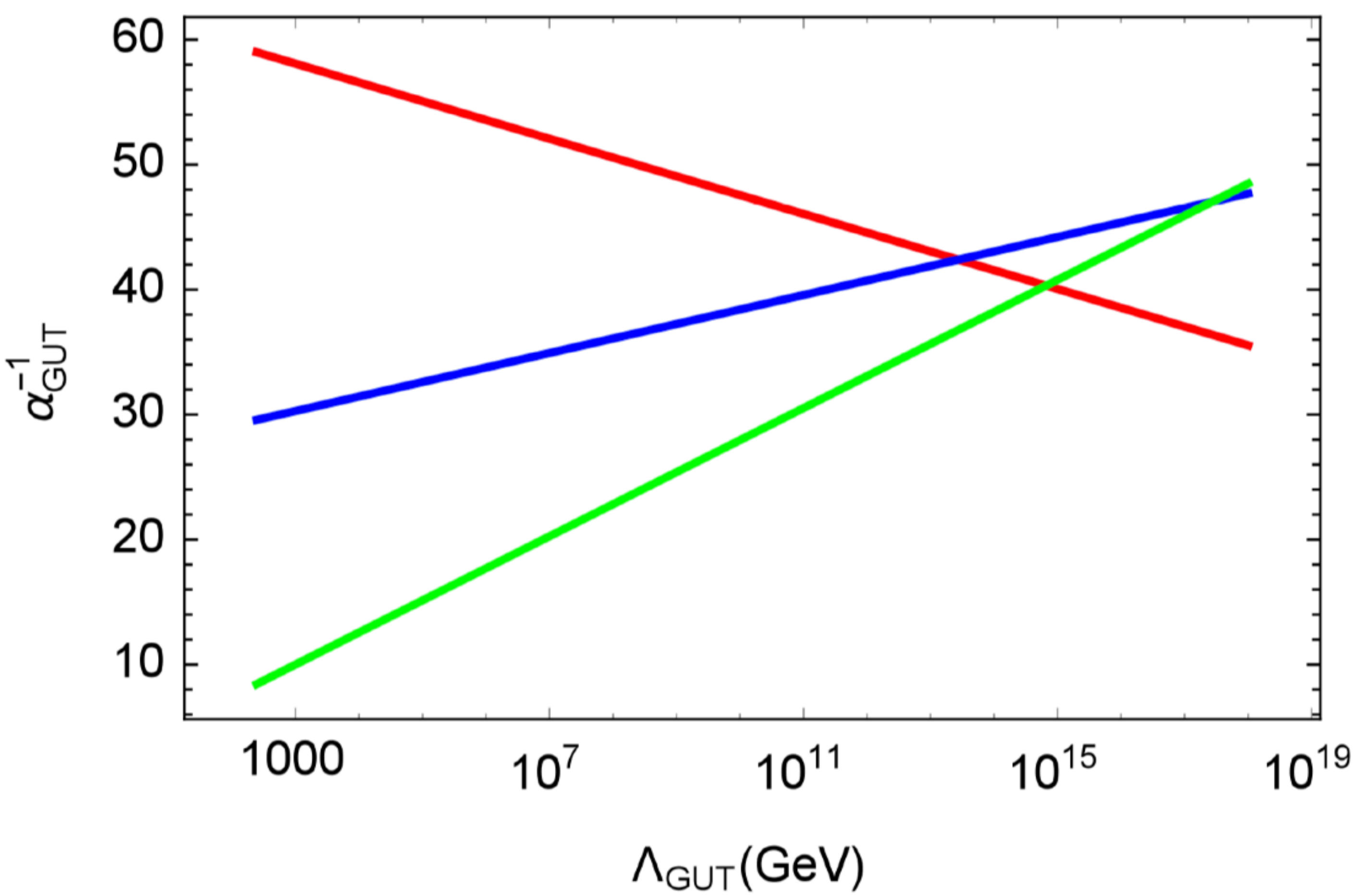
Georgi, Glashow

- Gauge couplings do not unify

α_1^{-1}

α_2^{-1}

α_3^{-1}



$$\alpha_i = \frac{g_i^2}{4\pi}$$

Grand Unified Theories (GUTs)

$SU(5)_{GG}$

- Gauge couplings do not unify
- Wrong fermion mass relations

$$\mathcal{L}_Y \supset Y_1\, \bar{5}\, 10\, 5_H^* + Y_3\, 10\, 10\, 5_H \epsilon_5 + \text{h.c.}$$

$$M_d = M_e^T = Y_1 \frac{v^*}{2},$$
$$M_u = 4(Y_3 + Y_3^T)\frac{v}{2}$$

Grand Unified Theories (GUTs)

$SU(5)_{GG}$

- Gauge couplings do not unify
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$$\mathcal{L}_Y \supset Y_1\, \bar{5}\, 10\, 5_H^* + Y_3\, 10\, 10\, 5_H \epsilon_5 + \text{h.c.}$$

@ GUT scale

$$\boxed{M_d = M_e^T} = Y_1 \frac{v^*}{2},$$

$$M_u = 4(Y_3 + Y_3^T)\frac{v}{2}$$

Grand Unified Theories (GUTs)

$SU(5)_{GG}$

- Gauge couplings do not unify
- Wrong fermion mass relations
- Neutrinos are massless (as in the SM)

$$\mathcal{L} \supset \cancel{Y_\nu \, \bar{5} \, \bar{5} \, 5_H}$$

Not allowed by Gauge Symmetry

Grand Unified Theories (GUTs)

$SU(5)_{GG}$

- Gauge couplings do not unify
- Wrong fermion mass relations
- Neutrinos are massless (as in the SM)

	$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification
Renorm. $SU(5)$ extension?			

Grand Unified Theories (GUTs)

$SU(5)_{GG}$

$\otimes\ U(1)_{PQ}$

Wise, Georgi, Glashow

- Gauge couplings do not unify
- Wrong fermion mass relations
- Neutrinos are massless (as in the SM)

$$24_H \xrightarrow{U(1)} 24_H^* \quad \Rightarrow \quad 24_H \supset \frac{1}{\sqrt{2}}|\Sigma_0|e^{\frac{ia(x)}{v_\Sigma}}$$

	$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
Renorm. $SU(5)$ extension?				

Grand Unified Theories (GUTs)

$SU(5)_{GG}$
$\otimes\ U(1)_{PQ}$

- Gauge couplings do not unify
- Wrong fermion mass relations
- Neutrinos are massless (as in the SM)
- **Extra content for PQ!**

$$\mathcal{L} \supset Y_5\,\bar{5}\,10\,5^*_H + Y_{10}\,10\,10\,5'_H\,\epsilon_5 + \lambda\,5^*_H 24^2_H 5'_H + \text{h.c.}$$

	$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
$5'_H$				✓

Wise, Georgi, Glashow

Grand Unified Theories (GUTs)

$SU(5)_{GG}$
$\otimes\ U(1)_{PQ}$

- Gauge couplings do not unify
- Wrong fermion mass relations
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- **Extra content for PQ!**

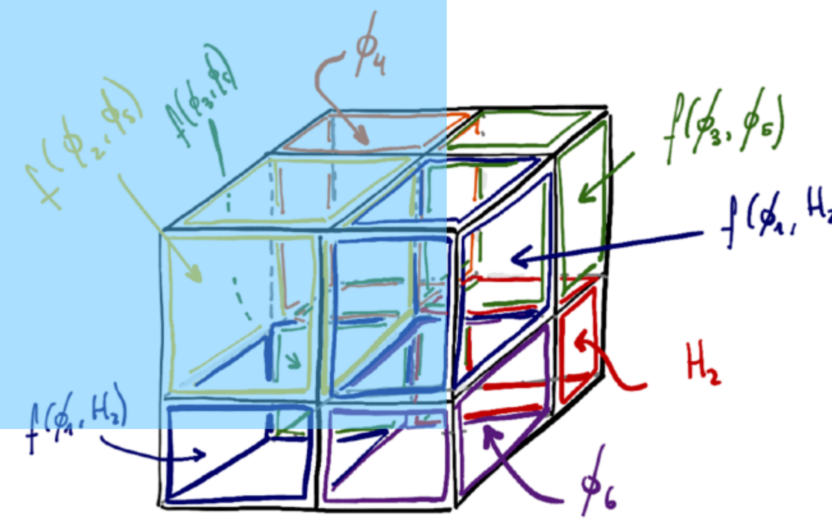
$$\mathcal{L} \supset Y_5\, \bar{5}\, 10\, 5_H^* + Y_{10}\, 10\, 10\, 5'_H\, \epsilon_5 + \lambda\, 5_H^* 24_H^2 5'_H + \text{h.c.}$$

	$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
$5'_H$	×	×	×	✓

Wise, Georgi, Glashow

GUTs and PQ?

	$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
45_H			✓ ✓ ✓ ✓	



$$45_H \sim \underbrace{(8,2,1/2)}_{\Phi_1} \oplus \underbrace{(\bar{6},1,-1/3)}_{\Phi_2} \oplus \underbrace{(3,3,-1/3)}_{\Phi_3} \oplus \underbrace{(\bar{3},2,-7/6)}_{\Phi_4} \oplus \underbrace{(3,1,-1/3)}_{\Phi_5} \oplus \underbrace{(\bar{3},1,4/3)}_{\Phi_6} \oplus \underbrace{(1,2,1/2)}_{H_2}$$

$$\mathcal{L}_Y = \bar{5}\,10\;(Y_1\;5_H^* + Y_2\;45_H^*)\; + \; 10\,10\;(Y_3\;5_H + Y_4\;45_H)\epsilon_5 + \text{h.c.}$$

$$V \supset \mu 5_H^* 24_H^2 45_H$$

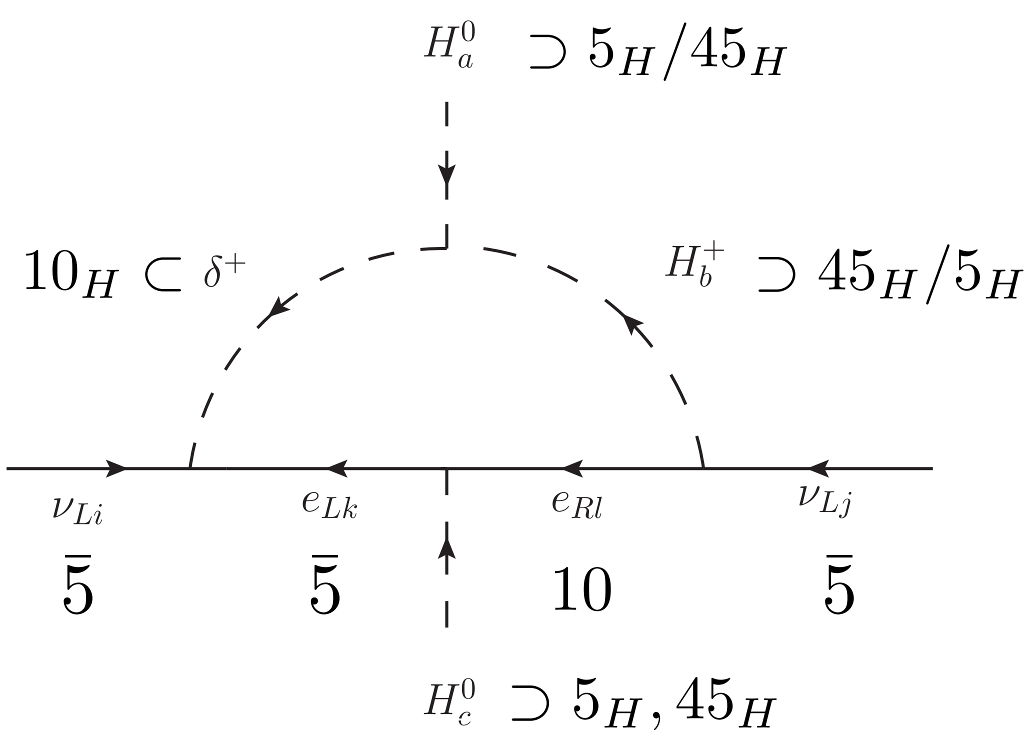
GUTs and PQ?

$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
	1_i	✓	×
45_H		✓	
		✓	
		✓	

$$\mathcal{L} \supset M_\nu 1_i 1_i + Y_\nu^i \, \bar{5} \, 1_i \, 5_H$$

GUTs and PQ?

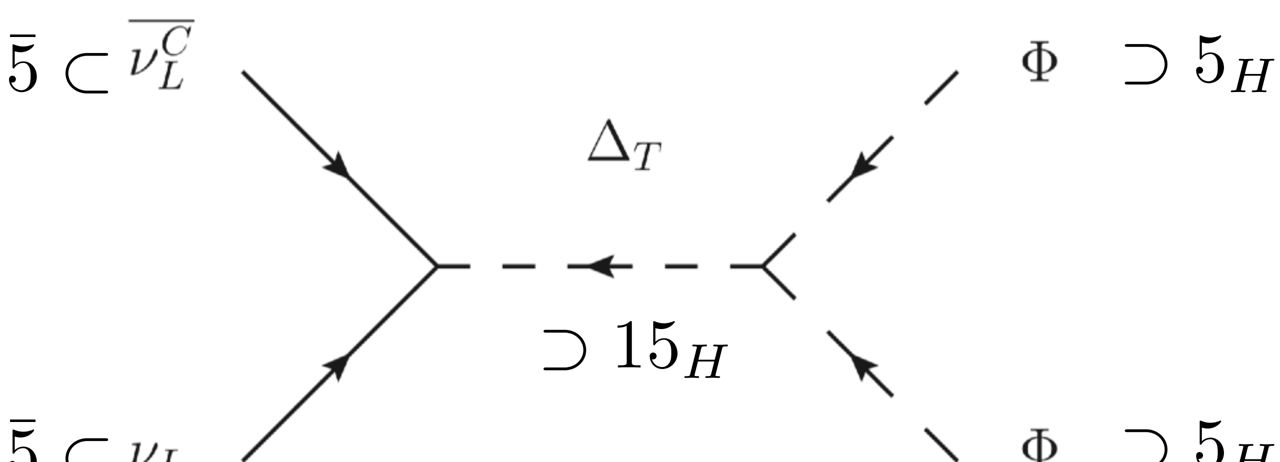
$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
45_H	1_i	✓	×
	10_H	✓	×
		✓	
		✓	



$$\mathcal{L} \supset \lambda\, \bar{5}\, \bar{5}\, 10_H + \bar{5}\, 10\, (Y_1^* 5_H^* + Y_2^* 45_H^*)$$
$$+ \mu\, 5_H 45_H 10_H^* + \text{h.c.}$$

GUTs and PQ?

$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
45_H	1_i	✓	×
	10_H	✓	×
	15_H	✓	×
		✓	



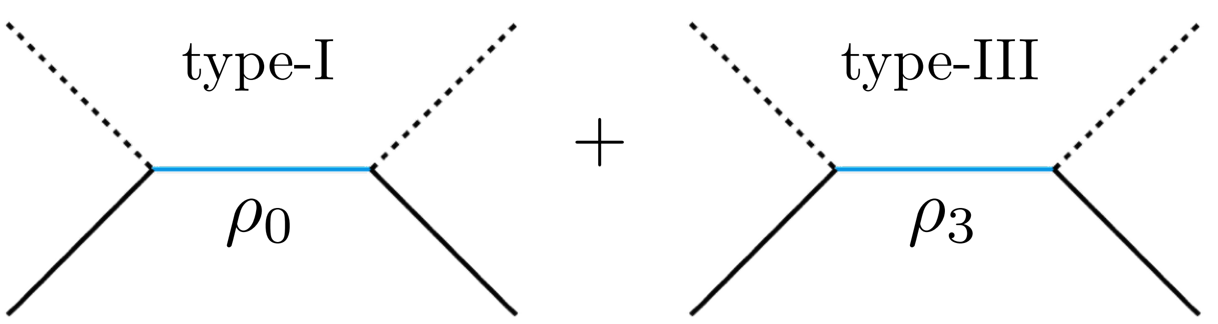
$$\mathcal{L} \supset Y_\nu \, \bar{5}\, \bar{5}\, 15_H + \mu \, 5_H^* 5_H^* 15_H + \text{h.c.}$$

$$M_\nu \sim \lambda Y_\nu \frac{\langle \Phi \rangle^2}{M_{\Delta_T}^2}$$

GUTs and PQ?

$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
45_H	1_i	✓	×
	10_H	✓	×
	15_H	✓	×
	24	✓	✓

$$24 = \begin{pmatrix} \rho_8 - \frac{1}{\sqrt{15}}\rho_0 & \rho_{(\bar{3},2)} \\ \rho_{(3,2)} & \rho_3 + \frac{3}{2\sqrt{15}}\rho_0 \end{pmatrix}$$



$$\mathcal{L} \supset \lambda \mathrm{Tr}\{24^2 24_H\} + \text{h.c.}$$

GUTs and PQ?

