PHY117 HS2023

Week 10, Lecture 1
Nov. 21st, 2023
Prof. Ben Kilminster

preliminary formula sheet for exam in exercises folder

what's left to do:
- magnetic field
- waves
- electromagnetic waves
- optics

PHY117 Formula Sheet

Let us know if you find mistakes!

Mechanics

Velocity	$\vec{v} = \frac{d\vec{r}}{dt}$
Speed	$\mathrm{v} = \lvert\vec{v}\rvert$
Acceleration	$\vec{a} = \frac{d\vec{v}}{dt}$
Acceleration components	$a_r = \frac{v^2}{r}$ and $a_T = \frac{d\lvert\vec{v}\rvert}{dt}$
Position	$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$
Velocity	$v^2 = v_0^2 + 2a\Delta x$ and $v(t) = v_0 + at$
Newtons second law:	$\sum \vec{F} = m\,\vec{a}$
Newtons third law	$\vec{F}_{12} = -\vec{F}_{21}$
Gravitational force	$\vec{F}_g = m\vec{g}$
Gravitational force law	$\vec{F}_g = \frac{Gm_1m_2}{r^2}$
Newtons second law of rotation:	$\sum \tau = I\alpha$
Centripetal force	$F_r = \frac{m\,v^2}{r} = mr\omega^2$
Angular positon:	$\Delta s = r\Delta\theta$
Angular velocity:	$\omega = \frac{d\vartheta}{dt} = \frac{v}{r}$ and $\omega = 2\pi/T$
Angular acceleration:	$\alpha = d\omega/dt$
Angular momentum:	$\vec{L} = \vec{r} \times \vec{p}$ and $\vec{L} = I\vec{\omega}$
Torque:	$\vec{\tau} = \vec{r} \times \vec{F}$ and $\vec{\tau} = \frac{d\vec{L}}{dt}$
Impulse:	$\vec{F}\Delta t = \Delta\vec{p} = m\Delta\vec{v}$
	$\Delta p = \int_0^T F(t)\mathrm{d}t = \bar{F}T$
Momentum	$\vec{p} = m\vec{v}$ and $\vec{F} = d\vec{p}/dt$
Spring force	$F_s = -kx$
Static friction	$F_f = \mu_s F_N$
Kinetic friction	$F_f = \mu_k F_N$
Mechanical equilibrium	$\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$
Precession frequency	$\omega_p = rmg/I\omega$

Energy and work

Hydrostatic

Pressure	$P = \frac{F}{A}$
Compressibility	$B = \frac{-P}{\frac{\Delta V}{V}}$
Pressure distribution in liquids	$P = P_0 + \rho gh$
Capillarity	$\Delta h = \frac{2\gamma cos\theta_c}{\rho gr}$
Buoyancy	$F_b = \rho_l V_{\mathrm{dis}} g$
Bouyancy in centrifuge	$F_b = m_l\omega^2 r$
Centrifugal "force"	$F_c = m_o\omega^2 r$

Hydrodynamics

Flow rate	$I_V = \frac{\Delta V}{\Delta t} = Av$ v: homogeneous velocity
Continuity equation	$I_V =$ constant $(v_1A_1 = v_2A_2)$
Bernoulli's equation	$p + \frac{\rho}{2}v^2 + \rho gh =$ constant
Toricelli's outflow law	$v = \sqrt{2gh}$
Resistance in pipe	$\mathrm{R} = \frac{8\eta L}{\pi R^4}$
Flow resistance	$\Delta P = I_V R$

Solidity

Stress	stress $= F/A$
Strain	strain $= \Delta L/L$
Youngś modulus	Y =stress/ strain
Moment of inertia	$I = \sum mr^2$
bars	$I_z = \frac{ab^3}{12}$, a, b: Side lengths
round profile	$I_z = \frac{\pi}{4}R^4$, R: Radius

Gases

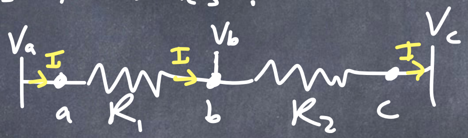
Ideal gases

K in 3D $K = \frac{3}{2}NkT$

LAST WEEK:

