Electronic properties of fractal lattices

Marta Brzezińska, Titus Neupert

Introduction

- Fractals exhibit **self-similarity** and **scale invariance**; these features are often desirable for technological applications (e.g. antennae and capacitor designs) as well as fundamental research (quantum interference, optical transmission)
- Fractals have non-integer Hausdorff dimension, which leads to fundamentally different physical properties; $d_H = lnA/lnL$
- **Motivation**: What kind of phases can artificial fractal lattices host?
  What is the effect of disorder on such systems?

Model

We consider TB model describing spinless fermions in an external magnetic field

$$H = t \sum_{\langle i,j \rangle} e^{iA_{ij}} c_i^\dagger c_j + h.c.$$

with on-site disorder term $\frac{W}{2} \sum_i c_i^\dagger c_i$

Disorder and level spacing distribution

Let $s = (\lambda_{i+1} - \lambda_i) / \langle s \rangle$ be normalized spacing between eigenvalues. If the (weakly) disordered system is not time-reversal invariant, the level spacing distribution P(s) is given by Wigner-Dyson distribution

$$P_{GUE}(s) = \frac{32}{\pi^2} s^2 e^{-4s^2/\pi}$$

Localized states (strong disorder) follow the Poisson distribution .

$$P(s) = e^{-s}$$

Local markers

Real-space quantities allow to capture properties of disordered systems.

- the Chern number over finite N x N mesh, regardless of boundary conditions

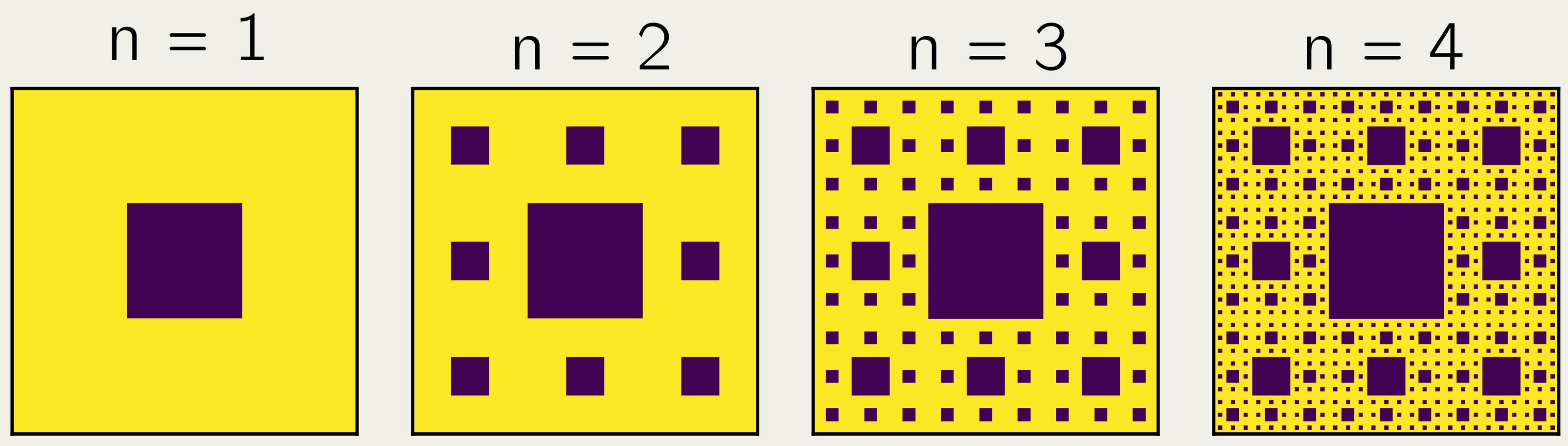
$$C = -\frac{2\pi i}{N^2} \sum_{\mathbf{n},\alpha} \langle \mathbf{n}, \alpha | P[-i[\hat{x}_1, P], -i[\hat{x}_2, P]] | \mathbf{n}, \alpha \rangle$$

