Towards NNLO accuracy for ε′/ε

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Outline

- CP Violation
- ε'/ε Anatomy
- NNLO calculation
- Future improvements
- Conclusions
- Discussion

Big mystery of the Universe

Very early in the Universe might expect equal numbers of baryons & anti-baryons.

However, today the Universe is matter dominated.
(no evidence for anti-galaxies, etc.)

How did this happen?

Matter-antimatter asymmetry

A. Baryon violating interactions

B. CP violation [Andrei Skharov. '67]

C. Thermal non-equilibrium situation

CP violation

CP violation is an essential aspect of our understanding of the Universe.

There are two places in the SM where CP violation enters:

a. The PMNS matrix
b. The CKM matrix

To date CP violation has been observed only in the quark sector & the SM is unable to account for the observed matter-antimatter asymmetry in the Universe.

We need more CP violation

(new sources of CP violation at high energy scales)

CP violation in Kaons

Two possible explanations of CP violation in the kaon system:

A. K_L is a superposition of CP states:

Indirect CP violation: parameter eK

B. CP is violated in the decay of K_L:

Direct CP violation: parameter e'

Defining the CP violation ratios

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | \mathcal{H}_{eff} | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_{eff} | K_S \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{H}_{eff} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_{eff} | K_S \rangle}$$

Indirect & Direct CP violation can be expressed

$$\varepsilon = (\eta_{00} + 2\eta_{+-})/3 \qquad \varepsilon' = (\eta_{+-} - \eta_{00})/3$$

Direct CP violation

A non-zero value of Re(e′/e) signals that direct CPV exists

$$\boxed{\mathrm{Re}(e'/e)=1/6(1-|\eta_{00}/\eta_{+-}|^2)}$$

The measured quantity is the double ratio of the decay widths

$$\mathrm{R} = \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^+\pi^-)\Gamma(K_S \to \pi^0\pi^0)}$$

(a long series of precision counting experiments)

From NA48 and KTeV collaborations:

$$\boxed{\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{exp}} = (16.6 \pm 2.3) \times 10^{-4}}$$

ε′/ε in the SM I

$$\langle \pi^0\pi^0 | \mathcal{H}_{\rm eff} | K^0 \rangle = A_0\, e^{i\delta_0} + A_2\, e^{i\delta_2}/\sqrt{2}$$
$$\langle \pi^+\pi^- | \mathcal{H}_{\rm eff} | K^0 \rangle = A_0\, e^{i\delta_0} - A_2\, e^{i\delta_2}/\sqrt{2}$$
$$\langle \pi^+\pi^0 | \mathcal{H}_{\rm eff} | K^0 \rangle = 3A_2^+\, e^{i\delta_2^+}/2$$

A_0 & A_2:
Isospin amplitudes
for isospin conservation

$$\boxed{A_I\, e^{i\delta_I} \equiv \langle (\pi\pi)_I | \mathcal{H}_{\rm eff} | K \rangle}$$

Normalise to K+ decay (ω_+,a)
and ϵ_K
expand in A_2/A_0 and CP violation

A_0, A_2 & A_2^+
from experiment
[Cirigliano. et. al. '11]

The CPV is parametrised as,

$$\frac{\varepsilon'}{\varepsilon} = -i\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|}e^{i(\delta_2-\delta_0-\phi_{\varepsilon_K})}\left[\frac{\mathrm{ImA_0}}{\mathrm{ReA_0}}(1-\hat{\Omega}_{\rm eff}) - \frac{1}{a}\frac{\mathrm{ImA_2}}{\mathrm{ReA_2}}\right]$$

[Buras, Gorbahn, Jäger, Jamin '15] [Cirigliano et. al. '11]

ε′/ε in the SM II

$$\boxed{\omega_+ = a\,\frac{\mathrm{ReA}_2}{\mathrm{ReA}_0} = (4.53 \pm 0.02) \times 10^{-2}}$$

From experiment

[Cirigliano et. al. '03]

$$\frac{\varepsilon'}{\varepsilon} = -\,i\,\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|}\,e^{i(\delta_2-\delta_0-\phi_{\varepsilon_K})}\left[\frac{\mathrm{ImA}_0}{\mathrm{ReA}_0}\left(1-\hat{\Omega}_{\mathrm{eff}}\right)-\frac{1}{a}\,\frac{\mathrm{ImA}_2}{\mathrm{ReA}_2}\right]$$

From experiment

Leading isospin breaking

[Cirigliano et. al. '03]

$$\boxed{A_I\,e^{i\delta_I} \equiv \langle(\pi\pi)_I|\mathcal{H}_{\mathrm{eff}}|K\rangle = \sum_i C_i\,\langle(\pi\pi)_I|Q_i|K\rangle}$$

First-ever calculation with controlled errors

[Blum et. al., Bai et. al. '15]

K->ππ decays

CP symmetry is broken by the complex phase appearing in the quark mixing matrix

$$V^{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

s-d $\lambda^5 \sim 10^{-4}$

The CP violation is small because of flavour suppression

Weak Effective Theory

Effective Hamiltonian at $\mu < m_c$

$$\boxed{\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} \big(z_i(\mu) + \tau y_i(\mu) \big) Q_i}$$

$$\tau \equiv - \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}}$$

Perturbative Wilson coefficients

Only the imaginary part of tau is responsible for CPV

(everything else is pure-real)

Theoretically very complicated multi-scale problem

(weak scale, bottom, charm, QCD scale)

Operators I

Current-Current:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}}(\bar{u}_\beta d_\alpha)_{\mathrm{V-A}}, \quad Q_2 = (\bar{s}u)_{\mathrm{V-A}}(\bar{u}d)_{\mathrm{V-A}}$$

Large coefficients, but CP-conserving (y=0).

Account for K->pipi decay rates.

QCD-Penguins:

$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V-A}}, \quad Q_4 = (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V+A}}, \quad Q_6 = (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}$$

$O(\alpha_s)$ but CP-violating (y=!0).

However, isospin-0 final state only.

Operators II

The operators Q_3, Q_4, Q_5, & Q_6 are pure I=1/2 operators

In the isospin limit: $\langle Q_3 \rangle_2 = \langle Q_4 \rangle_2 = \langle Q_5 \rangle_2 = \langle Q_6 \rangle_2 = 0$

EW-Penguins:

$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V+A}}, \quad Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V-A}}, \quad Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$O(\alpha_e)$ but can create isospin-2 state.
Needed for direct CPV!

$\mathrm{ImA}_2/\mathrm{ReA}_2$: (V-A)x(V-A)

Let us first consider only pure left-handed operators

$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$ $Q_9 = (s_\alpha \bar{d}_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V-A}$

$Q_2 = (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$ $Q_{10} = (s_\alpha \bar{d}_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A}$

Fierz identities & isospin limit imply

$$\langle Q_9 \rangle_2 = \langle Q_{10} \rangle_2 = 3/2 \, \langle Q_+ \rangle_2$$

with $\langle Q_\pm \rangle_I = (\langle Q_2 \rangle_I \pm \langle Q_1 \rangle_I)/2$.

The V-A contribution to the ratio I=2

$$\left(\frac{\mathrm{ImA}_2}{\mathrm{ReA}_2} \right)_{\mathrm{V-A}} = \mathrm{Im}\tau \, \frac{y_9 + y_{10}}{z_+}$$

Cancellation of matrix elements

is perturbatively calculable without non-perturbative input.

ImA₀/ReA₀: (V−A)x(V−A)

More operators contribute to $\mathrm{Im}A_0/\mathrm{Re}A_0$.

Fierz relations for (V−A)x(V−A) operators give:

$$\langle Q_4 \rangle_0 = \langle Q_3 \rangle_0 \; + \; 2 \, \langle Q_- \rangle_0$$

Using the theoretical definition for $\mathrm{Re}A_0$:

$$\left(\frac{\mathrm{ImA}_0}{\mathrm{ReA}_0} \right)_{\mathrm{V-A}} = \mathrm{Im}\tau \; 2 \; \frac{y_4}{1+q} \; z_- + \mathcal{O}(p_3)$$

Where q is the only hadronic input (numerically very small)

$$q = \frac{z_+(\mu) \langle Q_+(\mu) \rangle_0}{z_-(\mu) \langle Q_-(\mu) \rangle_0}$$

(V-A)x(V+A) Contributions

Q_6 & Q_8 give the leading contribution to $\mathrm{Im}A_0$ & $\mathrm{Im}A_2$, respectively

$$\left(\frac{\mathrm{ImA}_0}{\mathrm{ReA}_0}\right)_6 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\; y_6\,\frac{\langle Q_6\rangle_0}{\mathrm{ReA}_0}$$

$$\left(\frac{\mathrm{ImA}_2}{\mathrm{ReA}_2}\right)_8 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\; y_8^{\mathrm{eff}}\,\frac{\langle Q_8\rangle_2}{\mathrm{ReA}_2}$$

To reduce the error on non-perturbative input take the real parts from CP conserving data.