Resistors in series:

Note: opposite rules as for capacitors



Equivalent resistance $\boxed{R_{eq} = R_1 + R_2 + \ldots}$

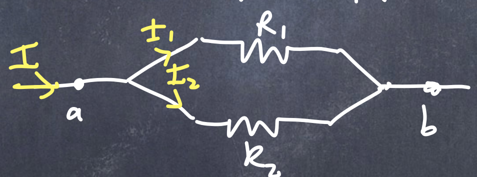
$V_b = V_a - IR_1$

$V_c = V_a - IR_1 - IR_2$

$I_a = I_b = I_c = I$

Potential decreases, current stays same.

Resistors in parallel:



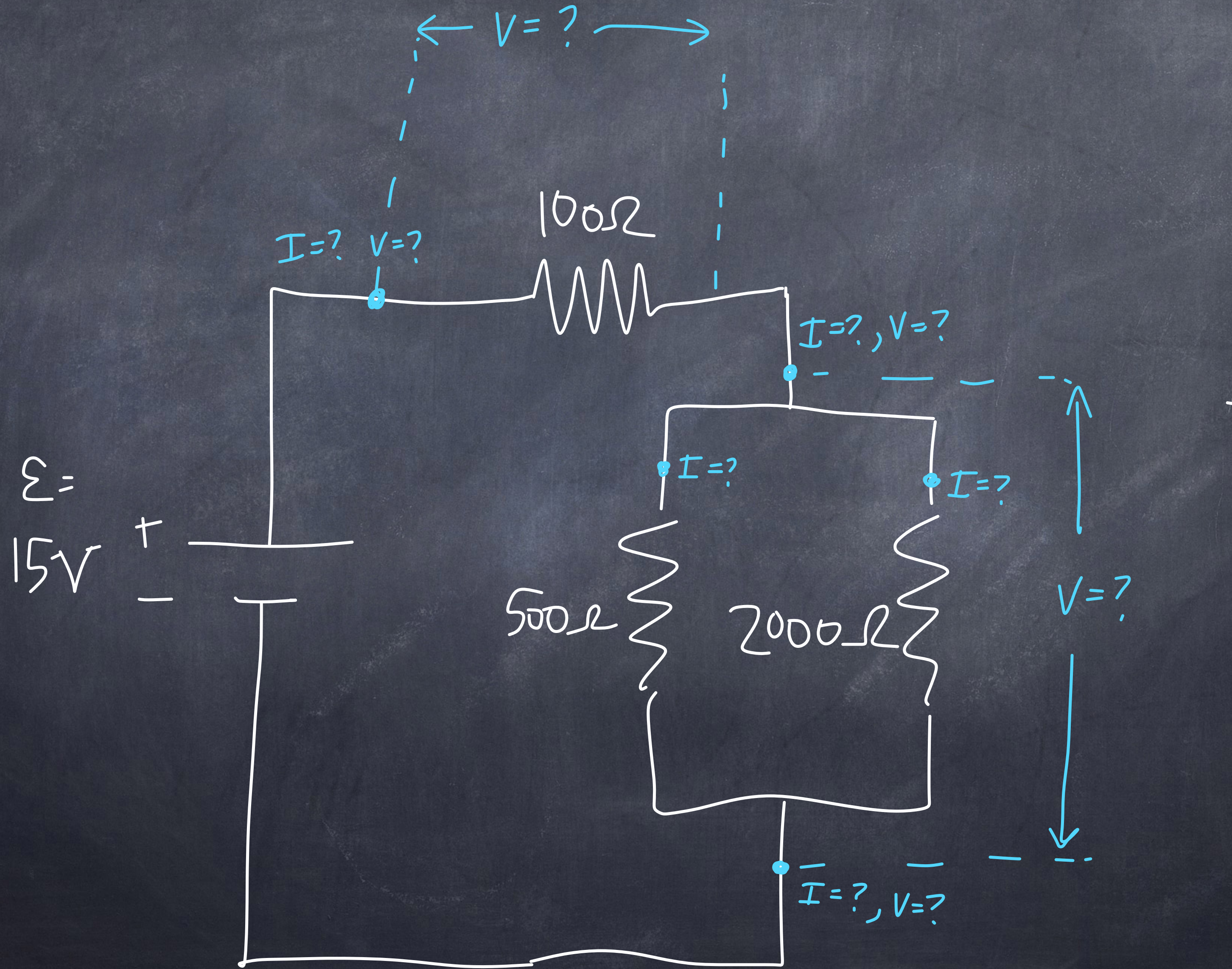
Equivalent resistance decreases. (More ways for current to flow)