$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
45_H	1_i	✓	×
	10_H	✓	×
	15_H	✓	×
	24	✓	✓
$5' + \bar{5}'$	1_i	×	✓
	15_H	×	✓
	24	×	✓

$$\mathcal{L} \supset Y_5 \, \bar{5}' 24_H 5'$$

GUTs and PQ?

$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$
45_H	1_i	✓	✗
	10_H	✓	✗
	15_H	✓	✗
	24	✓	✓
$5' + \bar{5}'$	1_i	✗	✓
	15_H	✗	✓
	24	✗	✓

$$\mathcal{L} \supset \cancel{M_{24}\mathrm{Tr}\{24^2\}} + \lambda \mathrm{Tr}\{24^2 24_H\} + \mathrm{h.c.}$$

GUTs and PQ?

	$Y_e \neq Y_d$	$M_\nu \neq 0$	Unification	$U(1)_{PQ}$	
	45_H	1_i	✓	✗	
		10_H	✓	✗	
		15_H	✓	✗	
KSVZ		24	✓	✓	Adjoint $SU(5)$
					Fileviez Pérez, 2007
	$5' + \bar{5}'$	1_i	✗	✓	
		15_H	✗	✓	
		24	✗	✓	

- Minimal GUT theory (# of Reps.) where the PQ-symmetry can be realized

Adjoint SU(5)

- PQ charges:

$$\bar{5} \to e^{-3i\theta}\bar{5}, \qquad 10 \to e^{+i\theta}10, \qquad 5_H \to e^{-2i\theta}5_H,$$
$$24_H \to e^{-10i\theta}24_H, \qquad 45_H \to e^{-2i\theta}45_H, \qquad 24 \to e^{+5i\theta}24.$$

- Charged fermion masses:

$$\mathcal{L} \supset \bar{5}\,10\,(Y_1\,5_H^* + Y_2\,45_H^*) + 10\,10\,(Y_3\,5_H + Y_4\,45_H)\epsilon_5 + \text{h.c.}$$

$$M_E = \frac{1}{2}(Y_1^T v_5^* - 6Y_2^T v_{45}^*),$$
$$M_D = \frac{1}{2}(Y_1 v_5^* + 2Y_2 v_{45}^*),$$
$$M_U = \frac{1}{2\sqrt{2}}(4(Y_3 + Y_3^T)v_5 - 8(Y_4 - Y_4^T)v_{45}).$$

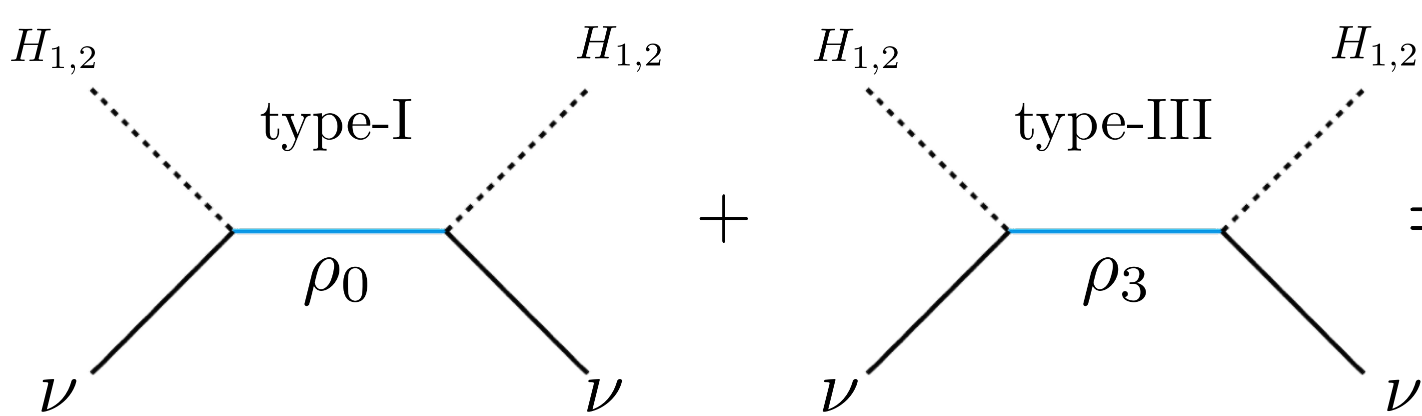
Adjoint SU(5)

- PQ charges:

$$\bar{5} \to e^{-3i\theta}\bar{5}, \qquad 10 \to e^{+i\theta}10, \qquad 5_H \to e^{-2i\theta}5_H,$$
$$24_H \to e^{-10i\theta}24_H, \qquad 45_H \to e^{-2i\theta}45_H, \qquad 24 \to e^{+5i\theta}24.$$

- Neutral fermion masses:

$$\mathcal{L} \supset h_1^i\, \bar{5}_i\, 24\, 5_H + h_2^i\, \bar{5}_i\, 24\, 45_H + \lambda\, \mathrm{Tr}\{24^2 24_H\} + \mathrm{h.c.}$$



$$= \left(\frac{\mathrm{L.C.}[h_1^i, h_2^j]}{M_{\rho_0}^2} + \frac{\mathrm{L.C.}[h_1^i, h_2^j]}{M_{\rho_3}^2} \right)$$

$$M_\nu = M_\nu^I + M_\nu^{III} + M_\nu^{cs}$$

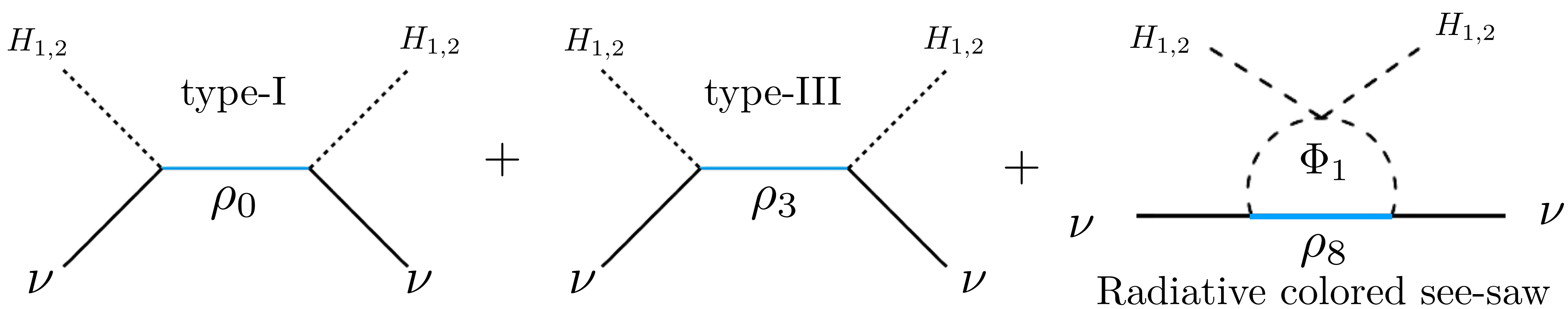
Adjoint SU(5)

- PQ charges:

$$\bar{5} \to e^{-3i\theta}\bar{5}, \qquad 10 \to e^{+i\theta}10, \qquad 5_H \to e^{-2i\theta}5_H,$$
$$24_H \to e^{-10i\theta}24_H, \qquad 45_H \to e^{-2i\theta}45_H, \qquad 24 \to e^{+5i\theta}24.$$

- Neutral fermion masses:

$$\mathcal{L} \supset h_1^i\, \bar{5}_i\, 24\, 5_H + h_2^i\, \bar{5}_i\, 24\, 45_H + \lambda\, \mathrm{Tr}\{24^2 24_H\} + \mathrm{h.c.}$$



$$M_\nu = M_\nu^I + M_\nu^{III} + M_\nu^{cs}$$

Adjoint SU(5)

- PQ charges:

$$\bar{5} \to e^{-3i\theta}\bar{5}, \qquad 10 \to e^{+i\theta}10, \qquad 5_H \to e^{-2i\theta}5_H,$$
$$24_H \to e^{-10i\theta}24_H, \qquad 45_H \to e^{-2i\theta}45_H, \qquad 24 \to e^{+5i\theta}24.$$

- Neutral fermion masses:

$$\mathcal{L} \supset \; h_1 \bar{5}\, 24\, 5_H + h_2\, \bar{5}\, 24\, 45_H + \lambda\, \mathrm{Tr}\{24^2 24_H\} + \mathrm{h.c.}$$

$$\cancel{M_{24}\mathrm{Tr}\{24^2\}}$$

- New fermion masses:

$$M_{\rho_0} = \frac{1}{3} M_{\rho_3}, \quad M_{\rho_8} = \frac{2}{3} M_{\rho_3}, \quad M_{\rho_{(3,2)}} = M_{\rho_{(\bar{3},2)}} = \frac{1}{6} M_{\rho_3}.$$

$$M_{24} \equiv M_{\rho_3}$$

Adjoint SU(5)

$$24_H \to \frac{v_\Sigma}{\sqrt{15}}\ \mathrm{diag}(-1,-1,-1,3/2,3/2)e^{ia(x)/v_\Sigma}$$

$$\boxed{\mathcal{L} \supset \lambda\,\mathrm{Tr}\{24^2\langle 24_H\rangle\} + \text{h.c.}}$$
$$= \frac{\lambda}{\sqrt{15}} v_\Sigma\, e^{ia(x)/v_\Sigma}\left(-\mathrm{Tr}\{\rho_8\rho_8\} + \frac{1}{2}\mathrm{Tr}\{\rho_{(\bar{3},2)}\rho_{(3,2)}\} + \frac{3}{2}\mathrm{Tr}\{\rho_3\rho_3\} + \frac{1}{2}\rho_0^2\right) + \text{h.c.}$$

Adjoint SU(5)

$$24_H \to \frac{v_\Sigma}{\sqrt{15}}\ \mathrm{diag}(-1,-1,-1,3/2,3/2)e^{ia(x)/v_\Sigma}$$

$$\boxed{\mathcal{L} \supset \mathrm{Tr}\{D_\mu \langle 24_H\rangle^\dagger D_\mu \langle 24_H\rangle\}} = \mathrm{Tr}\{(ig_{\mathrm{GUT}}[A_\mu, \langle 24_H\rangle])^\dagger (ig_{\mathrm{GUT}}[A^\mu, \langle 24_H\rangle]]\}$$

$$= \underbrace{g_{\mathrm{GUT}}^2 \frac{5}{6} v_\Sigma^2}_{M_{\mathrm{GUT}}^2} V_\mu^\dagger V^\mu$$

$$\boxed{\mathcal{L} \supset \lambda\, \mathrm{Tr}\{24^2 \langle 24_H\rangle\} + \mathrm{h.c.}}$$

$$= \frac{\lambda}{\sqrt{15}} v_\Sigma\, e^{ia(x)/v_\Sigma} \left(-\mathrm{Tr}\{\rho_8\rho_8\} + \frac{1}{2}\mathrm{Tr}\{\rho_{(\bar{3},2)}\rho_{(3,2)}\} + \frac{3}{2}\mathrm{Tr}\{\rho_3\rho_3\} + \frac{1}{2}\rho_0^2\right) + \mathrm{h.c.}$$

Adjoint SU(5)

- Same scalar field to break both theories ($SU(5)\otimes U(1)_{\rm PQ}$)

$$24_H \to \frac{v_\Sigma}{\sqrt{15}}\,\mathrm{diag}(-1,-1,-1,3/2,3/2)e^{ia(x)/v_\Sigma}$$

$$\boxed{\mathcal{L}\supset \mathrm{Tr}\{D_\mu\langle 24_H\rangle^\dagger D_\mu\langle 24_H\rangle\}} = \mathrm{Tr}\{(ig_{\rm GUT}[A_\mu,\langle 24_H\rangle])^\dagger(ig_{\rm GUT}[A^\mu,\langle 24_H\rangle]\}$$

$$= \underbrace{g_{\rm GUT}^2\frac{5}{6}v_\Sigma^2}_{M_{\rm GUT}^2}V_\mu^\dagger V^\mu$$

$$\boxed{\mathcal{L}\supset \lambda\,\mathrm{Tr}\{24^2\langle 24_H\rangle\}+\mathrm{h.c.}}$$

$$=\frac{\lambda}{\sqrt{15}}v_\Sigma\,e^{ia(x)/v_\Sigma}\left(-\mathrm{Tr}\{\rho_8\rho_8\}+\frac{1}{2}\mathrm{Tr}\{\rho_{(\bar 3,2)}\rho_{(3,2)}\}+\frac{3}{2}\mathrm{Tr}\{\rho_3\rho_3\}+\frac{1}{2}\rho_0^2\right)+\mathrm{h.c.}$$

$$\Rightarrow M_{\rm GUT}=g_{\rm GUT}\sqrt{\frac{5}{6}}v_\Sigma$$

$$\Rightarrow \mathcal{L}\supset\frac{g_s^2}{32\pi^2}\,\frac{N}{\underbrace{v_\Sigma}_{f_a}}\,a\,G_{\mu\nu}\tilde G^{\mu\nu}$$

GUT scale window $\longleftrightarrow$ Axion mass window

Adjoint SU(5)

GUT scale window $\longleftrightarrow$ Axion mass window

$M_{\rm GUT} \simeq (10^{15.06} - 10^{15.74})$ GeV

$m_a \simeq (3 - 13) \times 10^{-9}$ eV

Adjoint SU(5)

GUT scale window ⟷ Axion mass window

$M_{\mathrm{GUT}} \simeq (10^{15.06} - 10^{15.74})$ GeV

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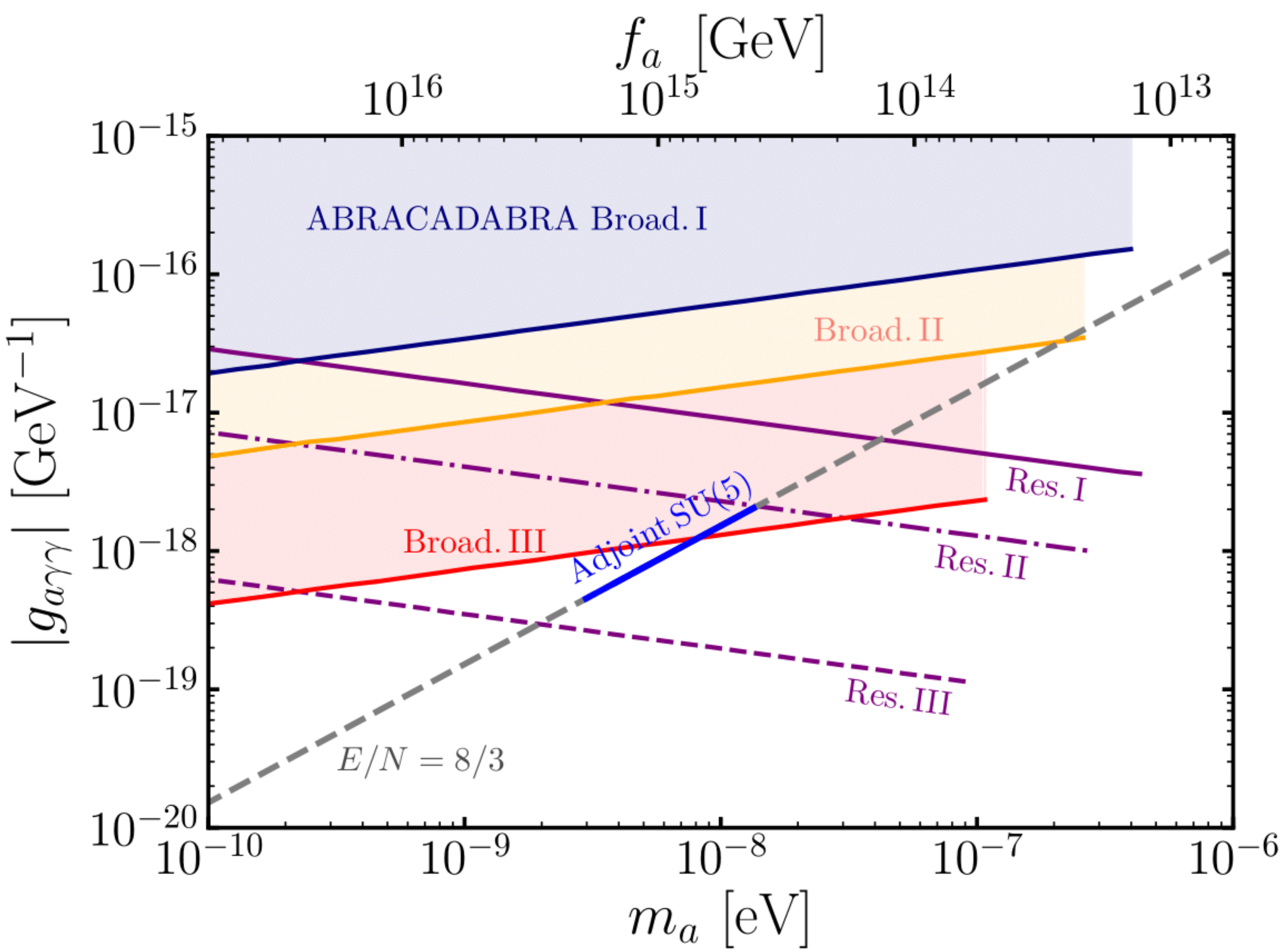


Res. I	$B_{\max} = 5\ T,\ V_B = 1\,\mathrm{m}^3$
Res. II	$B_{\max} = 20\,T,\ V_B = 1\,\mathrm{m}^3$
Res. III	$B_{\max} = 5\ T,\ V_B = 100\,\mathrm{m}^3$

SPOILER

The theory can be fully tested!