State of phenomenology

[Buras, Gorbahn, Jäger, Jamin '15]

$$(\varepsilon'/\varepsilon)_{\rm SM} = (1.9 \pm 4.5) \times 10^{-4}$$

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

2.9σ
discrepancy

The error is completely dominated by the non-perturbative sector

[Blum et. al., Bai et. al. '15]

Perturbative error are only estimates

Quantit	Error	Quantity	Error
$B_6^{(1/2)}$	4.1	$m_d(m_c)$	0.2
NNLO	1.6	q	0.2
$\hat{\Omega}_{\rm eff}$	0.7	$B_8^{(1/2)}$	0.1
p_3	0.6	p_{72}	0.1
$B_8^{(3/2)}$	0.5	p_{70}	0.1
p_5	0.4	$\alpha_s(M_Z)$	0.1
$m_s(m_c)$	0.3		
$m_t(m_t)$	0.3	All unites in 10^{-4}	

Why does a single matrix element dominate the error?

ReA0 & ReA2 known from CP-conserving data

Fierz Identities 7 independent operators

Better control over <Qi>$_2$ on lattice

EWP suppressed in I=0 (α/α_s)

QCDP cannot create I=2

Colour hierarchies between matrix elements, Wilson coefficients

QCDP Dominate ImA_0

ImA_2 due to EWP

Broken by QED & mu =! md Estimated separately

Why ε′/ε is so small?

The prediction of ε′/ε very sensitive to interplay between QCD (Q_6) & electroweak (Q_8) penguin operators

$$\varepsilon'/\varepsilon = 10^{-4}\left[\frac{\mathrm{Im}\lambda_t}{1.4\times10^{-4}}\right]\left[a(1-\hat{\Omega}_{\mathrm{eff}})(\,-4.1(8)\boxed{+}\,24.7\,B_6^{(1/2)})\;+\;1.2(1)\boxed{-}10.4\,B_8^{(3/2)}\right]$$

[Blum et. al., Bai et. al. '15] B_6=0.57(19) & B_8=0.76(5)

Cancellation between QCD & EW penguin operators.

Electroweak operators are very sensitive to new physics.

Is New Physics there?

Are we missing important contributions in the SM?

New physics might not be the reason of the tension

$\langle Q_i \rangle$ off ?

Missing SM ?
QCD corrections

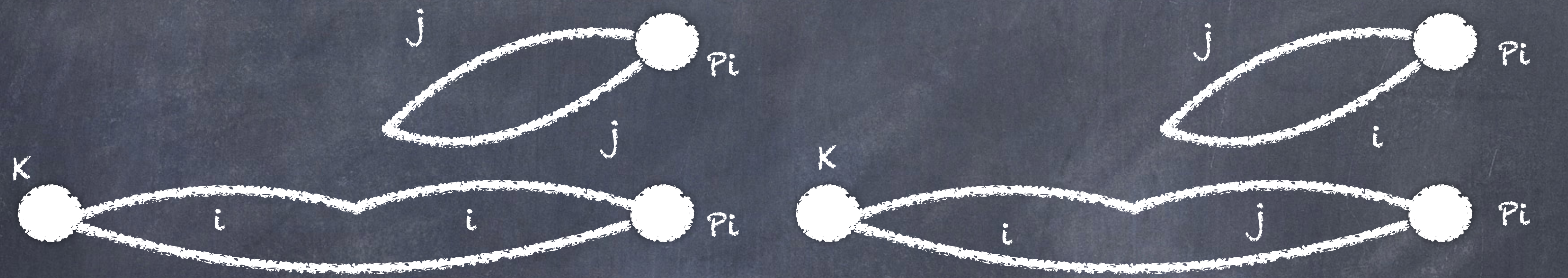
Missing SM ?
EW corrections

Missing ?
QED corrections

Deeper understanding of the SM is crucial

Long distance I=2

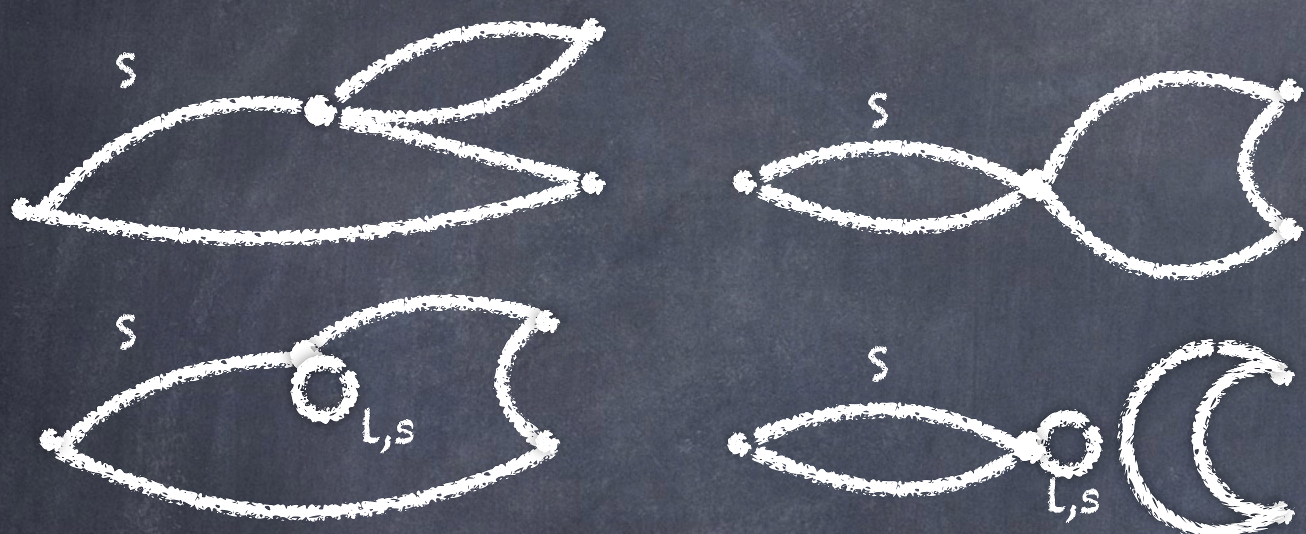
There are only three operators which contribute to A_2
and only two types of diagrams



The major challenge here it is to ensure that
the pions have physical momenta

Long distance I=0

The calculation of A_0 is more challenging than the evaluation of A_2



Challenges

Vacuum subtraction

Ground-state two-pion energy

$$\boxed{C^i_{K,\pi\pi}(t_K, T_Q, T_{\pi\pi}) = \langle 0 | J_{\pi\pi}(t_{\pi\pi})\ Q_i(t_Q)\ J_K(t_K) | 0 \rangle}$$

$\pi\pi$ phase shift from 2015 results: $\delta_0 = (23.8 \pm 4.9 \pm 2.2)^{\circ}$

Compared with dispersion theory result 34°

Puzzle resolved by adding more interpolating operators for $\pi\pi$ states $\delta_0 = (30.0 \pm 1.5 \pm 3)^{\circ}$

Why is important to compute e′/e at NNLO?

The theory prediction for e′/e only at NLO at the moment.
& higher order dimensional operators are not included in the error estimate (expected to be small)

$$\mathcal{O}(p^2/m_c^2) = (m_K - 2m_\pi)^2/(2m_c)^2$$

1. Prospects for improvement on <Qi> are good.

   Controlling other sources of uncertainties will become important soon.

2. Higher order corrections could have a huge impact on e′/e.

3. The convergence of perturbation theory at m_c is not clear.

Status of ϵ'/ϵ at NNLO

Energy	Fields	Order	Paper
μ_W	g, γ, W, Z, h u, d, s, c, b, t	NNLO Q_1-Q_6, Q_{89} NNLO EWP	[Misiak, Bobeth, Urban] [Gambino, Buras, Haisch]
RGE	g, γ, u, d, s, c, b,	NNLO Q_1-Q_6, Q_{89}	[Gorbahn, Haisch]
μ_b	g, γ, u, d, s, c, b	NNLO Q_1-Q_6	[Gorbahn, Brod]
RGE	g, γ, u, d, s, c	NNLO Q_1-Q_6, Q_{89}	[Gorbahn, Haisch]
μ_c	g, γ, u, d, s, c	NLO Q_1-Q_{10}	[Buras, Jamin, M.E.L]
RGE	g, γ, u, d, s	NNLO Q_1-Q_6, Q_{89}	[Gorbahn, Haisch]
$\mu_{lattice}$	g, u, d, s	NLO Q_1-Q_{10}	[Blum et. al., Bai et. al. '15]

NNLO corrections

NNLO weak Hamiltonian only known above bottom mass.