$I = I_1 + I_2$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots}$$

voltage drop $V_a - V_b$ is same across both paths: $V_{ab} = I_1 R_1 = I_2 R_2$

what are the requested voltages + currents ?



First, rules:

To calculate voltages + currents in a circuit, we can follow Kirchoff's rules:

i) Assume any direction for the current. If I is negative, it means the current is moving opposite the assumption.

ii) Any complete loop around a circuit has a total potential change of zero.
(Potential difference between 2 points is always the same, no matter which path)

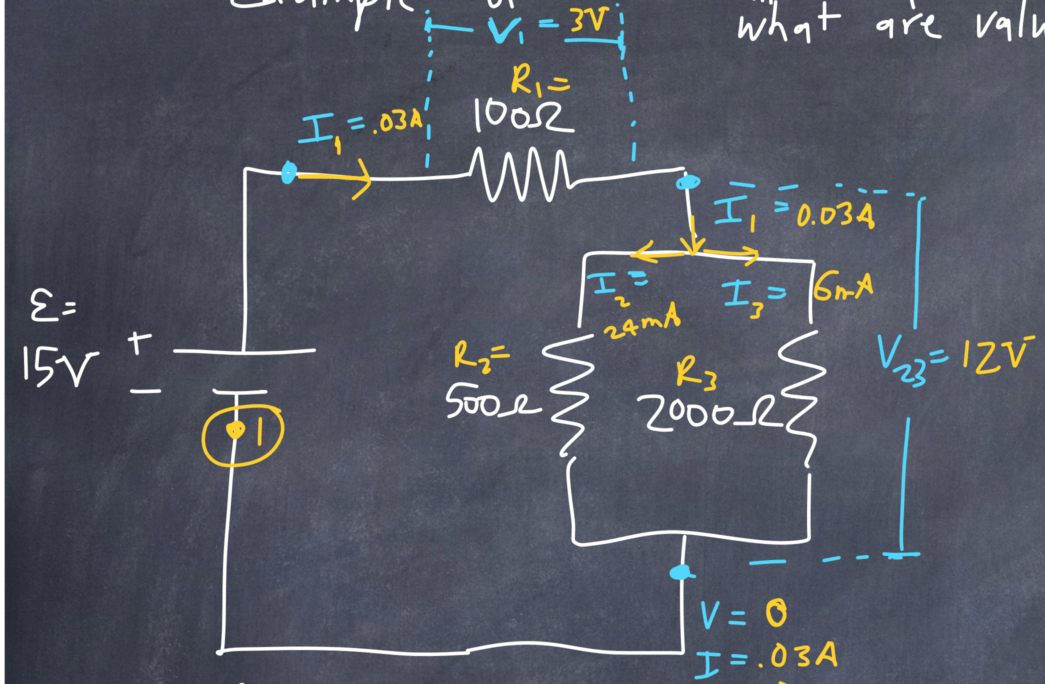
iii) For a battery, if the potential increases, − to +, then add it. If go from + to −, subtract it.

iv) The sum of currents into a junction must equal the sum of currents out of the junction.

v) Simplify using R_{eq} formulas

Example of circuit with resistors in parallel + series:

what are values of labeled currents + voltages?