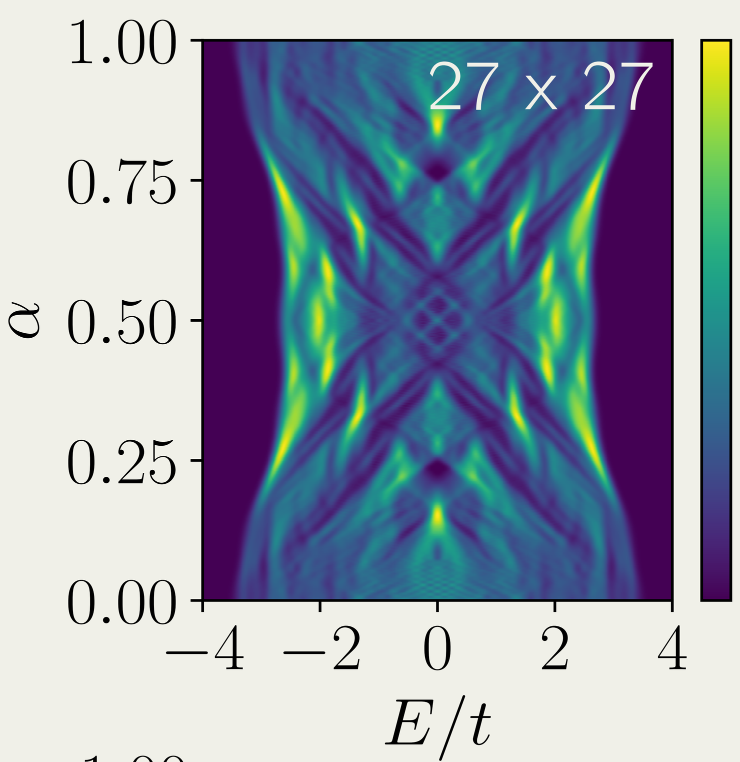
- edge-locality marker, which measures the localization of each energy eigenstate

$$\mathfrak{B}_\lambda = \sum_{\mathbf{r} \in \text{edge}} |\psi_\lambda(\mathbf{r})|^2$$