(from $B \to X_s$ gamma)

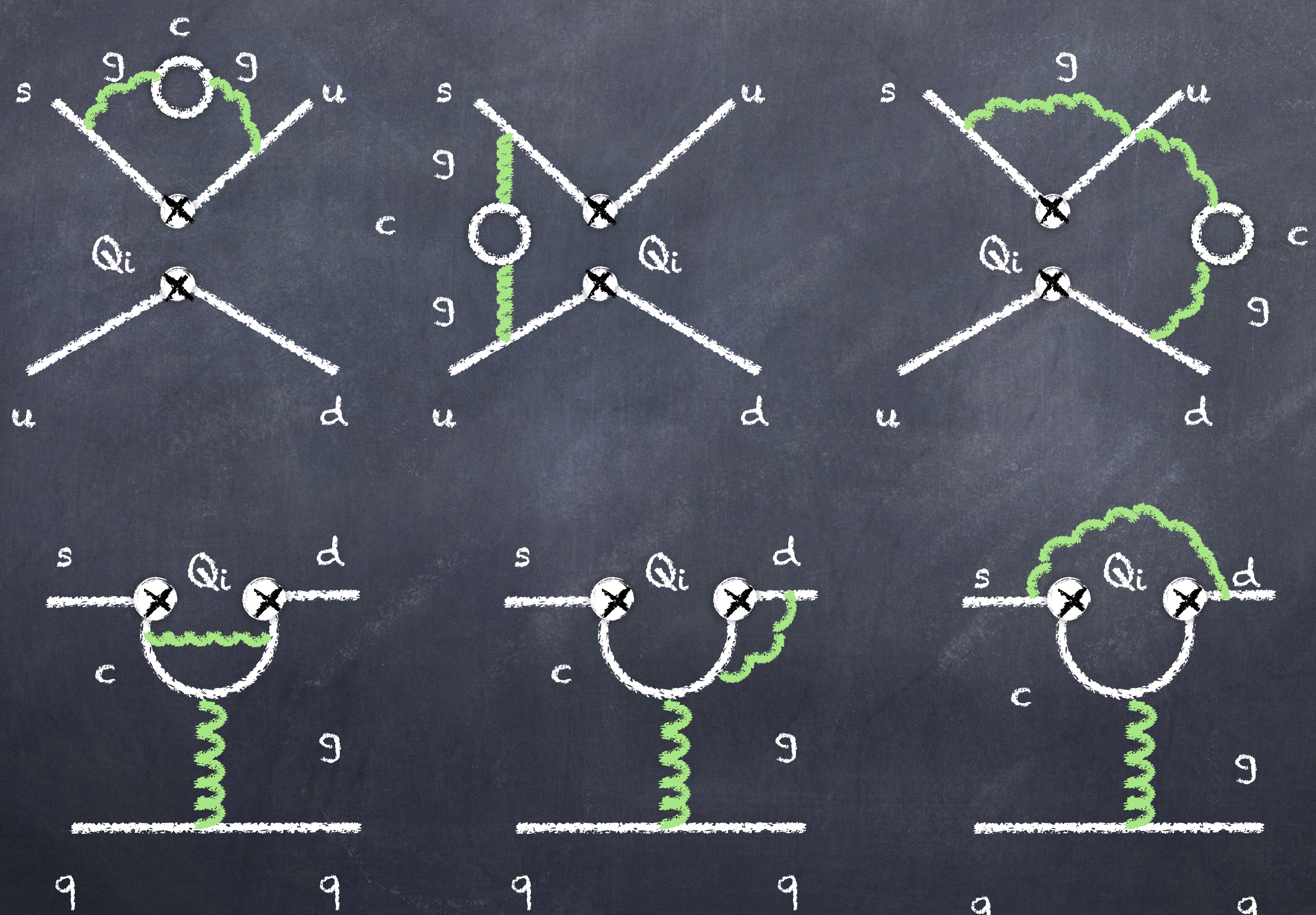
Analysis of ε'/ε requires bottom & charm threshold corrections & also NNLO mixing of QCD into EWP.

These threshold corrections are determined through a matching of the effective theories with n_f and n_f+1 flavours.

$$\boxed{A_{\mathrm{eff}}^{(n_f+1)} = A_{\mathrm{eff}}^{(n_f)}}$$

Charm matching at NNLO

Calculation of two-loop diagram with inserted operators



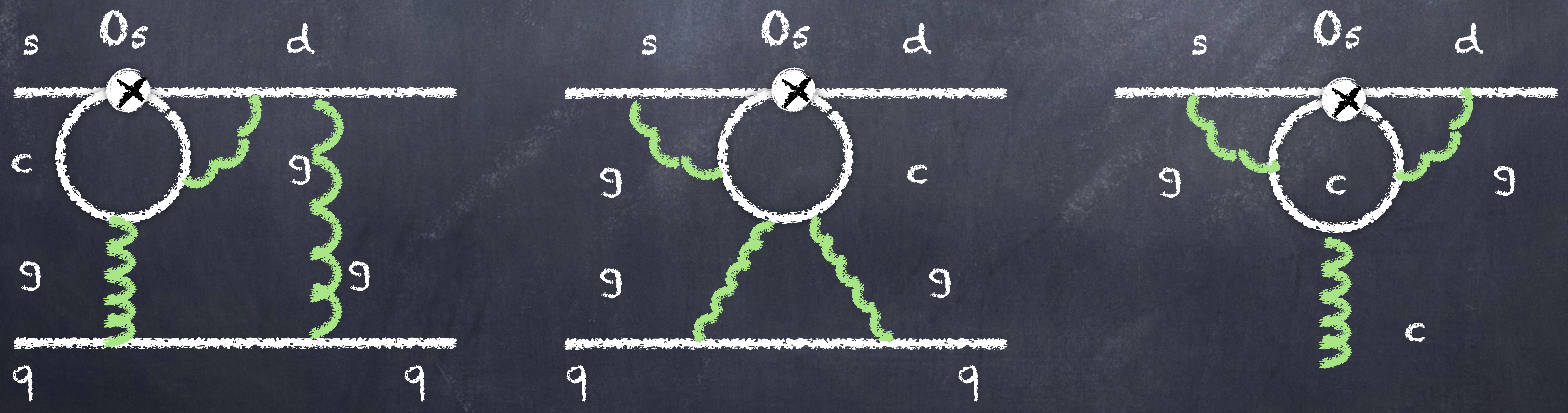
Operator basis for NNLO

The traditional basis requires the calculation of traces with γ_5

Issues with the treatment of the γ_5 in D-dimensions

Higher order calculations can be significantly simplifies if we use a different operator basis

$O_5 = (s_i\, \gamma_\mu \gamma_\nu \gamma_\rho P_L\, d_j)_{V-A}\, (q_K\, \gamma^\mu \gamma^\nu \gamma^\rho\, q_L)$ No traces with γ_5



Relation to traditional basis not trivial in D-dimensions

Setup

To work in dimensional regularisation

To renormalise the theories in the MS-bar scheme

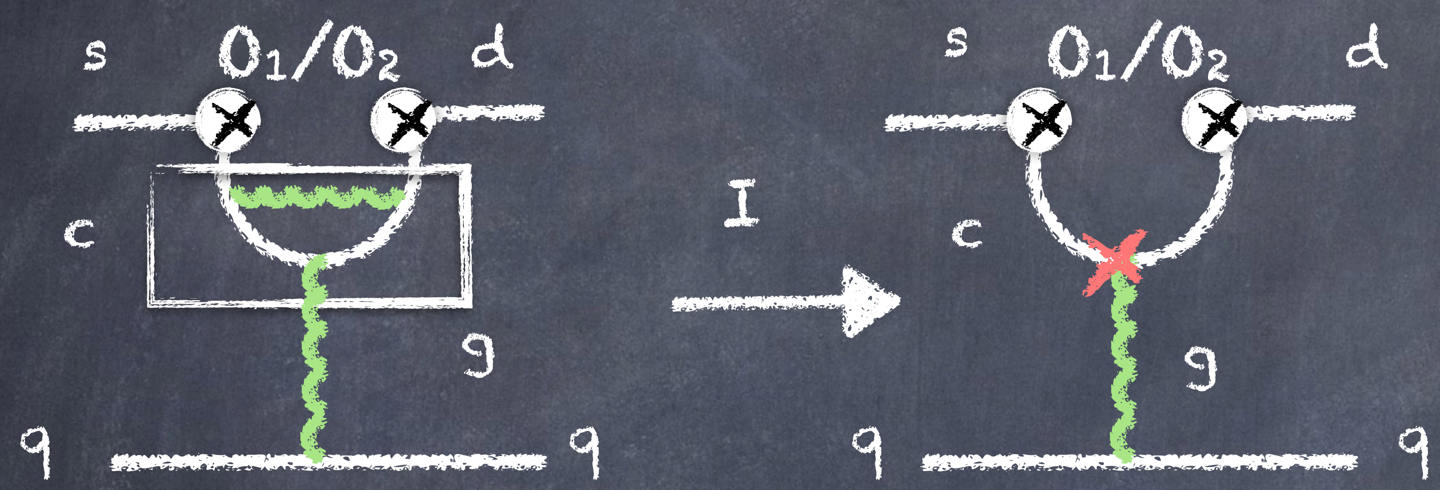
To expand the external momenta up to $O(k^2)$