1) Label resistors + currents

2) Calculate R_{eq} (parallel first, then series)

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{23}} = \frac{1}{500\Omega} + \frac{1}{2000\Omega} = \frac{5}{2000\Omega}$$

$$R_{23} = 400\Omega \qquad (<R_2, <R_3)$$

$$R_{123} = R_1 + R_{23} = 100\Omega + 400\Omega = 500\Omega$$

3) Loop from 1 : $\mathcal{E} - I_1(R_{123}) = 0$

$$I_1 = \frac{\mathcal{E}}{R_{123}} = \frac{15V}{500\Omega} = 0.03A$$

$$V_1 = I_1 R_1 = (0.03A)(100\Omega) = 3V$$

4) we also know that starting at 1 : $\mathcal{E} - V_1 - V_{23} = 0 \Rightarrow 15V - 3V - V_{23} = 0$

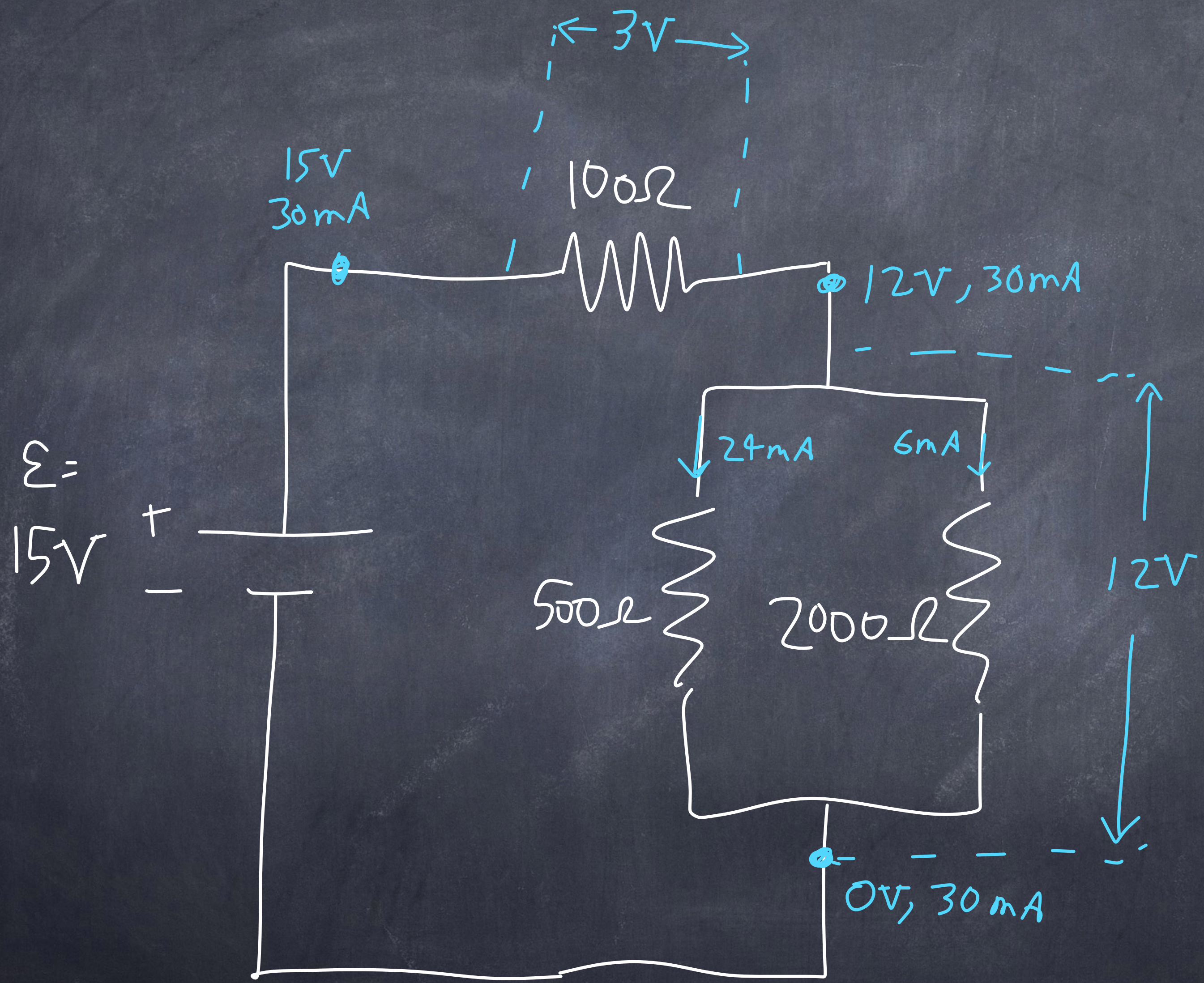
$$V_{23} = 12V$$

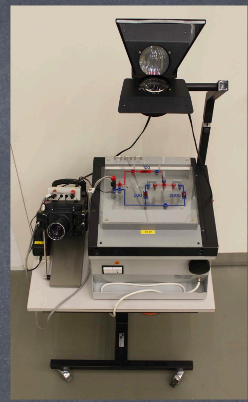
5) The voltage drop across $R_2 + R_3$ must be the same: $V_{23} = V_2 = V_3$