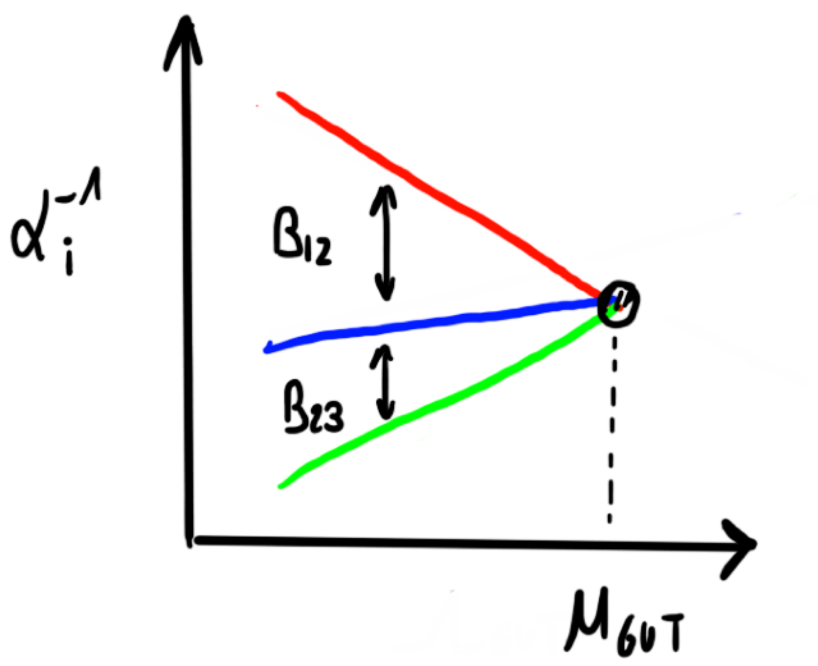
Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\mathrm{SM}} \ \mathrm{Ln}\frac{\mu}{M_Z} + \frac{1}{2\pi} \sum_I b_{iI} \ \Theta(\mu - M_I)\, \mathrm{Ln}\frac{\mu}{M_I}$$

Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\rm SM} \,{\rm Ln}\frac{\mu}{M_Z} + \frac{1}{2\pi}\sum_I b_{iI}\;\Theta(\mu - M_I)\,{\rm Ln}\frac{\mu}{M_I}$$

$$B_{ij} = (b_i - b_j)$$

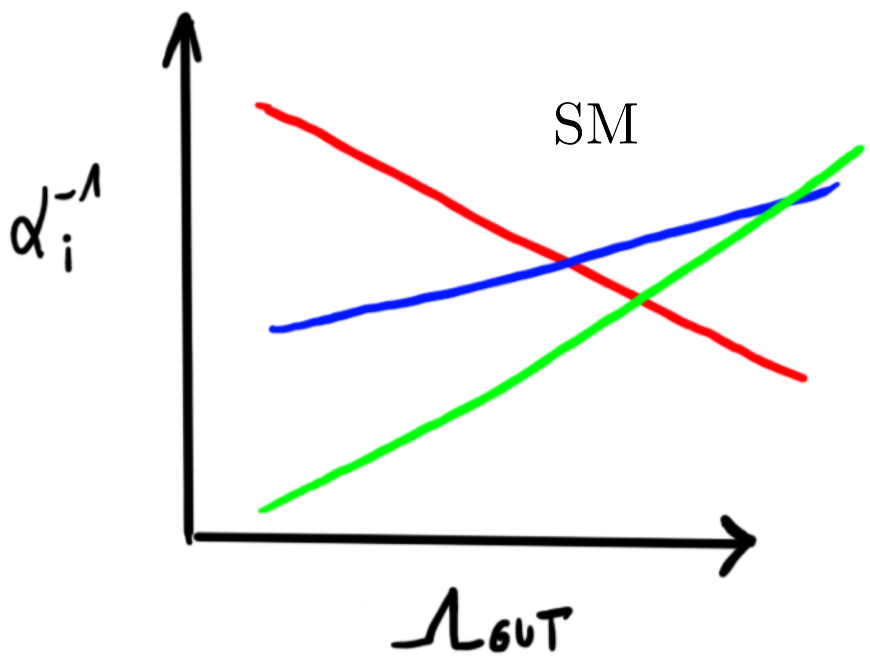


$$\frac{B_{23}}{B_{12}} = 0.718$$

$${\rm Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}$$

Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\rm SM}\ \mathrm{Ln}\frac{\mu}{M_Z}$$



$$B_{ij}^{\rm SM} = (b_i^{\rm SM} - b_j^{\rm SM})$$

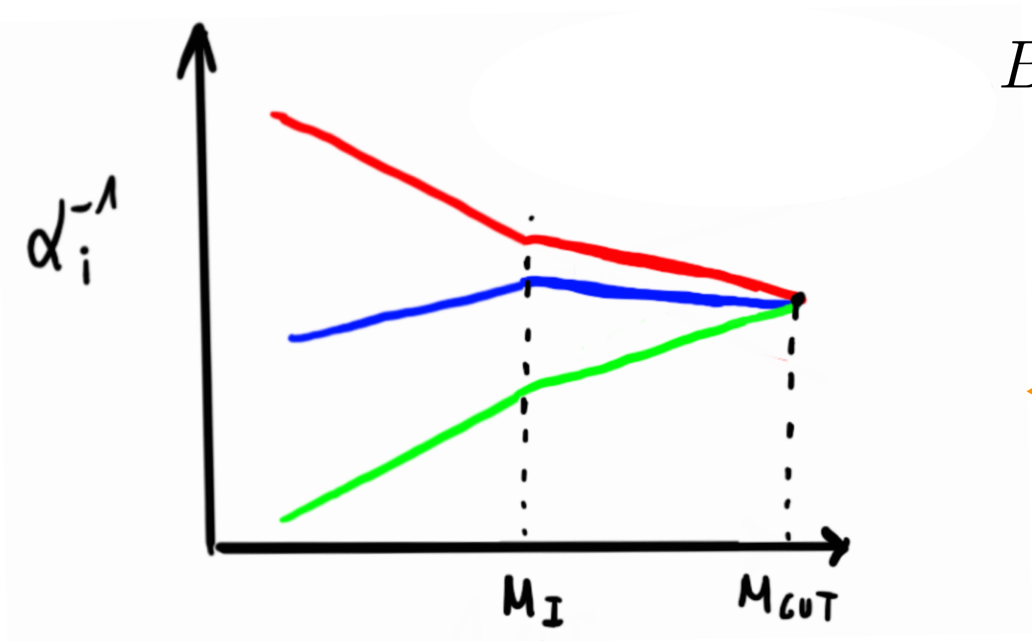
$$\xcancel{\frac{B_{23}}{B_{12}} = 0.718}$$

$$\xcancel{\mathrm{Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}}$$

Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\rm SM} \, {\rm Ln} \frac{\mu}{M_Z} + \frac{1}{2\pi} \sum_I b_{iI} \, \Theta(\mu - M_I) \, {\rm Ln} \frac{\mu}{M_I}$$

$$B_{ij}^I = (b_i^I - b_j^I) r_I \qquad\qquad r_I = \frac{{\rm Ln}(M_{\rm GUT}/M_I)}{{\rm Ln}(M_{\rm GUT}/M_Z)}$$



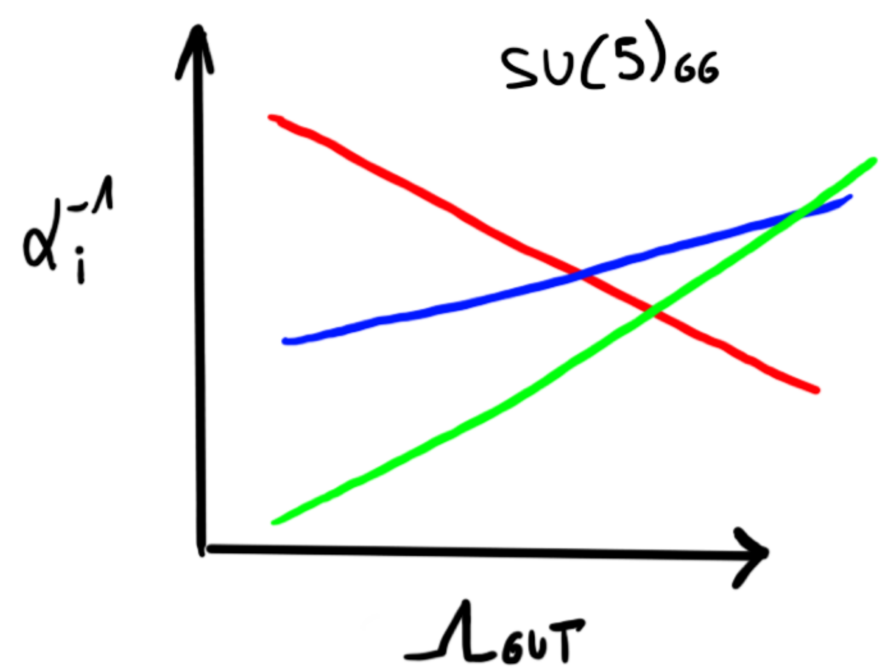
$r_I = 1$ $\qquad$ $r_I = 0$

$$\frac{B_{23}}{B_{12}} = 0.718$$

$$\text{Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}$$

Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\mathrm{SM}} \, \mathrm{Ln}\frac{\mu}{M_Z} + \frac{1}{2\pi}\sum_I b_{iI}\,\Theta(\mu - M_I)\,\mathrm{Ln}\frac{\mu}{M_I}$$



$$B_{ij}^I = (b_i^I - b_j^I)\, r_I \qquad\qquad r_I = \frac{\mathrm{Ln}(M_{\mathrm{GUT}}/M_I)}{\mathrm{Ln}(M_{\mathrm{GUT}}/M_Z)}$$

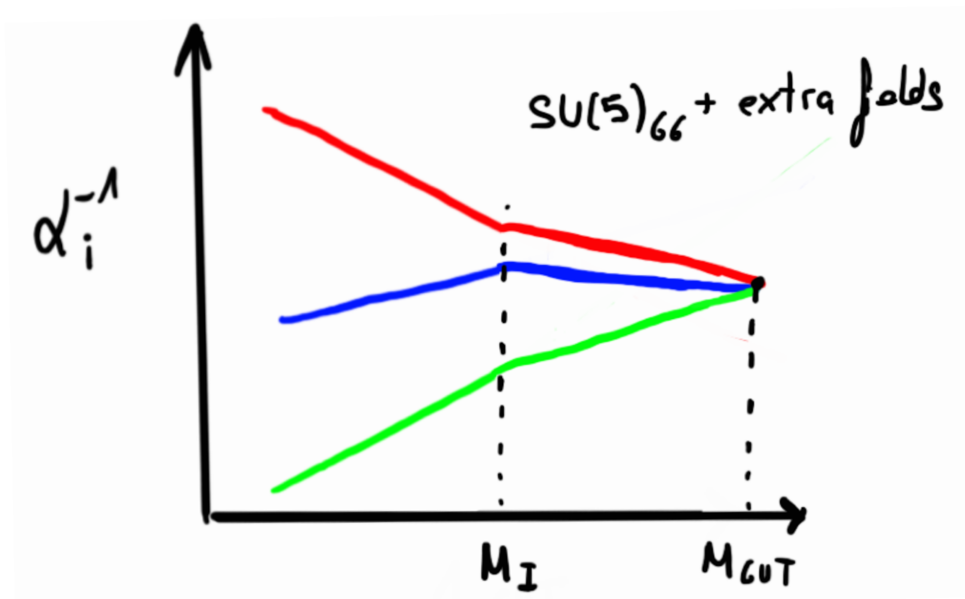
$$\cancel{\frac{B_{23}}{B_{12}} = 0.718}$$

$$\cancel{\mathrm{Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}}$$

$$\left(\frac{B_{23}}{B_{12}}\right)^{SU(5)_{GG}}_{\max} \sim 0.6$$

Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\rm SM}\ {\rm Ln}\frac{\mu}{M_Z} + \frac{1}{2\pi}\sum_I b_{iI}\ \Theta(\mu - M_I)\,{\rm Ln}\frac{\mu}{M_I}$$



$$B_{ij}^I = (b_i^I - b_j^I)\, r_I$$

$$\frac{B_{23}}{B_{12}} = 0.718$$

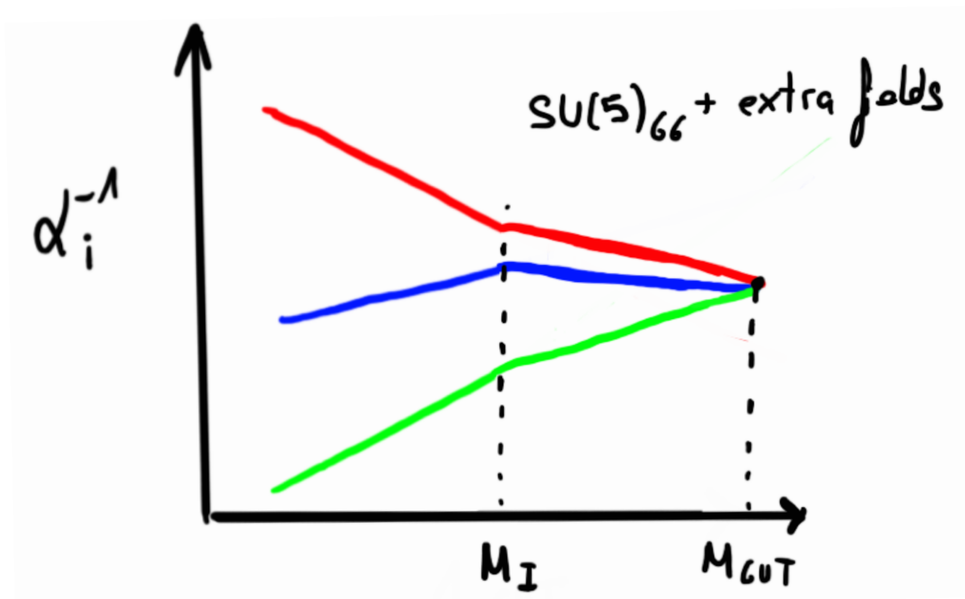
$${\rm Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}$$

$$\mathcal{L} \supset \cancel{M_{24}{\rm Tr}\{24^2\}} + \lambda {\rm Tr}\{24^2 24_H\} + {\rm h.c.}$$

	5_H		24			45_H						
B_{ij}	H_1	T	ρ_8	ρ_3	$\rho_{(3,2)} + \rho_{(\bar{3},2)}$	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	H_2
B_{12}	$-\frac{1}{15}$	$\frac{1}{15} r_T$	0	$-\frac{4}{3} r_{\rho 3}$	$\frac{4}{3} r_{32}$	$-\frac{8}{15} r_{\Phi_1}$	$\frac{2}{15} r_{\Phi_2}$	$-\frac{9}{5} r_{\Phi_3}$	$\frac{17}{15} r_{\Phi_4}$	$\frac{1}{15} r_{\Phi_5}$	$\frac{16}{15} r_{\Phi_6}$	$-\frac{1}{15} r_{H_2}$
B_{23}	$\frac{1}{6}$	$-\frac{1}{6} r_T$	$-2 r_{\rho 8}$	$\frac{4}{3} r_{\rho 3}$	$\frac{2}{3} r_{32}$	$-\frac{2}{3} r_{\Phi_1}$	$-\frac{5}{6} r_{\Phi_2}$	$\frac{3}{2} r_{\Phi_3}$	$\frac{1}{6} r_{\Phi_4}$	$-\frac{1}{6} r_{\Phi_5}$	$-\frac{1}{6} r_{\Phi_6}$	$\frac{1}{6} r_{H_2}$

Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\rm SM}\ {\rm Ln}\frac{\mu}{M_Z} + \frac{1}{2\pi}\sum_I b_{iI}\ \Theta(\mu - M_I)\,{\rm Ln}\frac{\mu}{M_I}$$



$$B_{ij}^I = (b_i^I - b_j^I) r_I$$

$$\frac{B_{23}}{B_{12}} = 0.718$$

$${\rm Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}$$

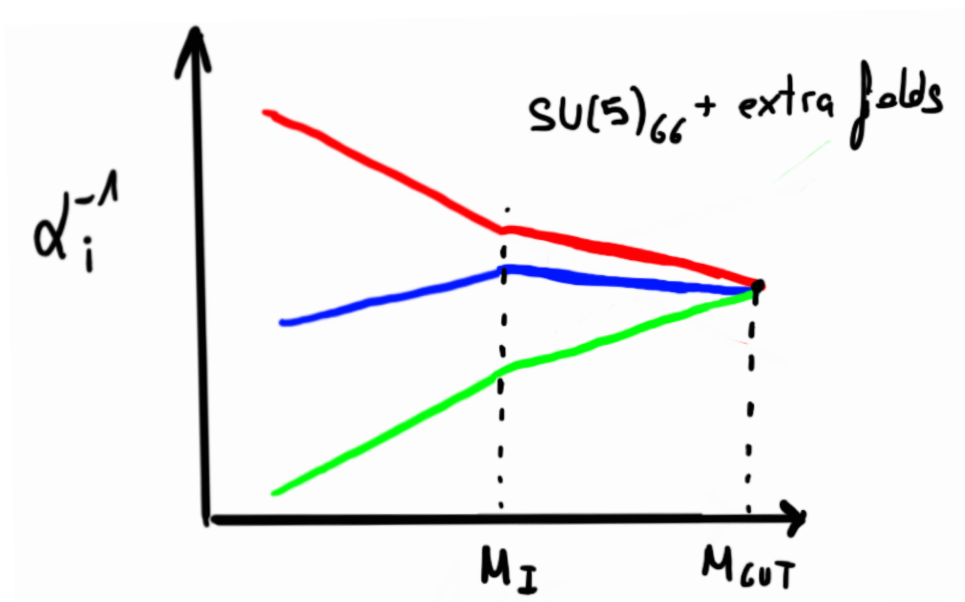
Help towards unification

$$\mathcal{L} \supset \cancel{M_{24}{\rm Tr}\{24^2\}} + \lambda{\rm Tr}\{24^2 24_H\} + {\rm h.c.}$$

	5_H		24			45_H						
B_{ij}	H_1	T	ρ_8	ρ_3	$\rho_{(3,2)} + \rho_{(\bar{3},2)}$	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	H_2
B_{12}	$-\frac{1}{15}$	$\frac{1}{15}r_T$	0	$-\frac{4}{3}r_{\rho 3}$	$\frac{4}{3}r_{32}$	$-\frac{8}{15}r_{\Phi_1}$	$\frac{2}{15}r_{\Phi_2}$	$-\frac{9}{5}r_{\Phi_3}$	$\frac{17}{15}r_{\Phi_4}$	$\frac{1}{15}r_{\Phi_5}$	$\frac{16}{15}r_{\Phi_6}$	$-\frac{1}{15}r_{H_2}$
B_{23}	$\frac{1}{6}$	$-\frac{1}{6}r_T$	$-2r_{\rho 8}$	$\frac{4}{3}r_{\rho 3}$	$\frac{2}{3}r_{32}$	$-\frac{2}{3}r_{\Phi_1}$	$-\frac{5}{6}r_{\Phi_2}$	$\frac{3}{2}r_{\Phi_3}$	$\frac{1}{6}r_{\Phi_4}$	$-\frac{1}{6}r_{\Phi_5}$	$-\frac{1}{6}r_{\Phi_6}$	$\frac{1}{6}r_{H_2}$

Window for the GUT scale

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\mu)} = \frac{1}{2\pi} b_i^{\mathrm{SM}}\ \mathrm{Ln}\frac{\mu}{M_Z} + \frac{1}{2\pi}\sum_I b_{iI}\ \Theta(\mu - M_I)\,\mathrm{Ln}\frac{\mu}{M_I}$$



$$B_{ij}^I = (b_i^I - b_j^I)\, r_I$$

$$\frac{B_{23}}{B_{12}} = 0.718$$

$$\mathrm{Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}$$

$$\mathcal{L} \supset \cancel{M_{24}\mathrm{Tr}\{24^2\}} + \lambda \mathrm{Tr}\{24^2 24_H\} + \mathrm{h.c.}$$

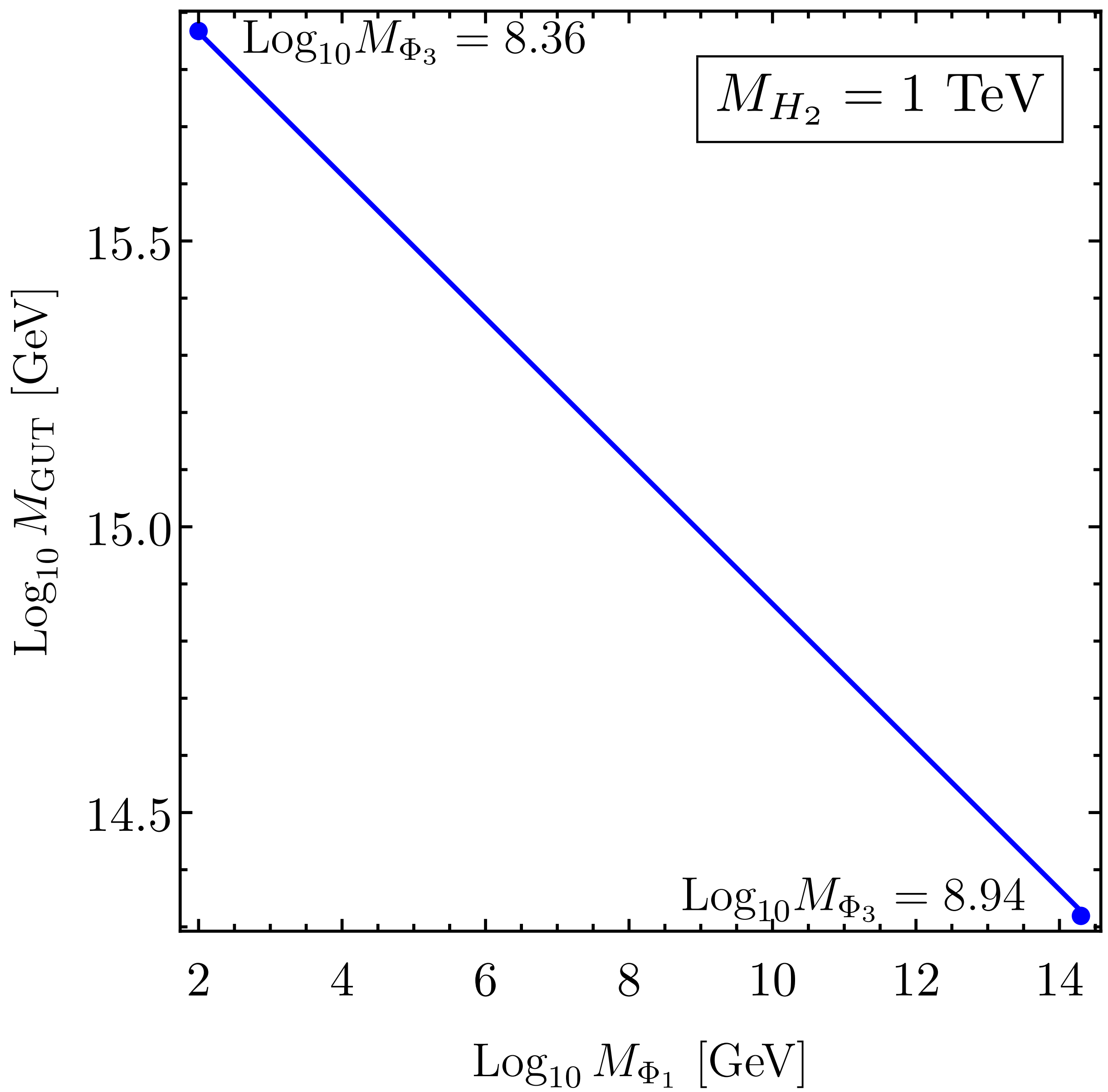
Help towards unification

Proton decay

	5_H		24			45_H						
B_{ij}	H_1	T	ρ_8	ρ_3	$\rho_{(3,2)} + \rho_{(\bar{3},2)}$	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	H_2
B_{12}	$-\frac{1}{15}$	$\frac{1}{15}r_T$	0	$-\frac{4}{3}r_{\rho 3}$	$\frac{4}{3}r_{32}$	$-\frac{8}{15}r_{\Phi_1}$	$\frac{2}{15}r_{\Phi_2}$	$-\frac{9}{5}r_{\Phi_3}$	$\frac{17}{15}r_{\Phi_4}$	$\frac{1}{15}r_{\Phi_5}$	$\frac{16}{15}r_{\Phi_6}$	$-\frac{1}{15}r_{H_2}$
B_{23}	$\frac{1}{6}$	$-\frac{1}{6}r_T$	$-2r_{\rho 8}$	$\frac{4}{3}r_{\rho 3}$	$\frac{2}{3}r_{32}$	$-\frac{2}{3}r_{\Phi_1}$	$-\frac{5}{6}r_{\Phi_2}$	$\frac{3}{2}r_{\Phi_3}$	$\frac{1}{6}r_{\Phi_4}$	$-\frac{1}{6}r_{\Phi_5}$	$-\frac{1}{6}r_{\Phi_6}$	$\frac{1}{6}r_{H_2}$

Window for the GUT scale

Allowed by unification constraints



$$\boxed{\frac{B_{23}}{B_{12}} = 0.718 \qquad \mathrm{Ln}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}}$$

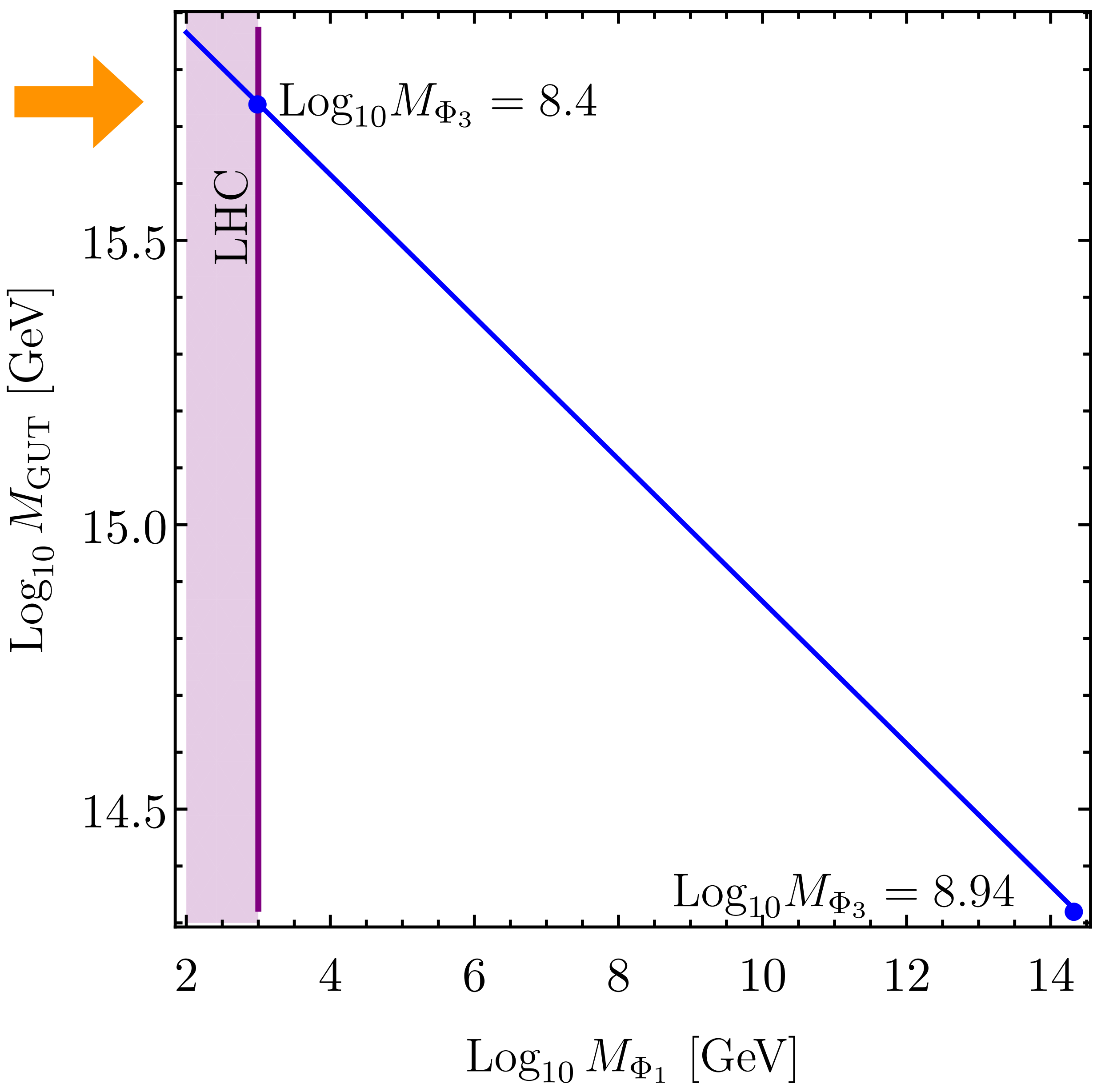
$$M_{\mathrm{GUT}} = f(M_{\Phi_1}, M_{\Phi_3}, M_{H_2})$$

$\Rightarrow$ only sensitive to splitting in **24**

$$\Phi_1 \sim (8, 2, 1/2)$$

Collider bounds

$\Phi_1 \sim (8, 2, 1/2)$

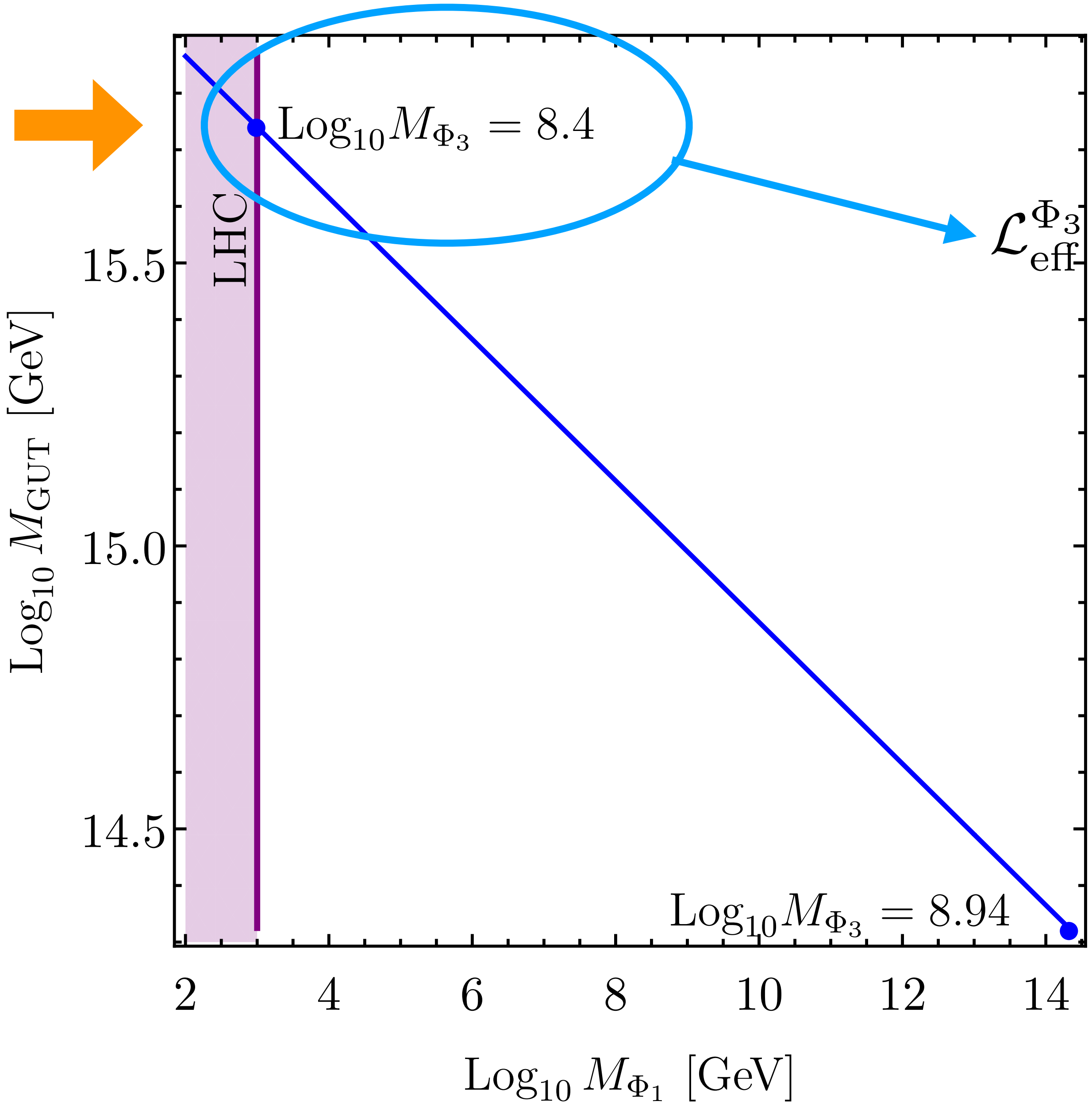


Miralles, Pich

$M_{\Phi_1} \geq 1$ TeV $\Rightarrow \boxed{M_{\mathrm{GUT}} \leq 10^{15.87}\ \mathrm{GeV}}$