To set the mass of the light quarks to zero

This introduces Infrared Divergences in the n_f+1 theory amplitude which have to be cancelled by the Ultra-Violet divergences in the n_f flavour theory

Renormalisation

2-loop diag. : 1/eps² & 1/eps poles

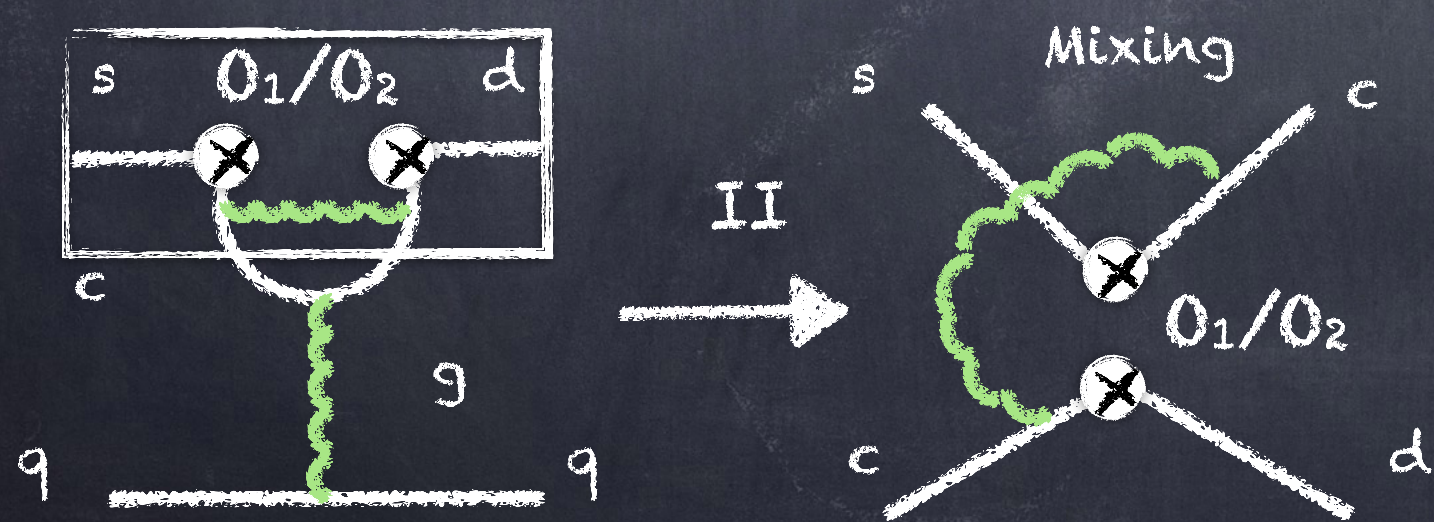


One-loop diag with
inserted counter-term

STEP I:

eps² pole is cancelled

eps⁻¹ is not fully cancelled



STEP II:

Mixing of cc required
to get a finite result

Running

Matrix elements are computed in the 3-flavour theory

& the perturbative corrections have the factorised structure:

$$C^{(3)}(\mu_L) = U^{(3)}(\mu_L, \mu_c)\,.\,M^{(34)}(\mu_c)\,.\,U^{(4)}(\mu_c, \mu_b)\,.\,M^{(45)}(\mu_b)\,.\,U^{(5)}(\mu_b, \mu_W)\,.\,C^{(5)}(\mu_W)$$

NNLO for the isospin-0 amplitude now complete

The short-distance contributions are μ- and scheme-dependent

But, observables do not depend on μ-scale or the scheme used.

$C_i^{(3)}(\mu_L)\langle Q_i\rangle^{(3)}(\mu_L)$ Cancellation!!

$\langle Q_i\rangle(\mu_L)$ are needed in the same scheme and for the same scale or ideally as a function of μ.

Conversion to the $\overline{\text{MS}}$ scheme

Perturbation theory is easiest and most transparent in dimensional regularisation with minimal subtraction.

What about the matrix elements?

STEP I: <Qi> are renormalised non-perturbatively in the RI-SMOM scheme.

STEP II: Match to the traditional operator basis in the continuum MS-bar renormalisation scheme using NDR:

$$\boxed{\langle Q_i \rangle^{\overline{\text{MS}}}(\mu_L) = \left[T^{(0)} + \alpha_s(\mu_L) T^{(1)}\right]_{ij} \langle Q_j \rangle^{\text{RI-SMOM}}}$$

Unknown master Feynman integrals from two loops.

More complicated than perturbative Wilson coefficients.

RGI-scheme I

Definition of the renormalised operators consistent with the scheme used in the calculation of the Wilson coefficients.

NDR-scheme, 't Hooft and Veltman-scheme, RI-scheme

In some cases, the differences between different schemes may be numerically large

To avoid all these problems, it is convenient to introduce a renormalisation group invariant definition of Wilson coefficients and composite operators

This relies on the fact that, $U(\mu,\mu_0) = u(\mu)u(\mu_0)^{-1}$

RGI-scheme II

$$\langle Q\rangle^{(3)}(\mu_L)C^{(3)}(\mu_L)=\langle Q\rangle(\mu_L)U^{(3)}(\mu_L,\mu_c)M^{(34)}(\mu_c)U^{(4)}(\mu_c,\mu_b)$$
$$\times\ M^{(45)}(\mu_b)U^{(5)}(\mu_b,\mu_W)C^{(5)}(\mu_W)$$

The contribution of running, $U(\mu,\mu_0)$, and matching, $M(\mu_q)$, can be factorised in terms of scheme & scale independent quantities:

$$\boxed{\langle Q\rangle^{(3)}(\mu_L)\ C^{(3)}(\mu_L) = \langle\hat{Q}\rangle^{(3)} . \hat{M}^{(34)} . \hat{M}^{(45)} . \hat{C}^{(5)}}$$

where,

$$\langle\hat{Q}\rangle = \langle Q\rangle(\mu_L) . u^{(3)}(\mu_L), \qquad \hat{M}^{(34)} = u^{(3)-1}(\mu_c) . M^{(34)}(\mu_c) . u^{(4)}(\mu_c)$$
$$\hat{C}^5 = u^{(5)-1}(\mu_W) . C^{(5)}(\mu_W), \qquad \hat{M}^{(45)} = u^{(5)-1}(\mu_b) . M^{(45)}(\mu_b) . u^{(5)}(\mu_b)$$

RGI-scheme III

In the RGI scheme:

1. hatted matrix elements satisfy d=4 Fierz identities

   missing $O(\alpha_s)$ corrections for the Fierz identities are also included.

2. All hatted quantities & also their products

$$\hat{C}^{(3)} = \hat{M}^{(34)} . \hat{M}^{(35)} . \hat{C}^{(5)}$$

are formally scheme and scale independent.

But they show residual μ dependence that is expected to reduce order by order & that is of the size of higher order corrections.