$$I_2 R_2 = V_2 = V_{23} \Rightarrow I_2 = \frac{12V}{500\Omega} = 24mA = .024A$$

$$I_3 R_3 = V_3 = V_{23} \Rightarrow I_3 = \frac{12V}{2000\Omega} = 6mA$$

Solution:

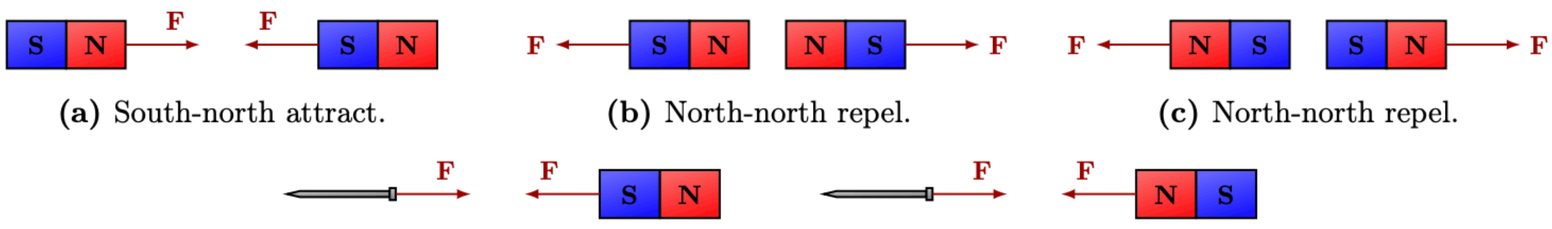




Magnetism

Basic observations:

Red = north



(a) South-north attract.

(b) North-north repel.

(c) North-north repel.

(d) South attracts the nail.

(e) North attracts the nail.

Figure 7.1: The magnetic force between two bar magnet depends on their orientation, but between a non-magnetic nail and bar magnet, orientation does not matter.

Magnetic Field labeled with $\bar{B}$

$\bar{B}$

(a) The magnetic field of bar magnet looks like the electric field of an electric dipole. The field lines close their loops inside the bar magnet.

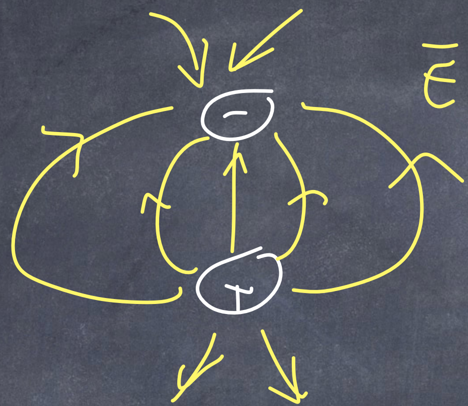
(b) Earth's magnetic field looks like that of a bar magnet. Magnetic compasses point to Earth's geographic north pole, the magnetic south pole.

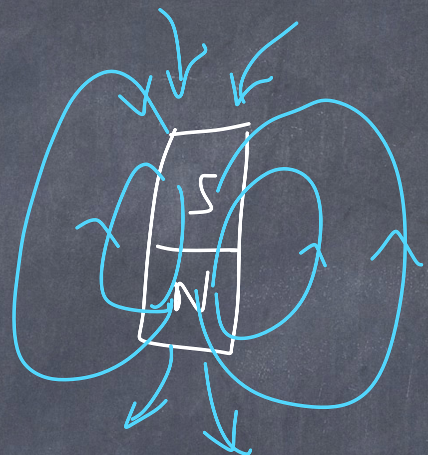
Figure 7.2: Bar magnets and the Earth create a magnetic dipole field (purple).