Collider bounds

$\Phi_3 \sim (3, 3, -1/3)$



Miralles, Pich

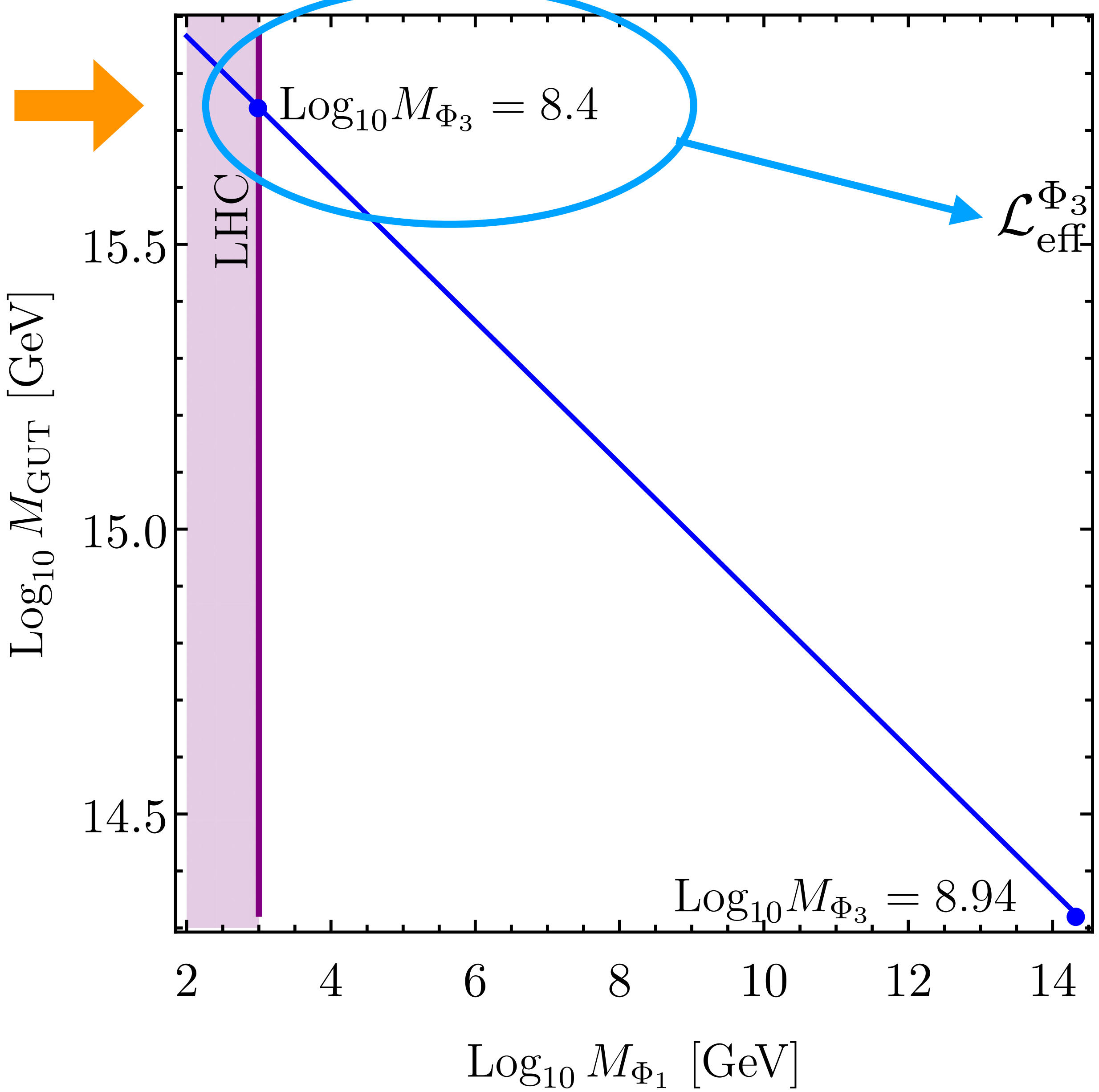
$$M_{\Phi_1} \geq 1 \text{ TeV} \Rightarrow \boxed{M_{\text{GUT}} \leq 10^{15.87} \text{ GeV}}$$

$$\mathcal{L}_{\text{eff}}^{\Phi_3} \supset \frac{1}{M_{\Phi_3}^2} (\overline{L_\alpha^c} Y_2 P_L Q_\beta) \epsilon^{\beta\gamma} (\overline{Q^{ac}} (Y_4 - Y_4^T) P_L Q_L)$$

$$\Gamma_p \sim Y_2^2 (Y_4 - Y_4^T)^2 \frac{M_p^5}{M_{\Phi_3}^4}$$

Collider bounds

$\Phi_3 \sim (3, 3, -1/3)$



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$$\Gamma_p \sim Y_2^2 (Y_4 - Y_4^T)^2 \frac{M_p^5}{M_{\Phi_3}^4}$$

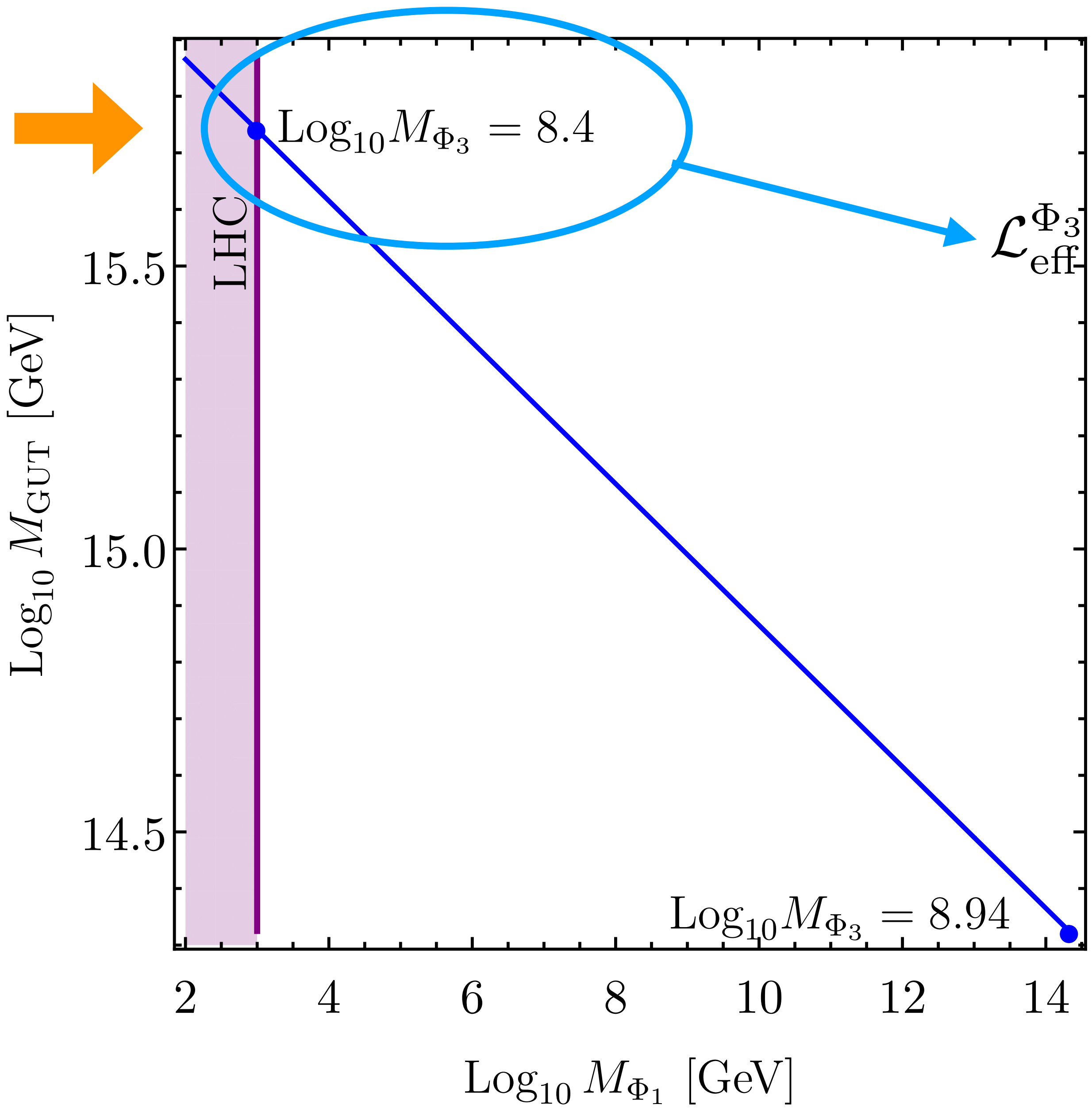
$$M_d = \frac{Y_1}{2} v_5^* + Y_2 v_{45}^*$$

$$M_e = \frac{Y_1^T}{2} v_5^* - 3 Y_2^T v_{45}^*$$

$$M_u = 2 (Y_3 + Y_3^T) \frac{v_5}{\sqrt{2}} - \cancel{4 (Y_4 - Y_4^T) \frac{v_{45}}{\sqrt{2}}}$$

Collider bounds

$\Phi_3 \sim (3, 3, -1/3)$



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$$M_d = \frac{Y_1}{2} v_5^* + Y_2 v_{45}^*$$

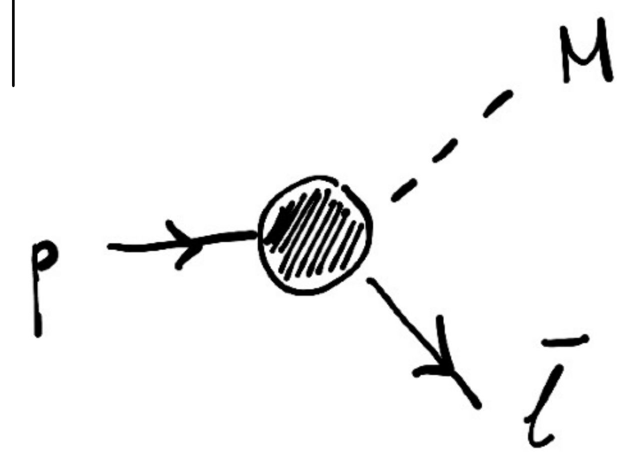
$$M_e = \frac{Y_1^T}{2} v_5^* - 3 Y_2^T v_{45}^*$$

$$M_u = 2 (Y_3 + Y_3^T) \frac{v_5}{\sqrt{2}} - 4 \cancel{(Y_4 - Y_4^T)} \frac{v_{45}}{\sqrt{2}}$$

$$Y_U \equiv (Y_3 + Y_3^T) = Y_U^T$$

Proton Decay

$$\Gamma(p \to M\bar{\ell}) = \frac{m_p}{32\pi}\left(1 - \left(\frac{m_p}{m_M}\right)^2\right)^2 \left|\sum_i \langle M|\mathcal{O}_i^{X,Y}|p\rangle\right|^2$$



Proton Decay

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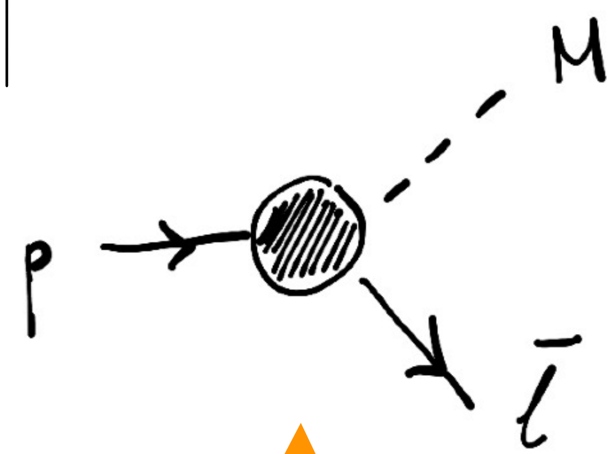


- Proton decay mediators:

$$\mathcal{L}_p \supset i\bar{\bar{5}}\,\gamma^\mu D_\mu \bar{5} + \frac{i}{2}\mathrm{Tr}\{\overline{10}\gamma^\mu D_\mu 10\}$$
$$- \quad M_{GUT}^2\left(\mathrm{Tr}\{X_\mu^{cT} C X^\mu + Y_\mu^{cT} C Y^\mu\}\right) + \mathrm{h.c.}$$

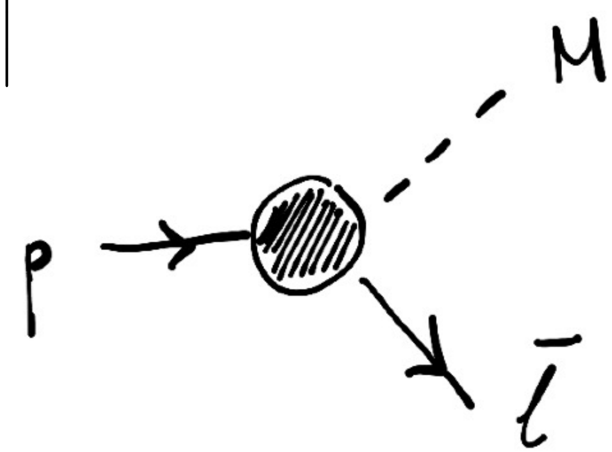
$$V_\mu \sim (3, 2, -5/6)$$

➡ Integrate X and Y fields out

$$V_\mu^c = \begin{pmatrix} X_\mu \\ Y_\mu \end{pmatrix} \begin{matrix} \to \mathcal{Q}_X = 4/3 \\ \to \mathcal{Q}_Y = 1/3 \end{matrix}$$

Proton Decay

$$\Gamma(p \to M\bar{\ell}) = \frac{m_p}{32\pi}\left(1-\left(\frac{m_p}{m_M}\right)^2\right)^2 \left|\sum_i \langle M|\mathcal{O}_i^{X,Y}|p\rangle\right|^2$$



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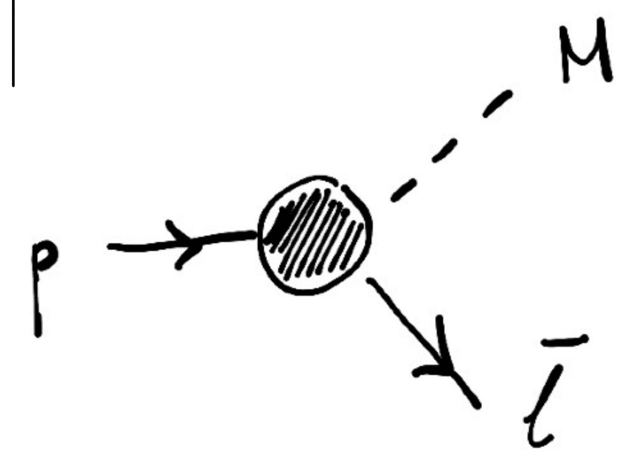
$$- \quad M_{GUT}^2\left(\mathrm{Tr}\{X_\mu^{cT} C X^\mu + Y_\mu^{cT} C Y^\mu\}\right) + \text{h.c.}$$

$$\Rightarrow \quad \mathcal{L}_{d=6}^p = \frac{g_{GUT}^2}{2M_{GUT}^2}\,\overline{u^c}\gamma_\mu Q^\alpha(\overline{e^c}\gamma^\mu \epsilon_{\alpha\beta} Q^\beta + \overline{d^c}\gamma^\mu \epsilon_{\beta\alpha} L^\beta) + \text{h.c.}$$

Proton Decay

$$\Gamma(p \to M\bar{\ell}) = \frac{m_p}{32\pi}\left(1-\left(\frac{m_p}{m_M}\right)^2\right)^2 \left|\sum_i \langle M|\mathcal{O}_i^{X,Y}|p\rangle\right|^2$$