Results at NNLO

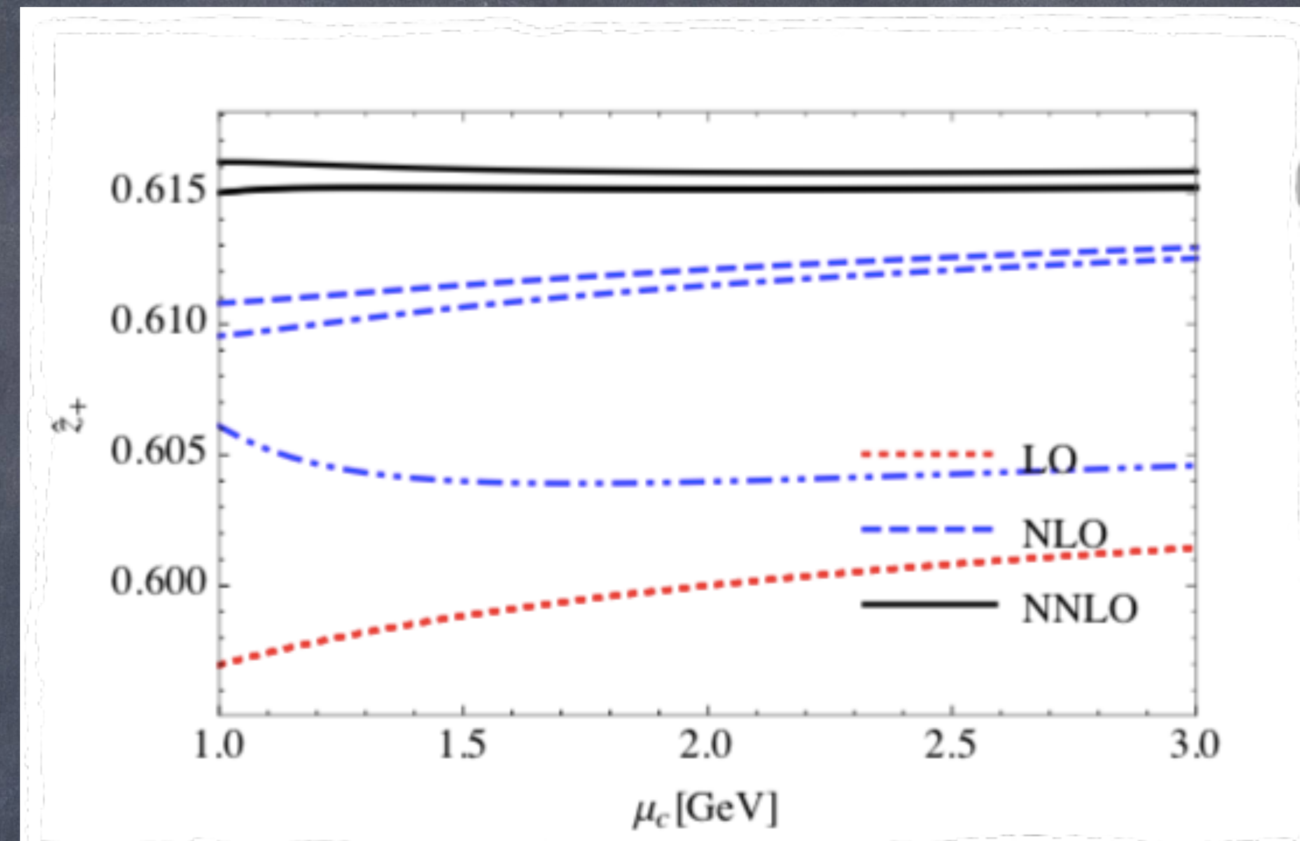
The real part of A_0 & A_2 is dominated by z_+ & z_-

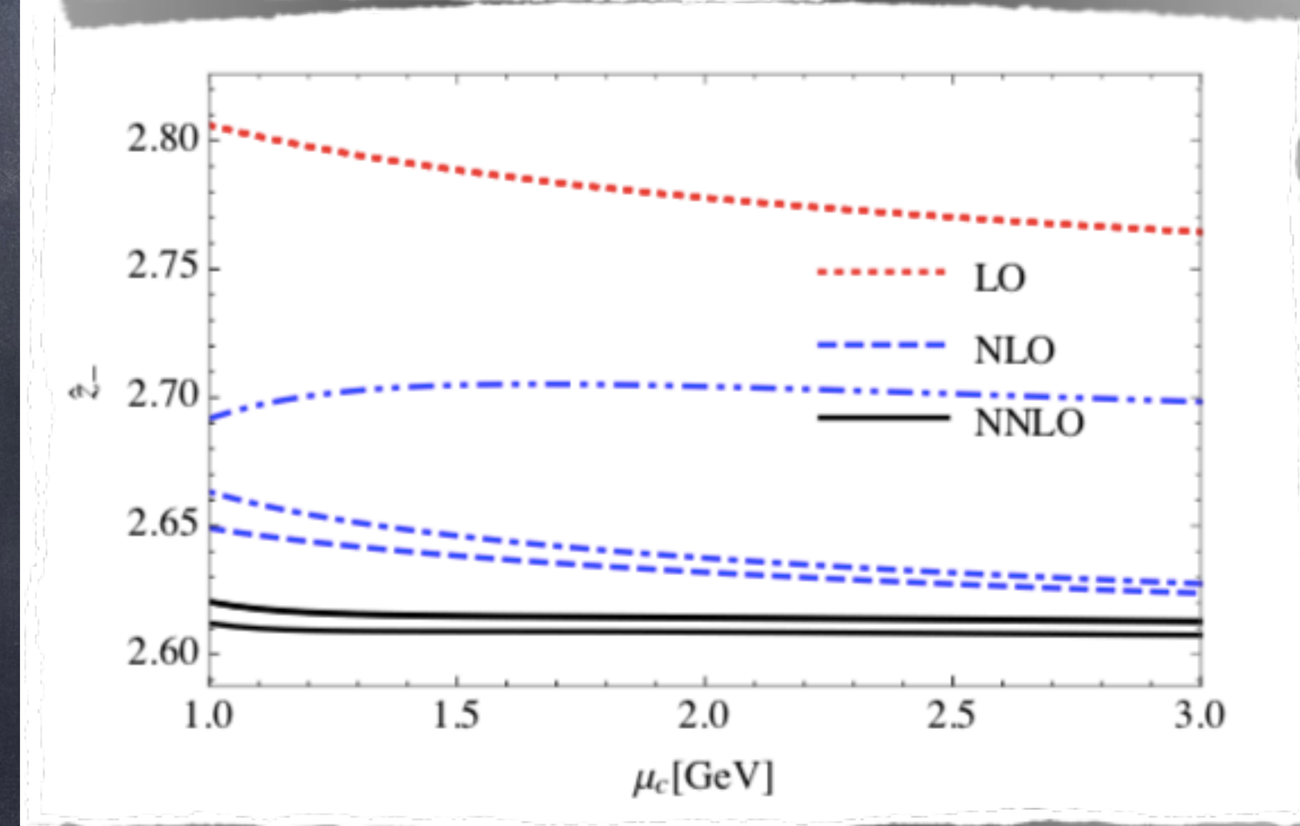
$$\mathrm{ReA}_2 = \hat{z}_+ \langle \hat{Q}_+ \rangle_2$$
$$\mathrm{ReA}_0 = \hat{z}_+ \langle \hat{Q}_+ \rangle_0 + \hat{z}_- \langle \hat{Q}_- \rangle_0$$