- B-field N → S outside the magnet, but $\bar{B}$-field is a complete loop (so $\bar{B}$ goes S→N inside magnet)
- Earth's geographic north pole is actually the magnetic south pole!

Your compass N points to magnetic south

This may remind you of the $\bar{E}$-field of a dipole.



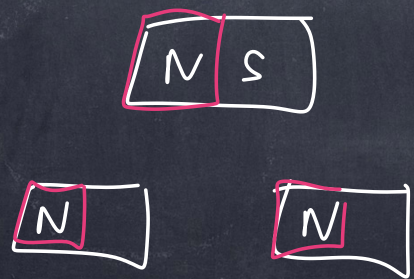


However, there are no magnetic charges!

No N charge, no S charge.

No single magnetic charge (magnetic monopole) has ever been observed.

If you break a magnetic in half:



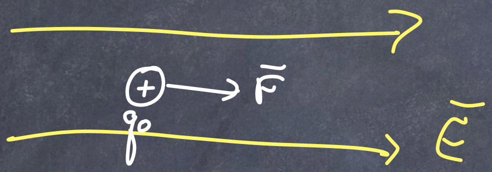
still have dipoles with N + S.


VisaLux
Favorit 400

One of the other differences is how $\bar{E}$ + $\bar{B}$ produce forces on an electric charge:

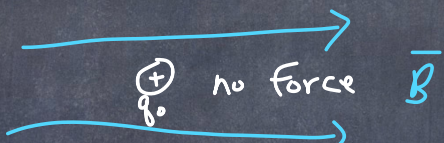
$$\bar{F}_E = q_0 \bar{E}$$

$$\bar{F}_B = q_0 \bar{V} \times \bar{B}$$

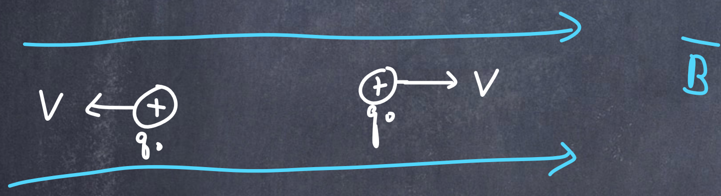


$\bar{F}_e = q\bar{E}$ electric force

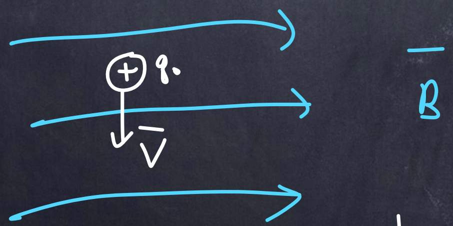
$\bar{F}$ is in the same direction as $\bar{E}$



If q_0 is not moving, $\bar{V}=0$,
then $\bar{F}$ is 0



If q_0 has $\bar{V} \parallel \bar{B}$, then F=0
($\bar{V}$ constant)



⊙ Force out of page.

If $\bar{V}$ + $\bar{B}$ are not completely parallel, then there is a force.

$$\bar{F}_B = q_0 \bar{V} \times \bar{B}$$

units for $B = \frac{F}{qV} = \frac{[N][s]}{[C][m]}$ = Tesla = T

$$1\,T = 1\,\frac{N \cdot s}{C \cdot m}$$

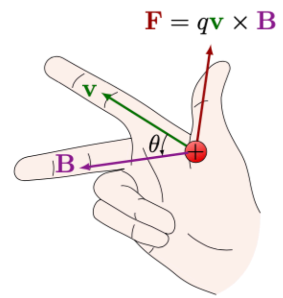
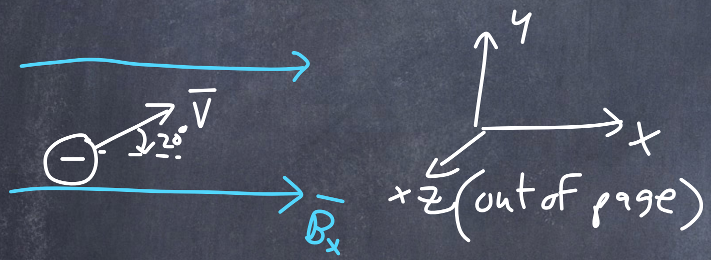


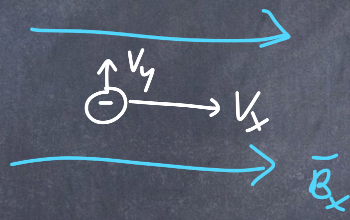
Figure 7.4: Right-hand rule for the magnetic force on a positive charge $q > 0$.