- Proton decay mediators:

$$\mathcal{L}_p \supset i\bar{\bar{5}}\,\gamma^\mu D_\mu \bar{5} + \frac{i}{2}\mathrm{Tr}\{\overline{10}\gamma^\mu D_\mu 10\}$$

$$- \quad M_{GUT}^2\left(\mathrm{Tr}\{X_\mu^{cT} C X^\mu + Y_\mu^{cT} C Y^\mu\}\right) + \text{h.c.}$$

$$\mathcal{L}_{d=6}^p = \frac{g_{GUT}^2}{2M_{GUT}^2}\,\overline{u^c}\gamma_\mu Q^\alpha(\overline{e^c}\gamma^\mu\epsilon_{\alpha\beta}Q^\beta + \overline{d^c}\gamma^\mu\epsilon_{\beta\alpha}L^\beta) + \text{h.c.}$$

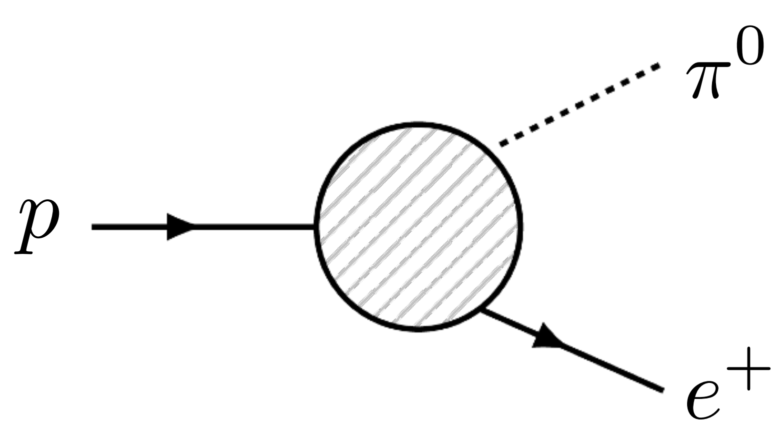
Running factor
$M_{GUT} \to \Lambda_{QCD}$

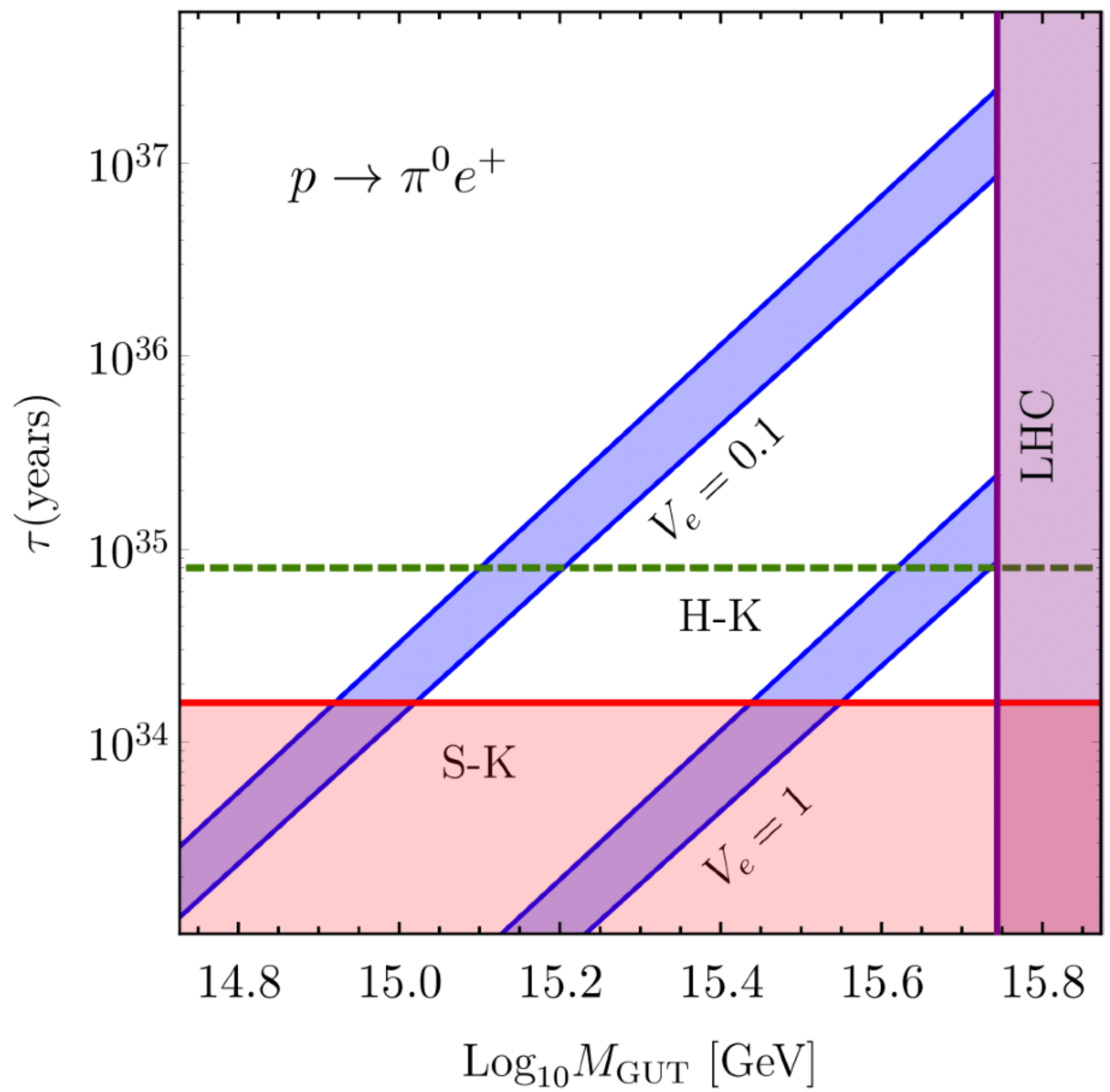
$$(x,y)_{R/L} \equiv x^T C P_{R/L}\, y$$

$$\Gamma(p \to M\bar{\ell}) = A_R^2 \frac{\pi\, m_p}{2}\left(1-\left(\frac{m_p}{m_M}\right)^2\right)^2 \left(\frac{\alpha_{GUT}}{M_{GUT}^2}\right)^2 \left|\sum_i c_i \langle M|\overline{\ell^c}(q_1 q_2)_{L/R}\; q_{3\,L/R}|p\rangle\right|^2$$

Proton Decay

$$\Gamma(p \to \pi^0 e^+) = \frac{\pi\, m_p}{2} \left(\frac{\alpha_{\rm GUT}}{M_{\rm GUT}^2} \right)^2 A_R^2 \, V_e \, |\langle \pi^0 | (ud)_R u_L | p \rangle|^2$$





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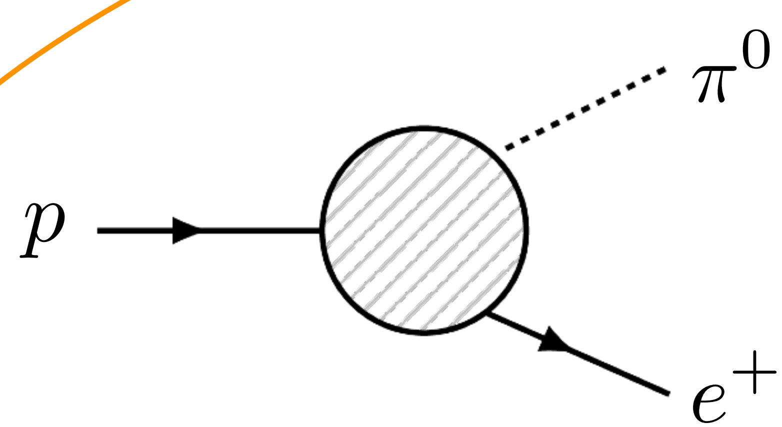
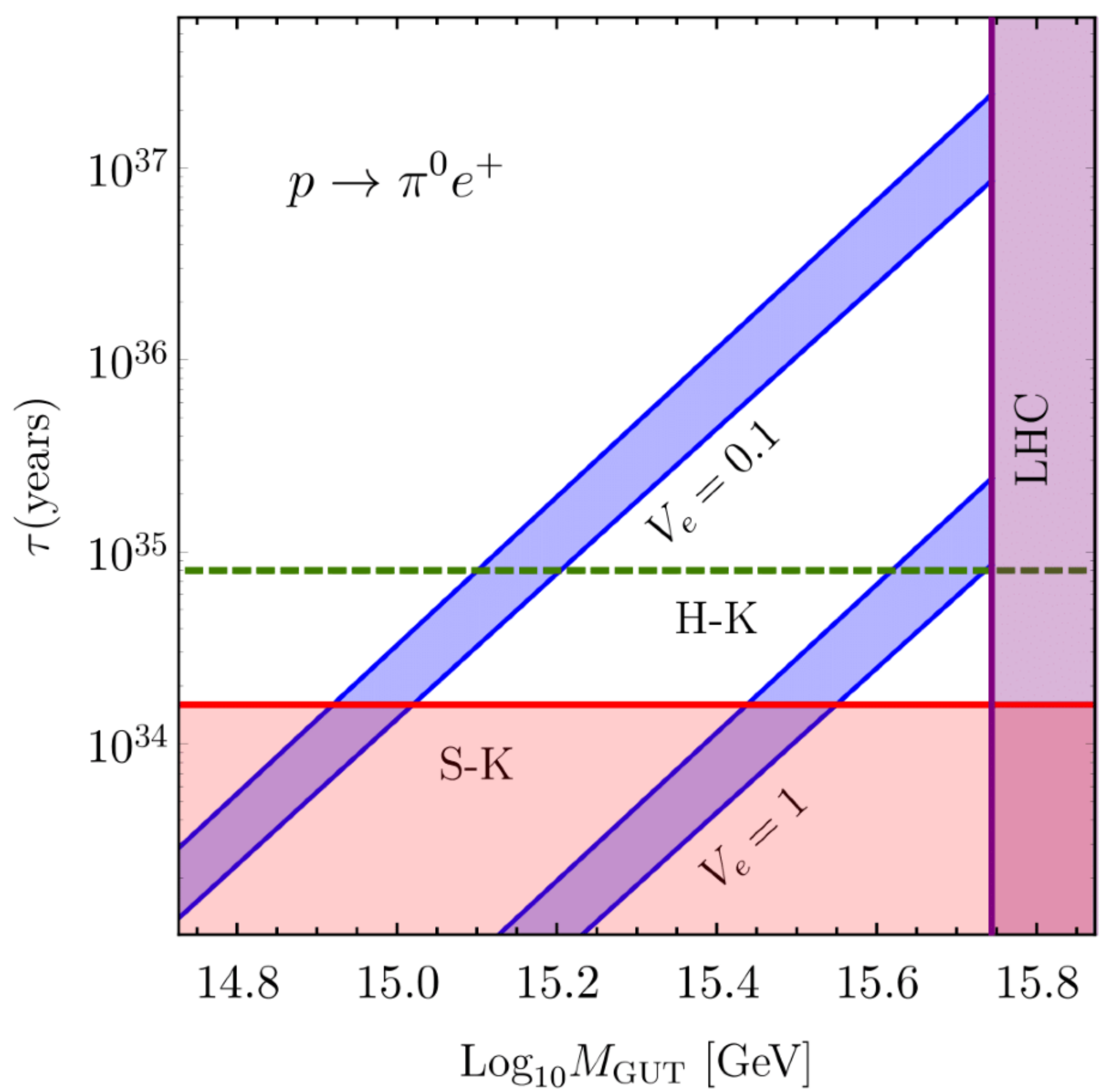


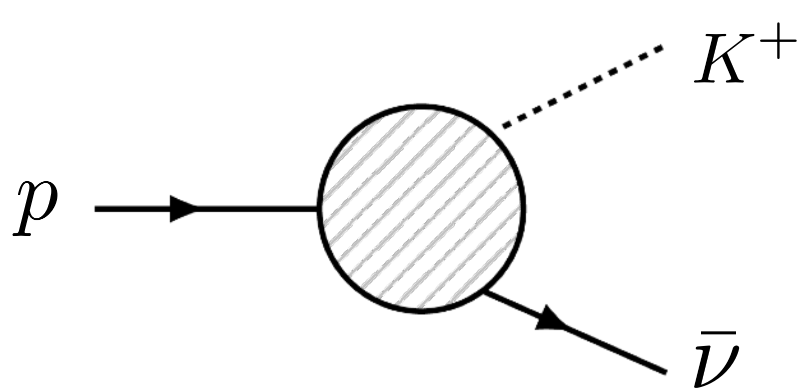
$$\alpha_{\rm GUT} = f(M_{\rm GUT}, M_{24})$$

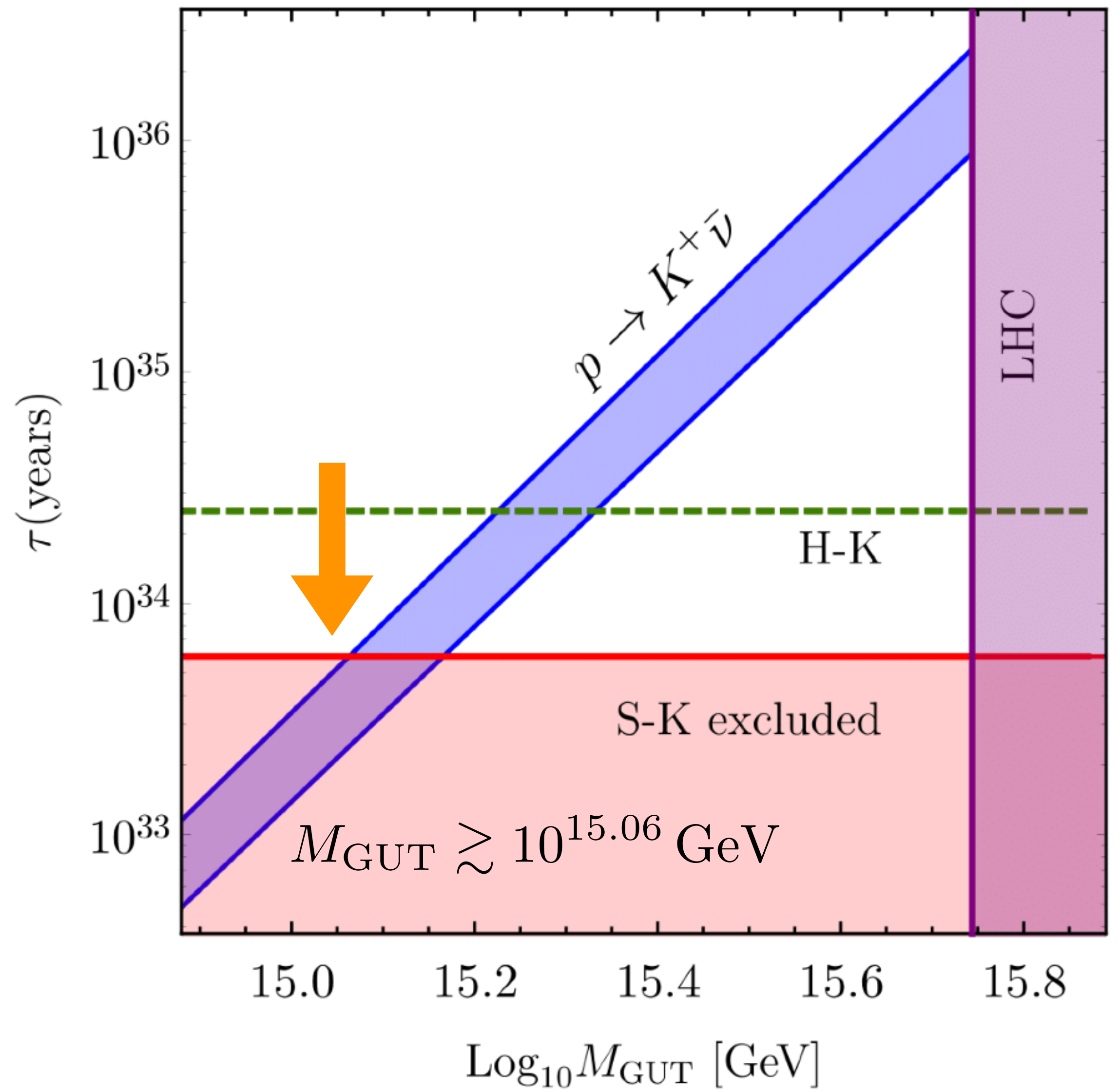
$$10^{4.51}\mathrm{GeV} \lesssim M_{24} \lesssim 10^{15}\mathrm{GeV}$$



Proton Decay

$$\Gamma(p \to K^+\bar{\nu}) = \frac{\pi\, m_p}{2}\left(\frac{\alpha_{\rm GUT}}{M_{\rm GUT}^2}\right)^2 \times \left(1 - \frac{m_{K^+}^2}{m_p^2}\right)^2 A_R^2 \big(|(V_1 V_{CKM})^{11}\langle K^+|(us)_R d_L|p\rangle|^2$$
$$+\,|(V_1 V_{CKM})^{12}\langle K^+|(ud)_R s_L|p\rangle|^2\big)$$