The residual μ_c dependence reduces order by order

At NLO there is still a dependence on the implementation of α_s running

Shift probably due to running down from M_Z



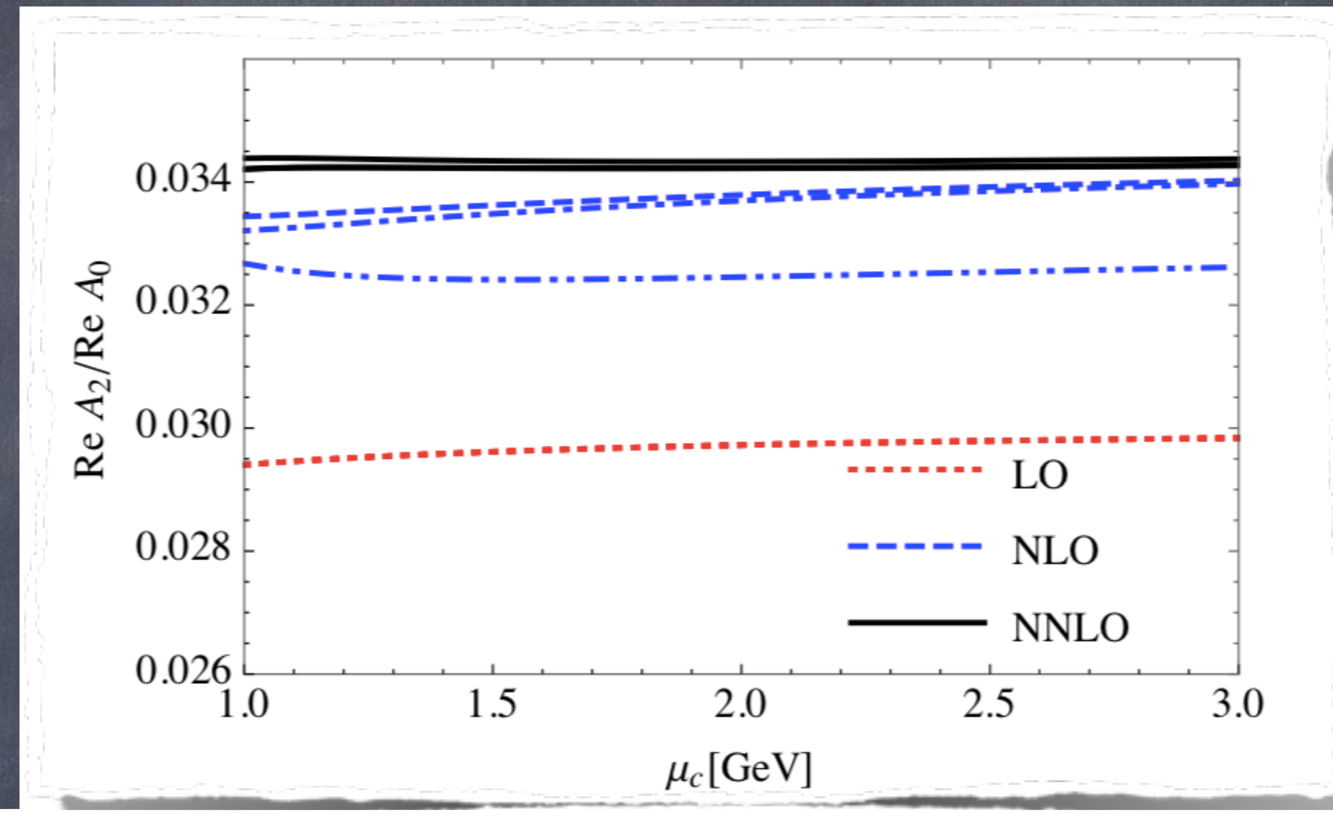


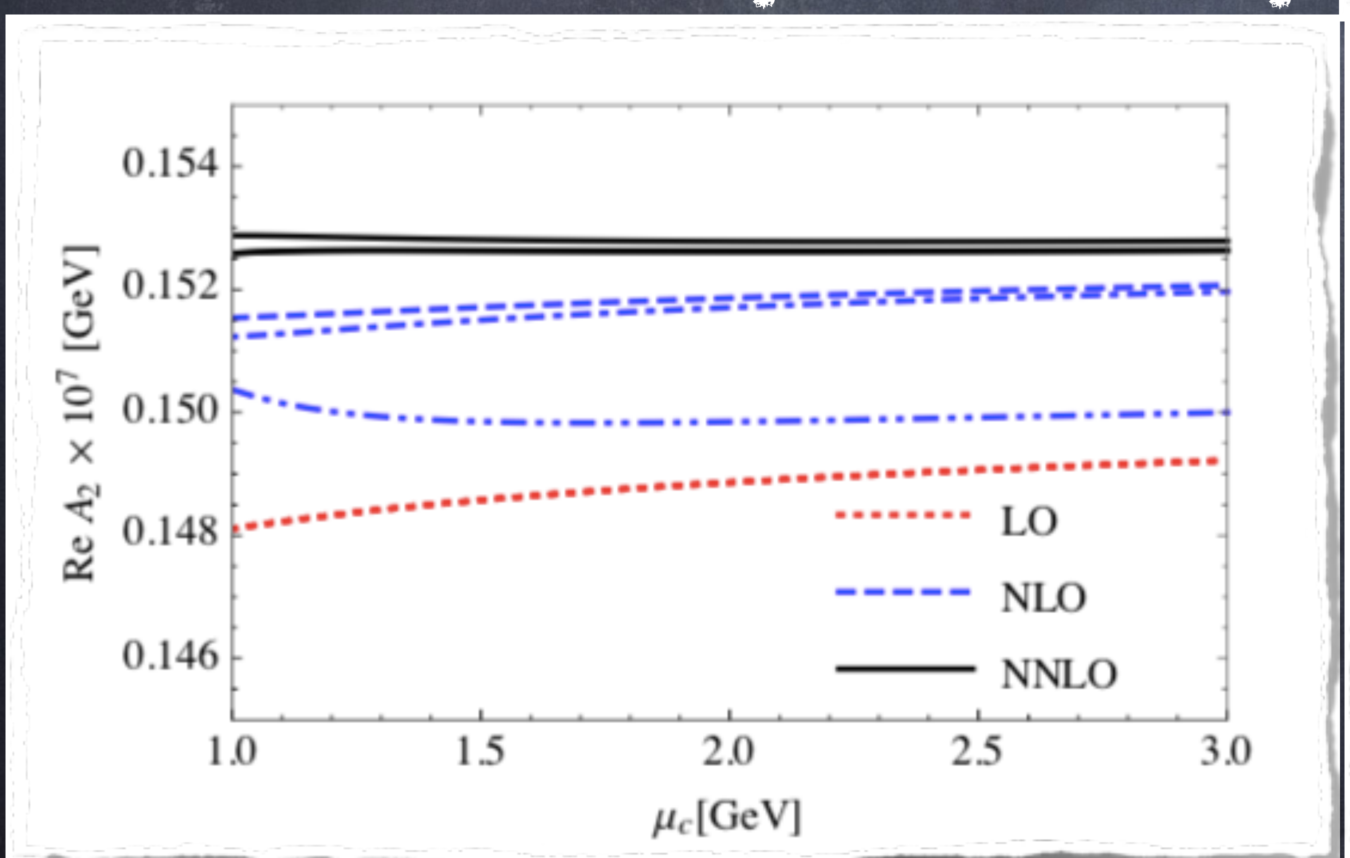
Impact onto ReA$_I$

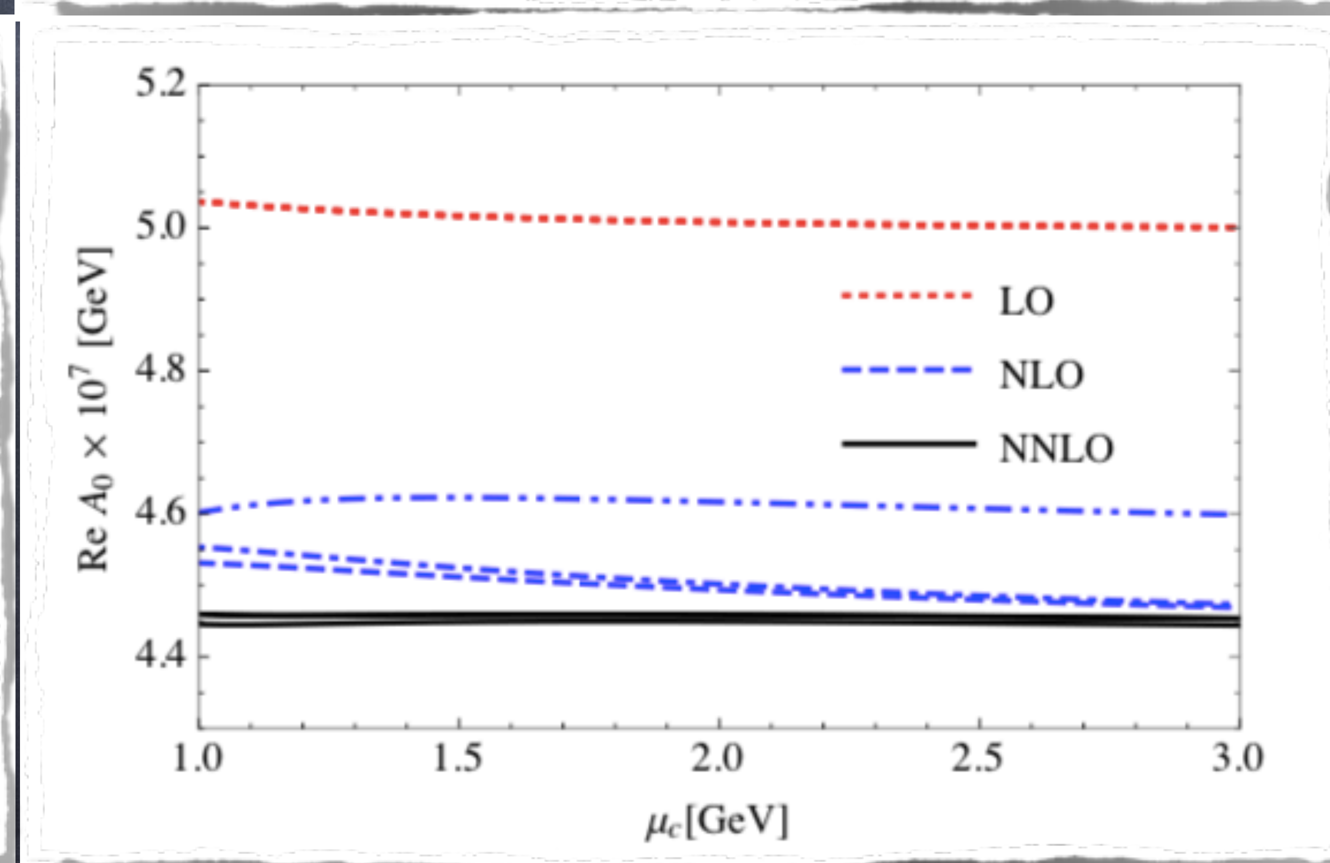
$$\mathrm{ReA}_2 = 1.48 \times 10^{-8}\mathrm{GeV}$$