Sierpinski carpet

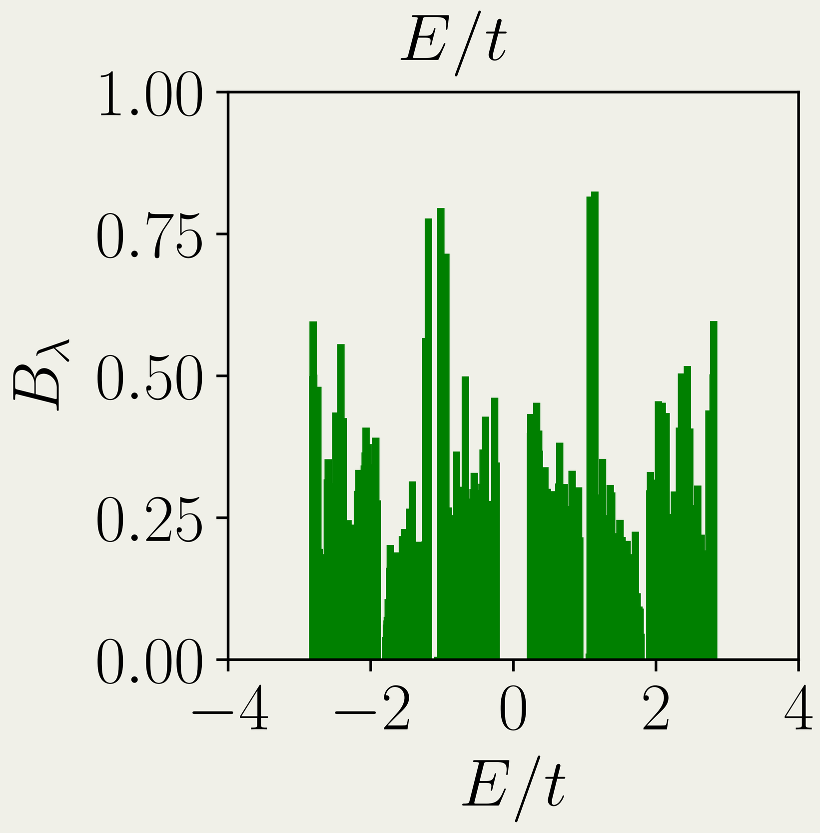
SC can be constructed iteratively



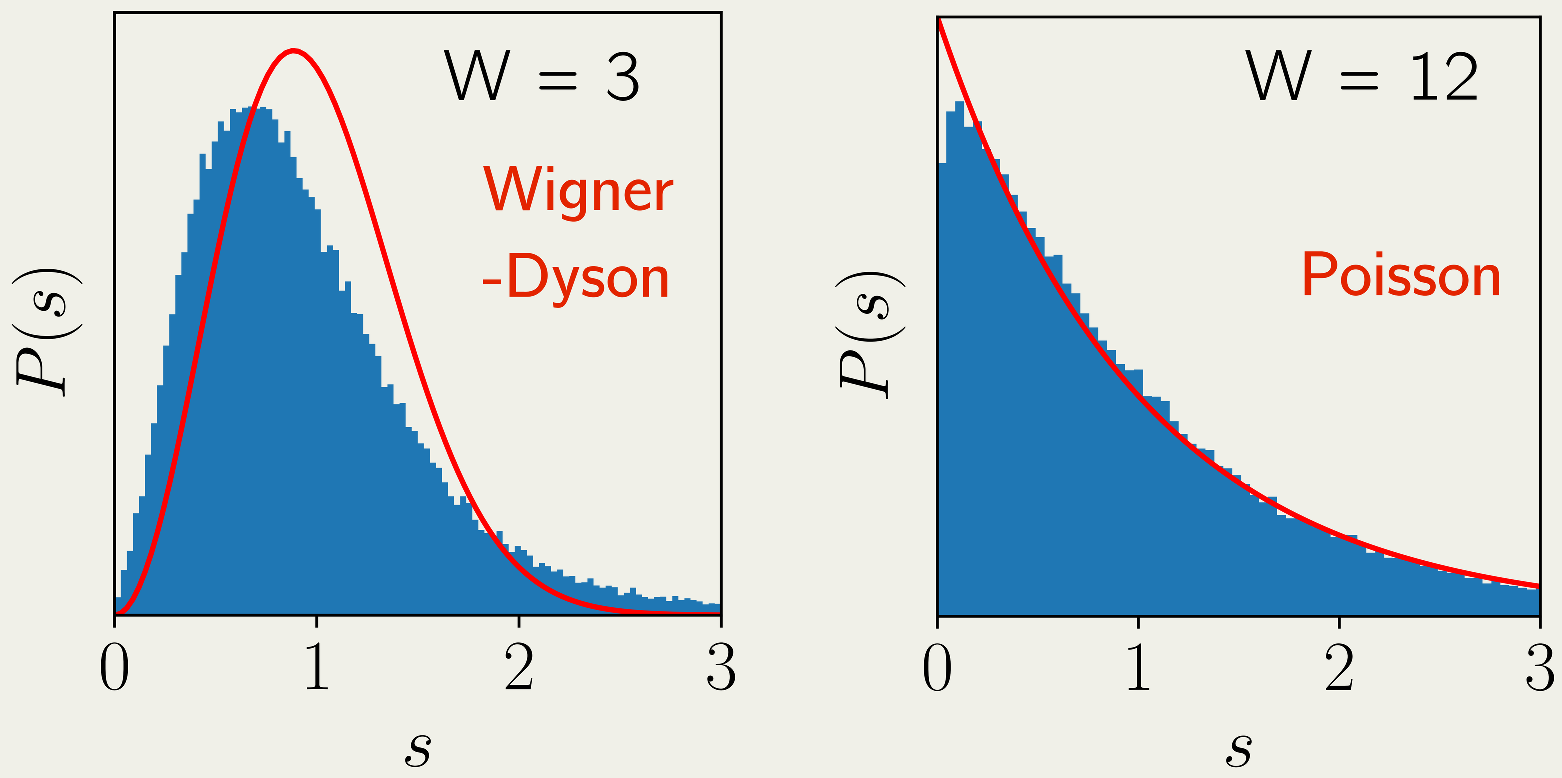


Density of states as a function of magnetic flux

- similar structure to Hofstadter butterfly



- states in the low-DOS regions are localized at the edges of the sample; results for $\alpha = \phi/2\pi \approx 0.24$



Current work

- studies of Pascal's triangle series
- application of the single-particle entanglement spectrum to fractal lattices