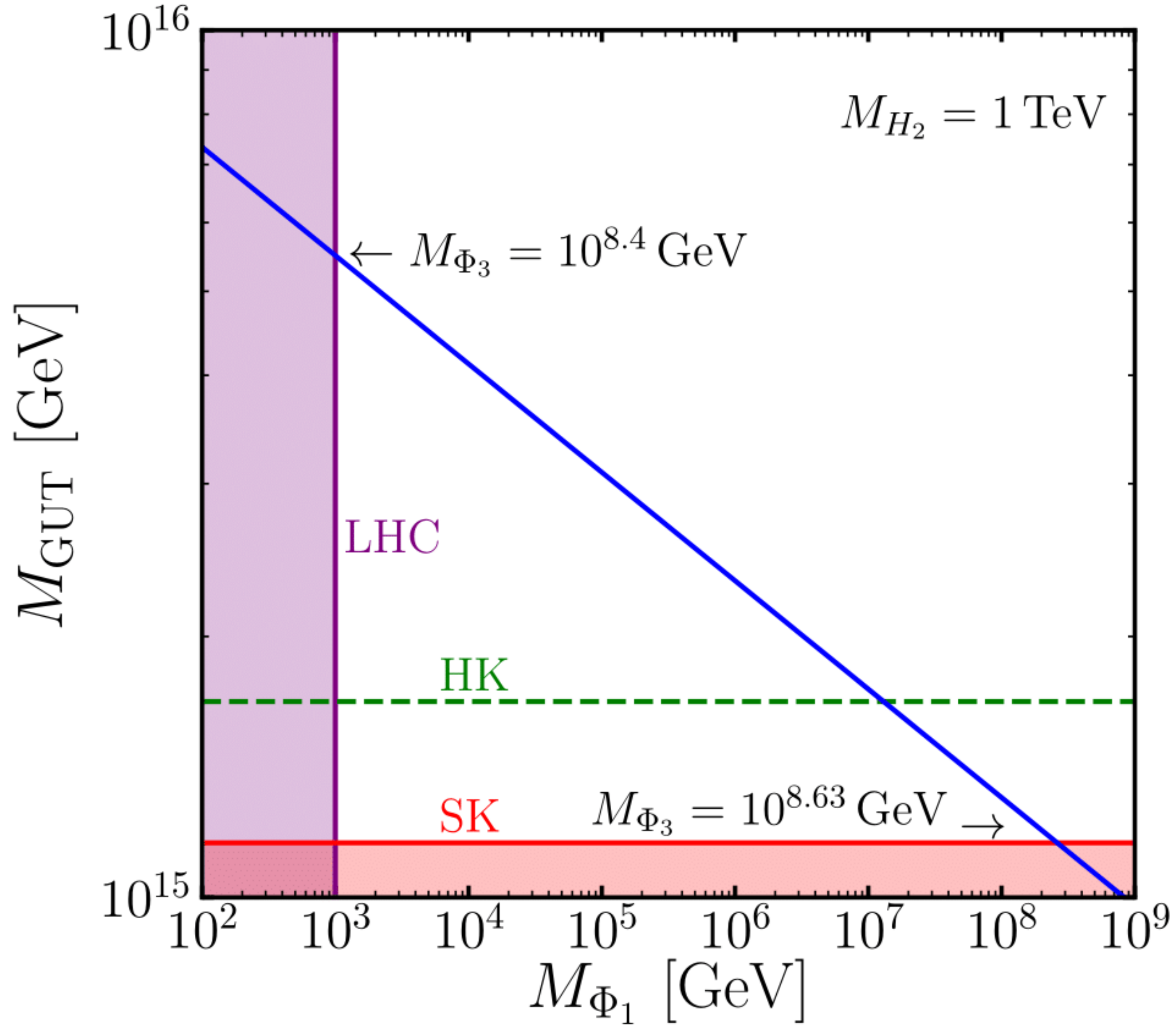


- Mixing matrices from fermions:

$$V_1 \equiv U_c^\dagger U$$

$$Y_U \sim (Y_3 + Y_3)^T = Y_U^T \Rightarrow V_1 \sim \mathbb{I}\, e^{I\delta}$$

Window for GUT Scale

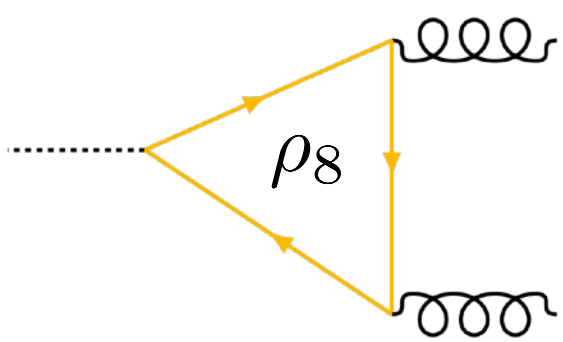


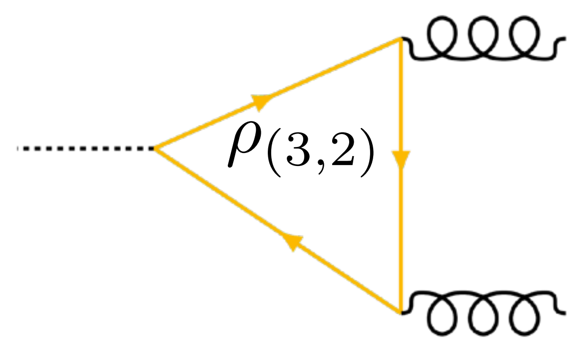
$$\boxed{M_{\rm GUT} = [10^{15.06} - 10^{15.74}]\ \text{GeV}}$$

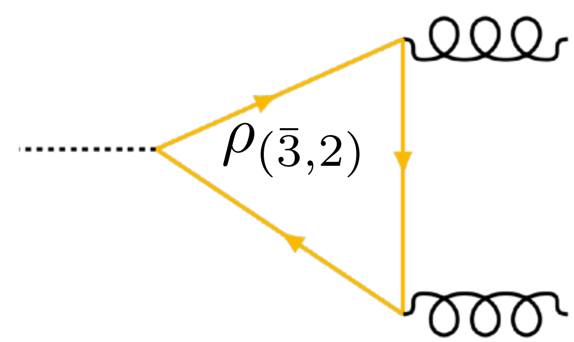
Axion couplings

$$\mathcal{L} \supset \frac{g_S^2}{32\pi^2}\frac{a}{v_\Sigma} N G\tilde{G} \equiv \frac{g_S^2}{32\pi^2}\frac{a}{f_a} G\tilde{G}$$

$$N = \sum_{\substack{\text{colored}\\ \text{chiral}\\ \text{fields}}} C_{\Psi_L^i} T_D[R_{\Psi_L^i}] \times \mathrm{mult}[\Psi_L^i] \; = \left(\underbrace{3 \times 1}_{\rho_8} + \underbrace{\frac{1}{2} \times 2}_{\rho_{(3,2)}} + \underbrace{\frac{1}{2} \times 2}_{\rho_{(\bar{3},2)}} \right) = 5$$





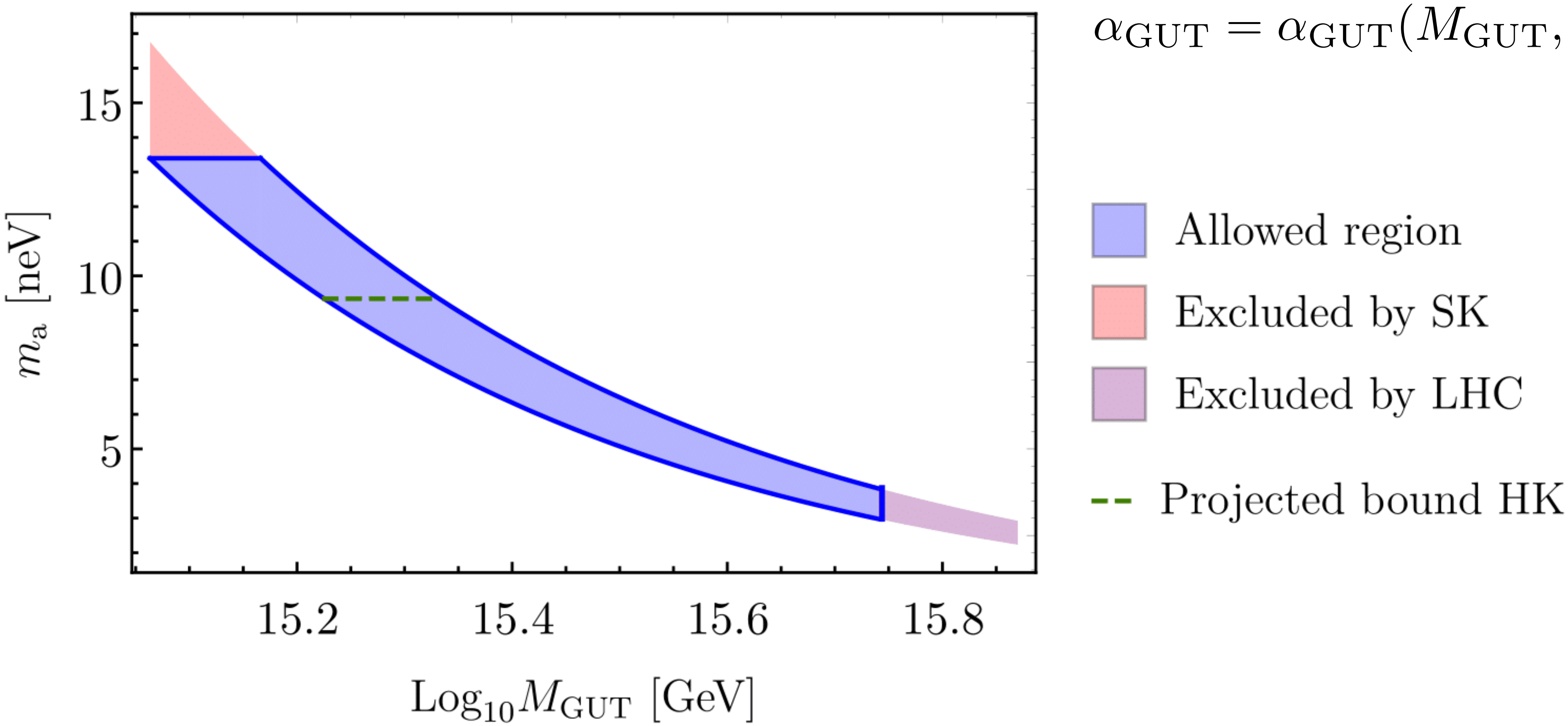


$$f_a \equiv \frac{N}{v_\Sigma} = \sqrt{\frac{6}{5\pi\alpha_{\mathrm{GUT}}}} \frac{M_{\mathrm{GUT}}}{10}$$

Axion Mass

$$m_a = 5.70(7) \times 10^{-6}\ \mathrm{eV} \left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right) = 5.70(7) \times 10^{-6}\ \mathrm{eV} \sqrt{\frac{5\pi\alpha_{\mathrm{GUT}}}{6}} \left(\frac{10^{13}\ \mathrm{GeV}}{M_{\mathrm{GUT}}}\right)$$

$$\alpha_{\mathrm{GUT}} = \alpha_{\mathrm{GUT}}(M_{\mathrm{GUT}}, M_{24})$$

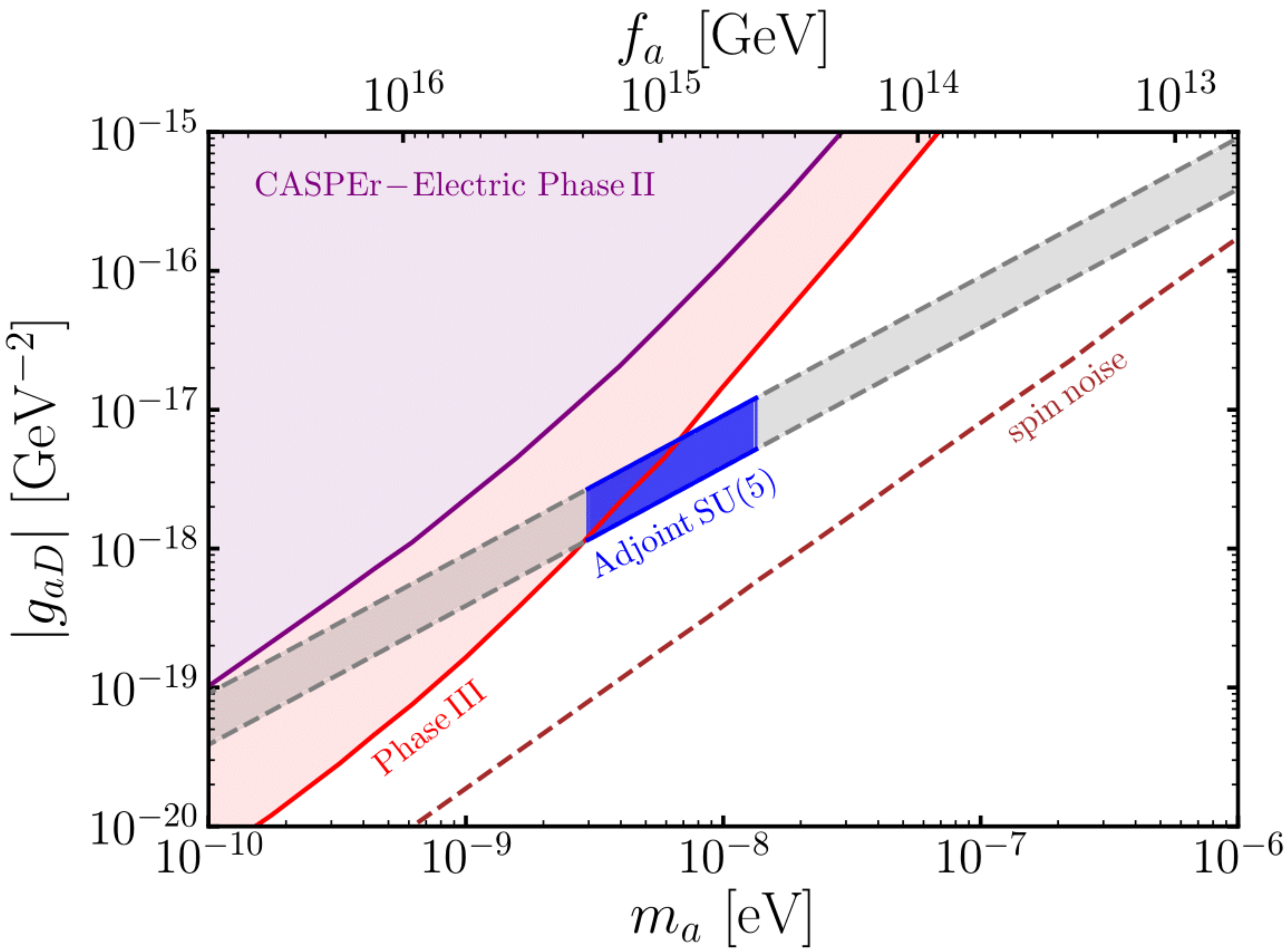


$$M_{\mathrm{GUT}} = [10^{15.06} - 10^{15.74}]\ \mathrm{GeV} \Rightarrow \boxed{m_a = (2.98 - 13.4) \times 10^{-9}\ \mathrm{eV}}$$

Axion-EDM coupling

$$\mathcal{L} \supset -\frac{i}{2} g_{aD}\, a(\overline{\Psi_N} \sigma^{\mu\nu} \gamma_5 \Psi_N) F_{\mu\nu}$$

$$d_n = g_{aD}\, a \approx 2.4 \times 10^{-16} \frac{a}{f_a} e \cdot \mathrm{cm}$$



Axion coupling to photons

$$g_{a\gamma\gamma} = \frac{\alpha_{\mathrm{EM}}}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right)$$