$$\mathrm{ReA}_0 = 33.2 \times 10^{-8}\mathrm{GeV}$$

Lattice input to ReA0 has still 20%/25% stat/sys. uncertainty

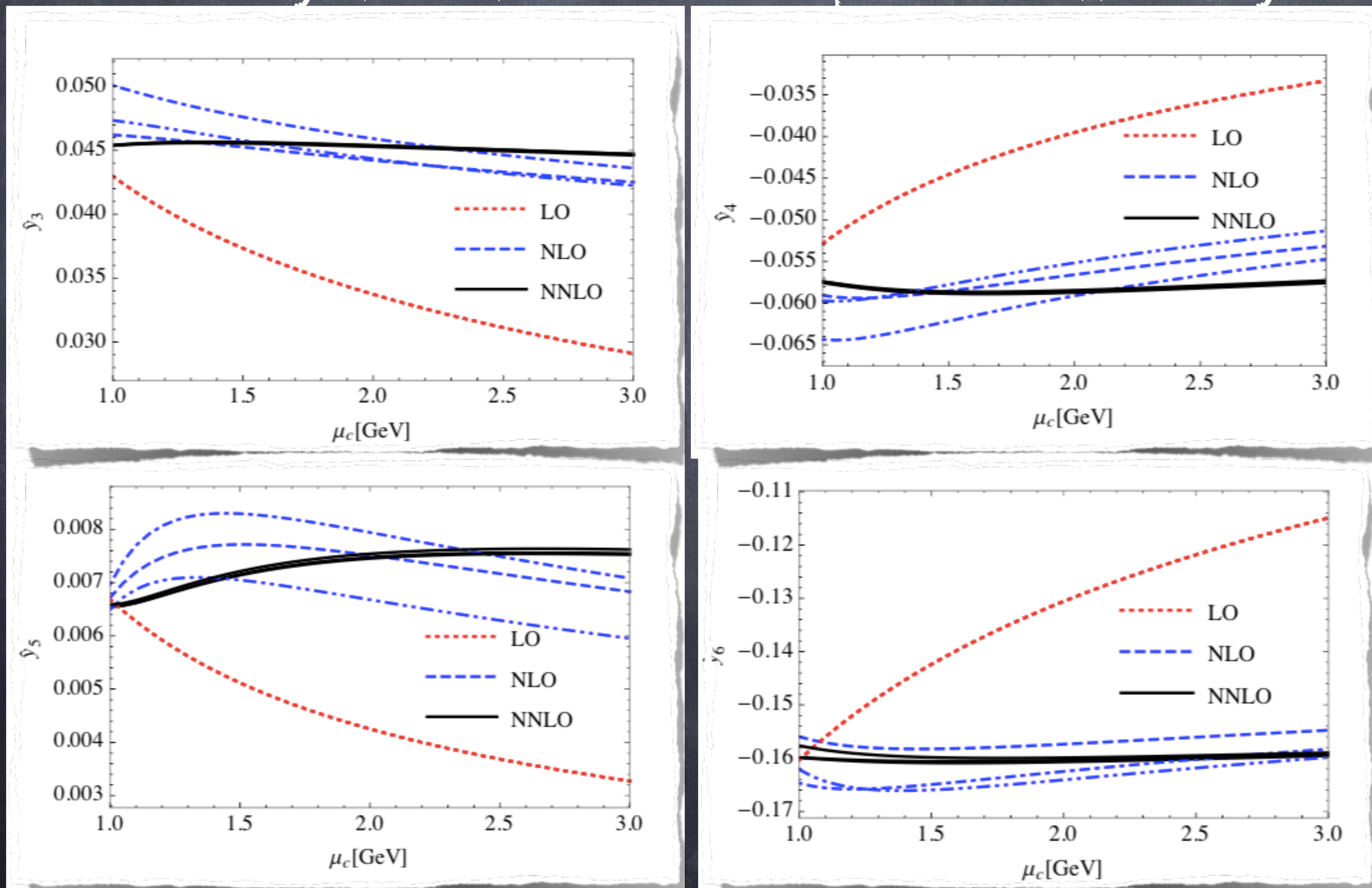




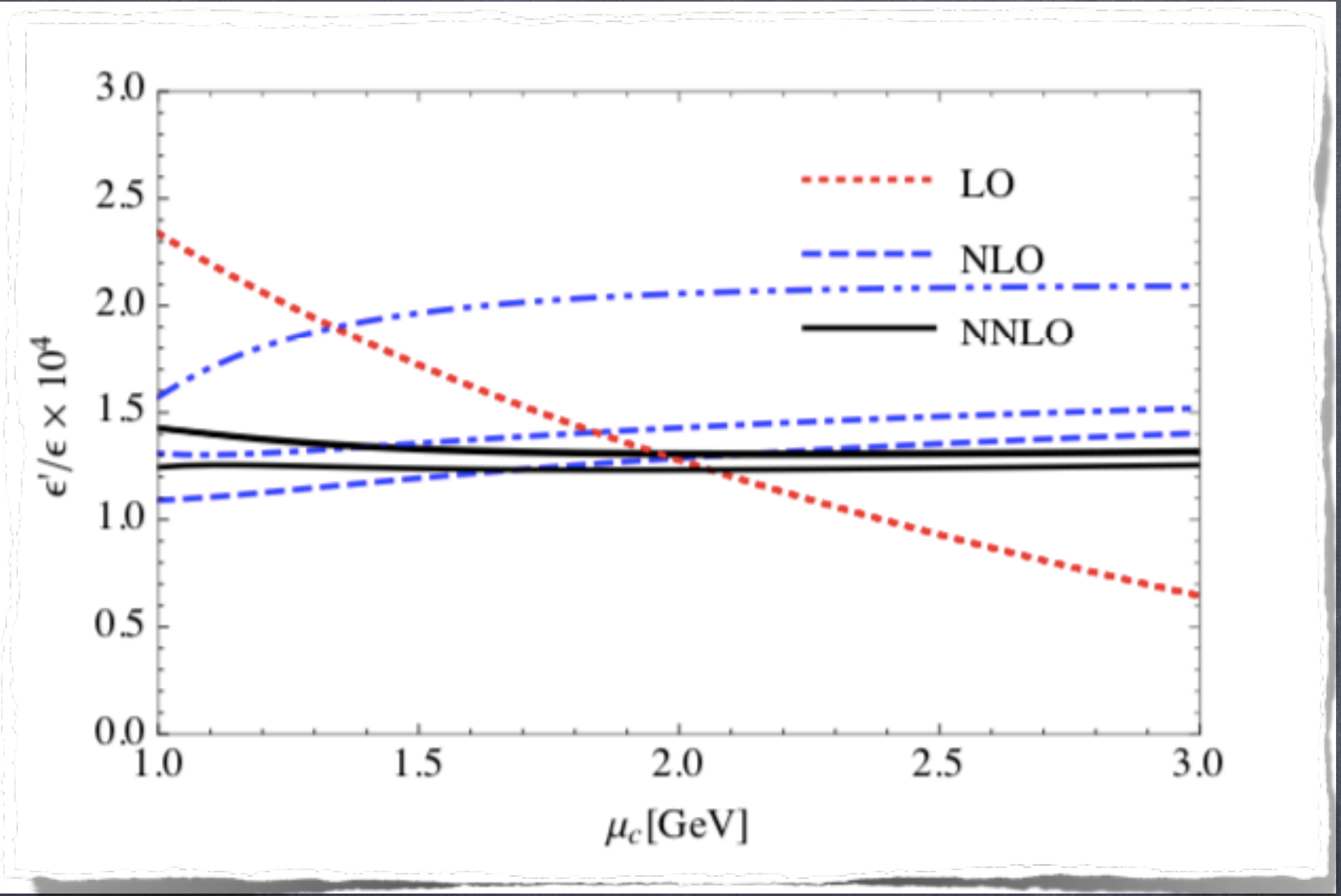


Results at NNLO

NNLO accuracy of ~2% for the most important coefficient y_6.



Impact onto e'/e



Uncertainty is significantly reduced by going to NNLO.

Tiny scale variation suggests negligible N³LO QCD effects.

There are still improvements: better as implementation & better incorporation of sub-leading corrections.