$|\bar{V} \times \bar{B}| = VB\sin\theta$

negative charges need left hand.

Consider negative charge in magnetic field. Force? Trajectory?

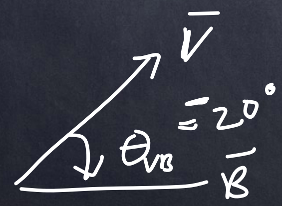


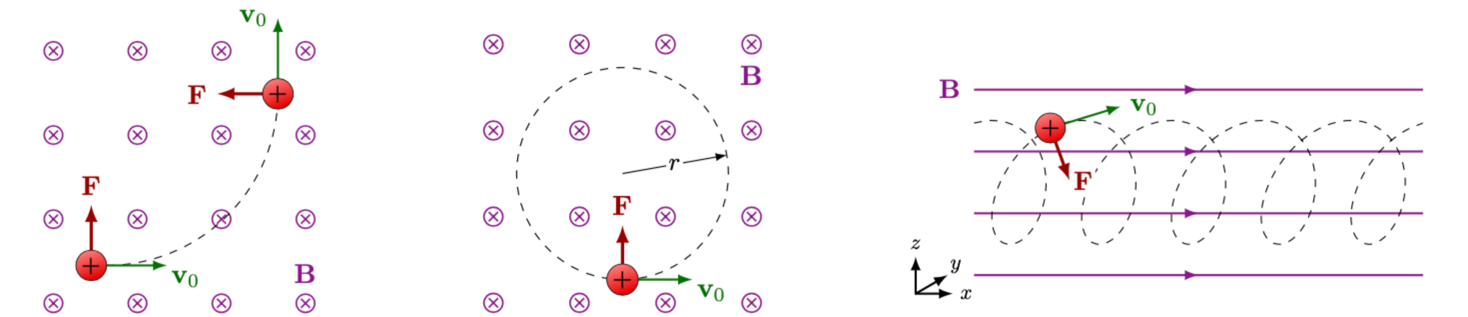


$\bar{F}$ is ⊙

Force results from $V_y \perp B_x$: $\bar{F} = \boxed{q}\,\bar{V} \times \bar{B} = \boxed{-e}\,VB\sin(-20^\circ)\,\hat{z}$ out of page.

$= eVB\sin 20^\circ\,\hat{z}$ out of page.