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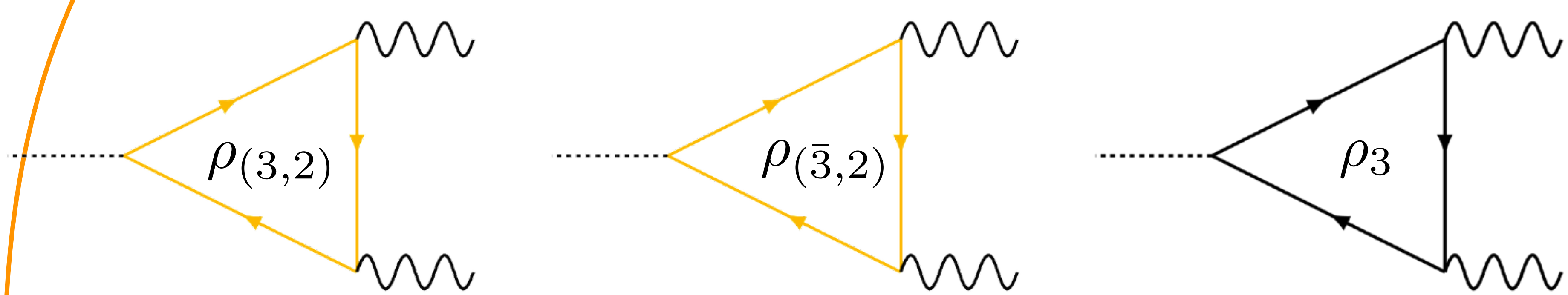
Complete SU(5) representation



$$E = \left(\underbrace{\left(\frac{4}{3}\right)^2 \times 3 + \left(\frac{1}{3}\right)^2 \times 3}_{\rho_{(3,2)}} + \underbrace{\left(\frac{4}{3}\right)^2 \times 3 + \left(\frac{1}{3}\right)^2 \times 3}_{\rho_{(\bar{3},2)}} + \underbrace{1^2 + (-1)^2}_{\rho_3} \right) = \frac{40}{3}$$

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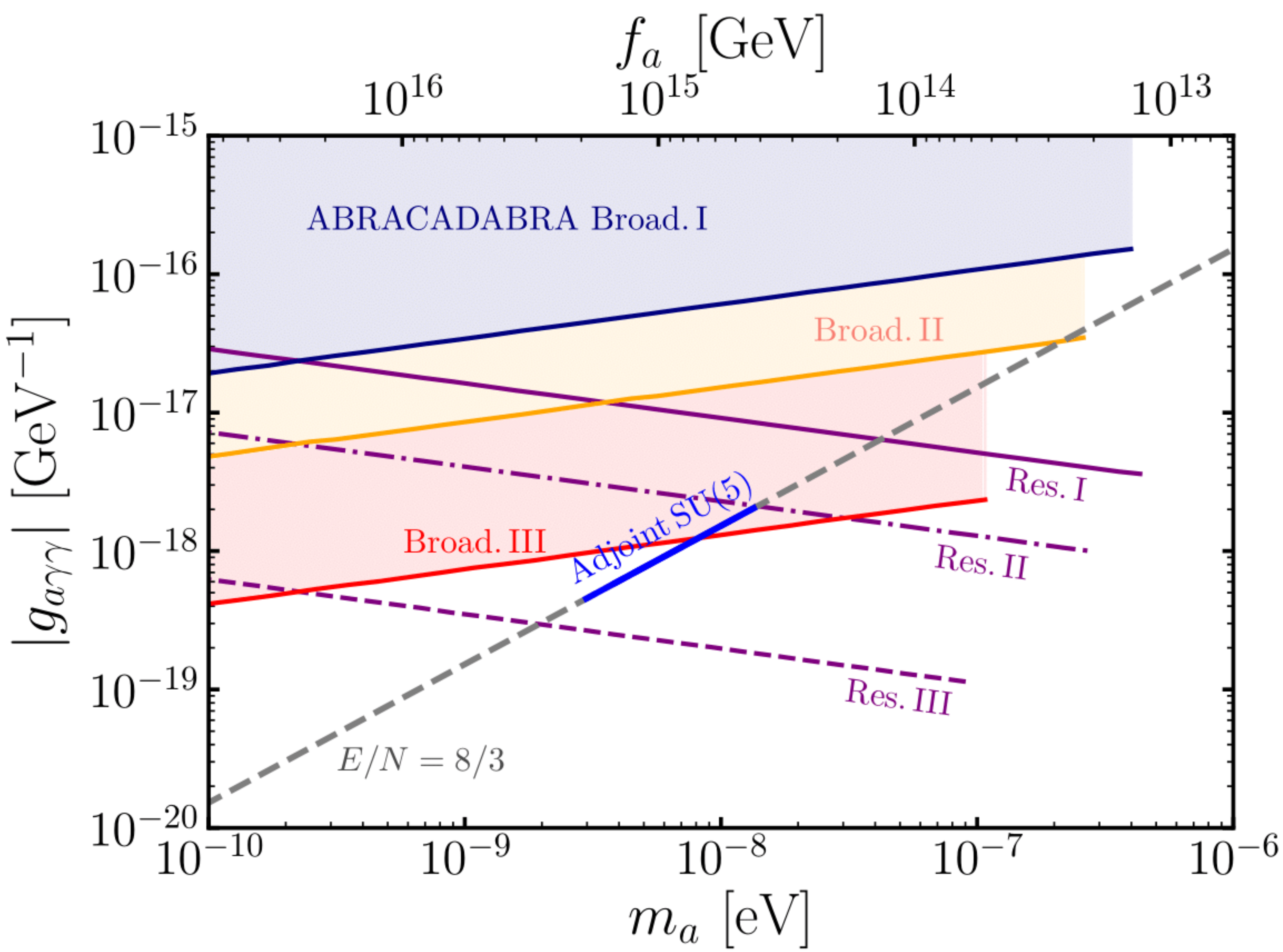

Res. I	$B_{\rm max} = 5\ \ T,\ V_B = 1\,{\rm m}^3$
Res. II	$B_{\rm max} = 20\,T,\ V_B = 1\,{\rm m}^3$
Res. III	$B_{\rm max} = 5\ \ T,\ V_B = 100\,{\rm m}^3$

Conclusions

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- The theory could be fully tested by the **ABRACADABRA** and the **CASPEr-Electric** experiments.

- Appealing theory!

Thank you for your attention!!

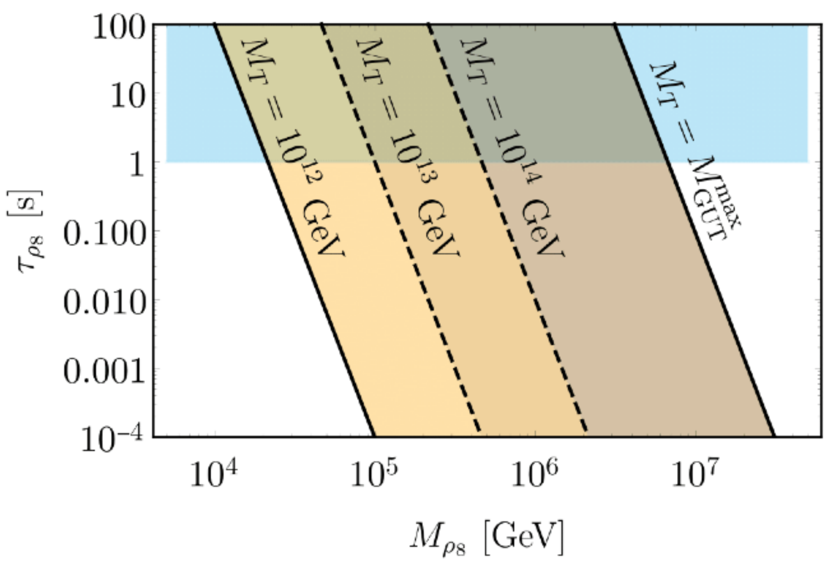
Alpha GUT

$B_{23}, B_{12} = 0$ ⇒ RGE only sensitive to splitting in **24**

BUT $b_i \neq 0 \Rightarrow \alpha_{\rm GUT} = \alpha_{\rm GUT}(M_{\rho_3})$

Range of M_{ρ_3}?

- Lower bound



BBN: $\tau_{\rho_8} \lesssim 0.1$ s $\Rightarrow M_{\rho_3} \gtrsim 10^{4.51}$ GeV

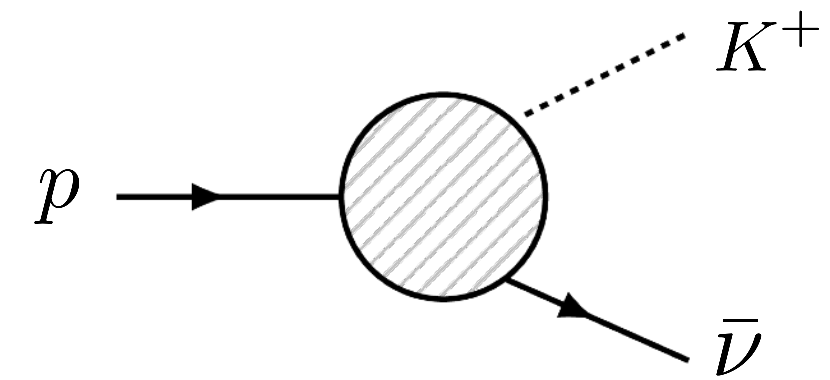
- Upper bound

$$M_\nu \simeq \frac{h_1^2 v_0^2}{M_{\rho_3}}$$

Perturbativity Yukawa:

$\Rightarrow M_{\rho_3} \lesssim 10^{15}$ GeV

Proton Decay



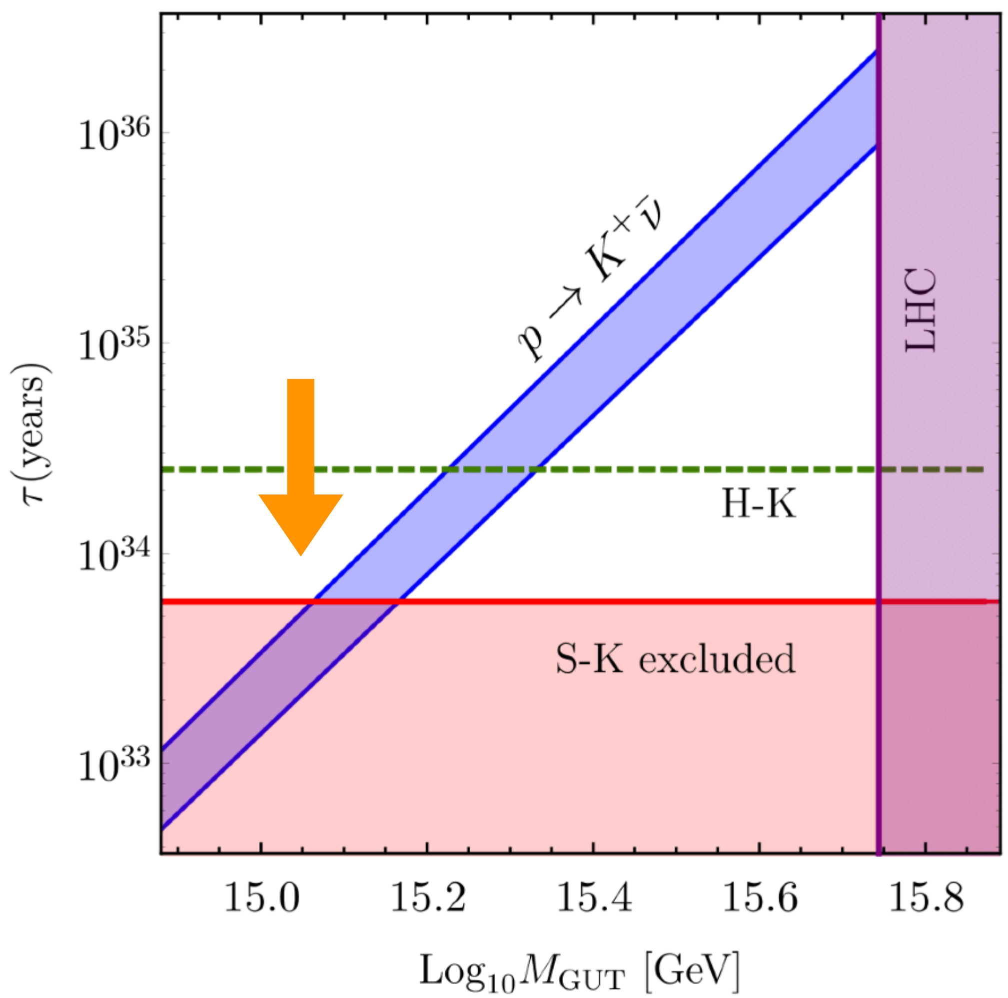
$$\Gamma(p \to K^+\bar{\nu}) = \frac{\pi\, m_p}{2}\left(\frac{\alpha_{GUT}}{M_{GUT}^2}\right)^2\left(1-\frac{m_{K^+}^2}{m_p^2}\right)^2 A_R^2$$
$$\times \sum_i \left|(V_1 V_{UD})^{11}(V_3 V_{EN})^{2i}\langle K^+|(us)_R d_L|p\rangle + (V_1 V_{UD})^{21}(V_3 V_{EN})^{1i}\langle K^+|(ud)_R s_L|p\rangle\right|^2$$
$$= \frac{\pi\, m_p}{2}\left(\frac{\alpha_{GUT}}{M_{GUT}^2}\right)^2\left(1-\frac{m_{K^+}^2}{m_p^2}\right)^2 A_R^2\left(|V_{CKM}^{11}\langle K^+|(us)_R d_L|p\rangle|^2 + |V_{CKM}^{12}\langle K^+|(ud)_R s_L|p\rangle|^2\right)$$

$$Y_U \sim (Y_3 + Y_3)^T = Y_U^T \Rightarrow V_1 \sim \mathbb{I}\, e^{I\delta}$$

- Mixing matrices:

$$V_1 \equiv U_c^\dagger U, \quad V_2 \equiv E_C^\dagger D, \quad V_3 \equiv D_C^\dagger E$$
$$V_{UD} \equiv U^\dagger D \equiv V_{CKM}, \quad V_{EN} \equiv E^\dagger N$$

$$A_c Y_A A \equiv Y_A^{\text{diag}}$$



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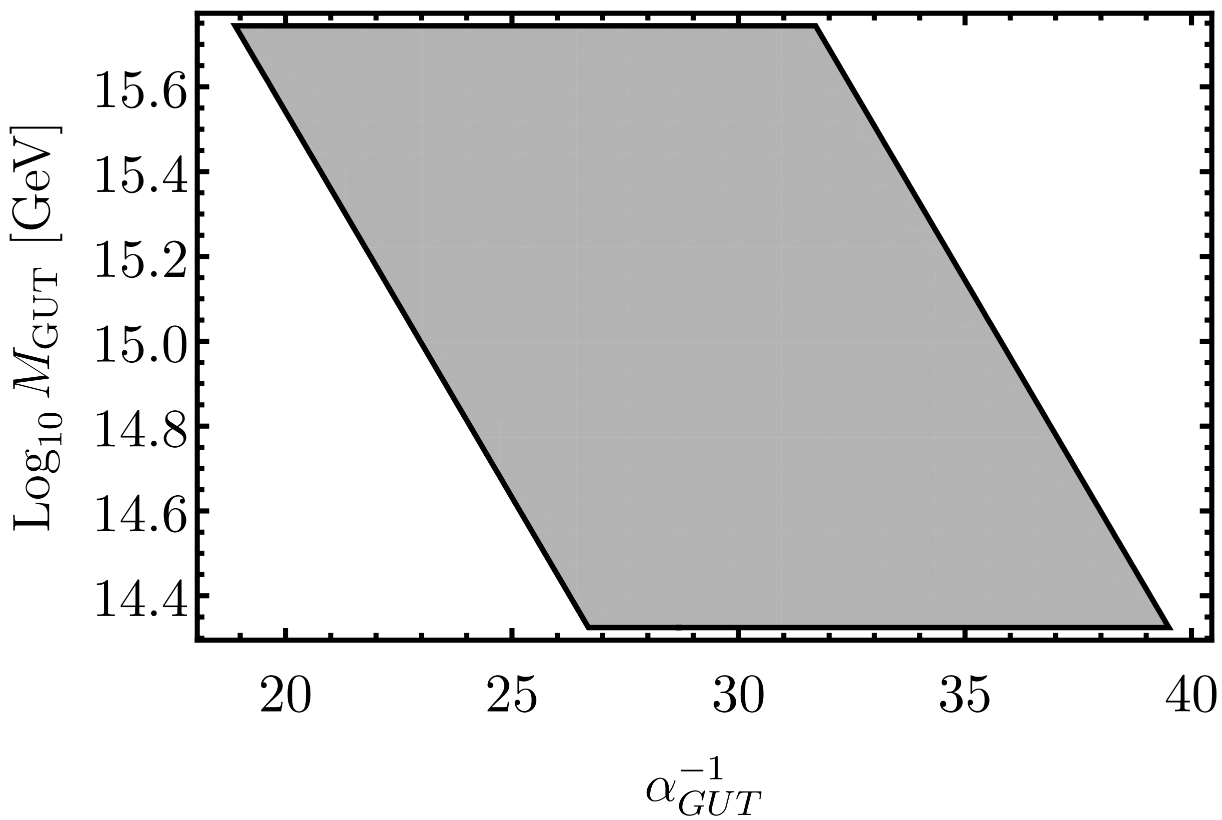

$$10^{4.51}\ \mathrm{GeV} \lesssim M_{\rho_3} \lesssim 10^{15}\ \mathrm{GeV}$$