Dynamical Charm

No evidence for a failure of perturbation theory at the charm scale.

$\boxed{\text{Non-perturbative Virtual-charm effects}}$?

Lattice simulations with dynamical charm are becoming feasible.

From our computed threshold corrections, we can provide an estimation of the four-flavour matrix elements.

$$\langle \hat{Q} \rangle^{(3)} \; \hat{C}^{(3)} = \langle \hat{Q} \rangle^{(3)} \,.\, \hat{M}^{(34)} \,.\, \hat{C}^{4} = \langle \hat{Q} \rangle^{(4)} \,.\, \hat{C}^{(4)}$$

$\hat{C}^{(4)}$ Available at NNLO (cc, QCDP) & NLO (EWP)

Phenomenology at nf=4

The formula for ε′/ε has to be modified at the 4-flavour theory.

There are two new operators, Q_1^c & Q_2^c, & the penguin operators contain charm quark.

The I=2 amplitude ratio is unchanged in form.

The I=0 ratio depends explicitly on the new operators:

$$\frac{\mathrm{ImA}_0}{\mathrm{ReA}_0} = \mathrm{Im}\tau \left[\frac{(2\ y_4 - \frac{1}{2}[3y_9 - y_{10}](1+2q_-^c)}{z_-(1+\tilde{q})} - \frac{q_-^c}{1+\tilde{q}} \right.$$
$$+ \frac{\frac{3}{2}[y_9 + y_{10}](1+q_+^c)\tilde{q}}{z_+(1+\tilde{q})} - \frac{q_+^c\tilde{q}}{1+\tilde{q}} + \frac{(y_3 + y_4 - \frac{1}{2}[y_9 + y_{10}])\tilde{p}_3}{z_-(1+\tilde{q})}$$
$$\left. + \frac{G_F}{\sqrt{2}} \frac{V_{ud}V_{us}^*}{\mathrm{ReA}_0} \Big(\langle Q_6 \rangle_0 (y_6 + p_5 y_5 + p_{8g} y_{8g}) + \langle Q_8 \rangle_0 (y_8 + p_{70} y_7 + p_{70\gamma} y_{7\gamma}) \Big) \right]$$

Isospin Breaking effects

The isospin limit is not very good: O(10%) corrections

Pions are not exact I=1 states	&	Electromagnetic effects cannot be neglected

This corrections are introduced via the parameters Ω_{eff} & a

A. The phases $\delta_{0,2}$ are still defined in the isospin limit.

Watson's theorem is only valid when isospin is conserved

B. One matches a QCDxQED evolution to a pure QCD lattice calculation

Electromagnetism in Lattice

Complicated, particularly QED effects (IR subtractions, real emission, lattice matching, ...)

A. Do not respect the two-amplitude structure

B. Violate Watson's theorem

Now conceptually understood on the lattice in QED perturbation theory. In practice need to

A. Define QED expansion of matrix element ratios

B. Carefully define & express observable at $O(\alpha_e)$

C. Disentangle QED RG evolution from matrix matrix element expansion, for matching short-distance and Lattice

Conclusions & Outlook

ϵ'/ϵ at NLO perturbation theory with RBC-UKQCD matrix elements shows a tension with the data.

Lattice results with improved stat. and syst. errors will be published soon.

New NNLO calculation of the non-EW-penguin part of the weak Hamiltonian removes large part of the perturbative uncertainty in ϵ'/ϵ.

ϵ'/ϵ can be expressed in terms of RGI objects, to achieve a fuller factorisation between perturbative and non-perturbative pieces.

Future goals

From a phenomenological perspective, the most important goal is reducing the error on $\langle Q_6 \rangle_0$.

If phenomenology is done appropriately, none of the other $\langle Q \rangle$ contribute above 1/4 or below of the current experimental error.

Apart from this, calculation of isospin breaking on the lattice, and interfacing with perturbation theory will be important.

Formalism can be extended to $n_f=4$ dynamical quarks.

EW NNLO including systematic treatment of $O(\alpha_e)$ (as well as m_u =! m_d) about the isospin limit are the next steps on perturbative side

What if RBC-UKQCD results are right?

a. SM value deviates by almost 3σ from experimental world average: $(\varepsilon'/\varepsilon)_{SM} \ll (\varepsilon'/\varepsilon)_{exp}$.

b. Destructive new-physics effects in ε'/ε are disfavoured.

this puzzle requires a NP contribution
even larger than the SM contribution

c. The large factor $1/\omega_+$ multiplying $\mathrm{Im}A_2$ renders ε'/ε sensitive to new physics in the $\Delta I=3/2$ transitions.

However, it is difficult to place a large effect into ε' without overshooting ε_K.

Main constraint: ϵ_K

The SM contributions to direct and indirect CPV depend on the CKM combination τ as

$$\epsilon'^{SM} \propto \mathrm{Im}\tau \text{ and } \epsilon_K^{SM} \propto \mathrm{Im}\tau^2$$

In new physics scenarios, with new sources of CPV replace τ with δ

$$\epsilon'^{NP} \propto \mathrm{Im}\delta \text{ and } \epsilon_K^{NP} \propto \mathrm{Im}\delta^2$$

For super-heavy new physics entering through loops, effects can only be relevant if $|\delta| \gg |\tau|$

But $\epsilon_K^{NP} \gg \epsilon_K^{SM}$ in contradiction with the experimental value. Need clever ideas to suppress ϵ_K!!

Possible New physics explanations

[Cirigliano et al., '16]

WR coupling

Generic
Z' models

[Buras et al., '15]

Chromo-magnetic
Operator

[Buras et al., '99]

[Bauer et al., '09]

[Constantinou et al., '14]

SUSY

[Kitahara et al., '16]

[Endo et al., '16]

[Crivellin et al., '17]

Z′ models

General models with tree-level Z and Z′ flavour violating exchanges.

The correlations of ε′/ε with other flavour observables allow to differentiate between models in which ε′/ε can be enhanced

Z-scenarios: Enhancement of ε′/ε, ϵ_K, $Br(K_L \to \pi^0 \nu \bar{\nu})$ & $Br(K^+ \to \pi^+ \nu \bar{\nu})$ only possible in the presence of both LH & RH flavour violating couplings

Z′-scenarios: the size of NP effects & the correlation between $Br(K_L \to \pi^0 \nu \bar{\nu})$ & $Br(K^+ \to \pi^+ \nu \bar{\nu})$ depends strongly on whether QCDP or EWP dominate NP contributions to ε′/ε

WR coupling

The enhancement in ε′/ε originates from right-handed charged-current interactions. (Tree level)

$$\mathscr{L}_{\mathrm{SM}} + \frac{g}{\sqrt{2}}\left[\xi_{ij}\ \bar{u}^i_R \gamma^\mu d^j_R\ W^+_\mu\right](1+\frac{h}{v})^2 + \mathrm{h.c}$$

To assume that ξ_{ud} and ξ_{us} have complex phases.

Right-handed scale of O(10^2 TeV) to explain the discrepancy.

Correlation with hadronic and atomic electric dipole moments.

Chromo-magnetic

Chromo-magnetic penguins

$$C_{\gamma,g}^{\mathrm{SM}} \sim \frac{\alpha_W(M_W)}{M_W}\frac{m_s}{M_W} \qquad C_{\gamma,g}^{\mathrm{NP}} \sim \frac{\alpha_s(M_{NP})}{M_{NP}}\delta_{LR}$$

can give large corrections to e'/e of form:

$$\begin{aligned}(\varepsilon'/\varepsilon)_{8g} &= 3\ B_{8g}\ \mathrm{Im}\left(C_{8g} - C'_{8g}\right)/(G_F m_K) \\ &= 520\ B_{8g}\ \mathrm{Im}(C_{8g} - C'_{8g})\mathrm{TeV}\end{aligned}$$

However, an enhancement of the SM by a factor of order 500 is necessary for a sizeable impact on e'/e

ϵ'/ϵ in MSSM

The MSSM has the required ingredients to explain ϵ' without conflict with ϵ_K despite $\delta \gg \tau$. [Kitahara et al., '16]

Mechanism

a. Enhancement of $\mathrm{Im}A_2$ due to strong isospin-breaking contributions. "Trojan penguin" [Grossman et al., '99]

(coupling differently to up and down quarks)

b. Suppression of the K-$\bar{K}$ mixing amplitude thanks to the Majorana nature of the gluinos [Crivellin et al., '10]

This is possible with squark and gluino masses in the range 3-7 TeV, far above the reach of LHC.

"and now here is my secret, a very simple secret: It is only with the heart that one can see rightly; what is essential is invisible to the eye"

-Le petite prince (Antoine de Saint-Exupéry)