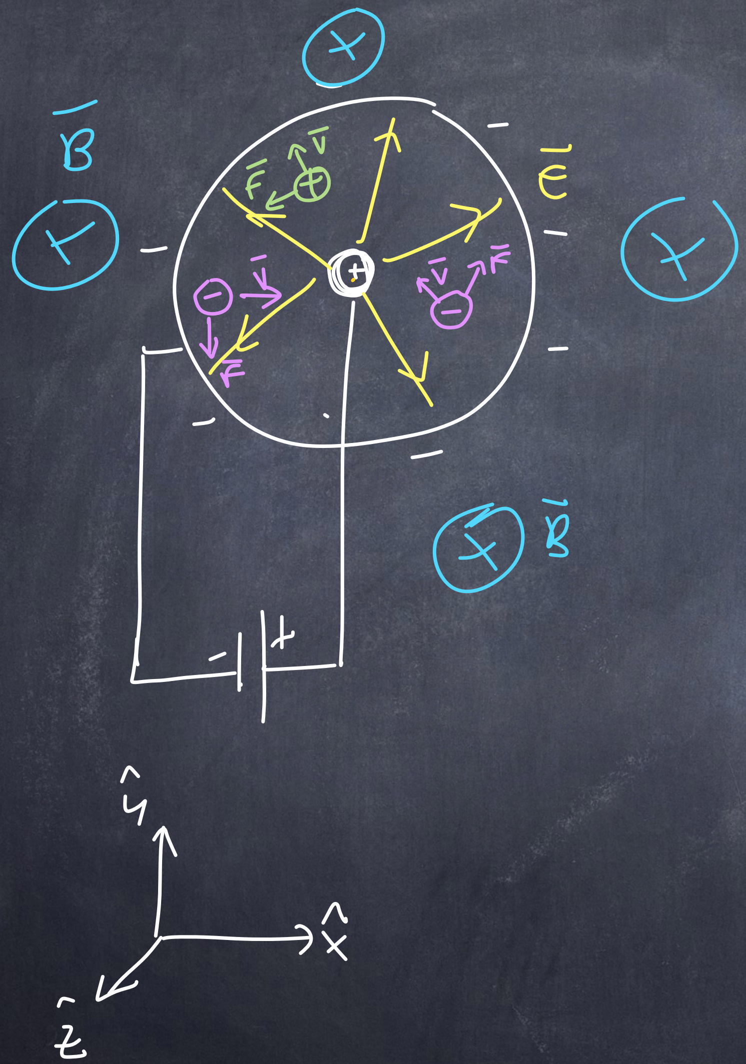


(a) Charge is bent in a magnetic field **B**.

(b) Charge with a constant velocity, perpendicular to **B** makes circles.

(c) Charge with constant velocity, not perpendicular to **B**, makes makes spirals.

Figure 7.5: Charge with a non-zero velocity, not parallel to a uniform magnetic field **B**, experiences a force perpendicular to the velocity and magnetic field.



⊕ = Na^+

⊖ = Cl^-

Both ⊕ + ⊖ charges cause force in the liquid that is counter-clockwise.

The motion of a charged particle in a $\bar{B}$-field is always of constant speed. (initial velocity component parallel to $\bar{B}$ doesn't change at all)

Circular motion when $\bar{V} \perp \bar{B}$, so $|\bar{V} \times \bar{B}| = VB\sin(90^\circ) = VB$

$$\Sigma F = ma$$

circular motion: $a = \frac{V^2}{r}$
(when constrained in a circle)

$$q\bar{V} \times \bar{B} = \frac{mV^2}{r}$$

$$q\cancel{V}B = \frac{mV^{\cancel{2}}}{r}$$

① $\boxed{r = \frac{mV}{qB}}$ radius of curvature of charge in B-field

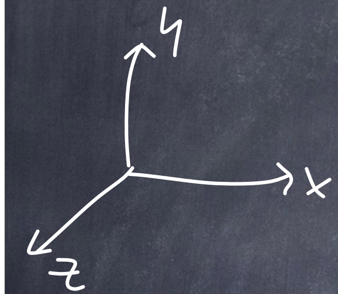
$$\omega = \text{angular velocity} = \frac{V}{r} = \frac{q\cancel{r}B}{m\cancel{r}} = \frac{qB}{m}$$

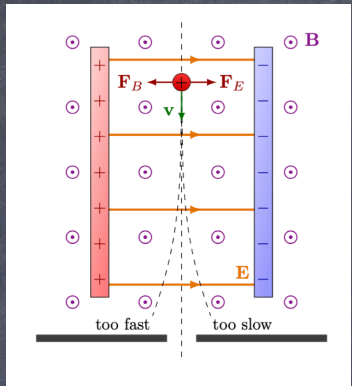
$\omega = 2\pi f$

f: frequency of rotation

$\Longrightarrow$ then $\boxed{f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}}$

"<u>velocity selector</u>" - we can balance the electric force & the magnetic force, but only at a specific velocity.





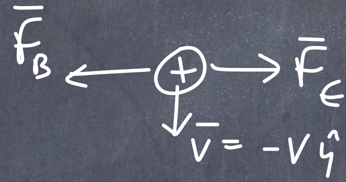
(a) Velocity selector.

$\bar{E}\,(\hat{x})$

$\bar{B}\,(\hat{z})$

$\bar{F}_E = qE\,\hat{x}$

$$\bar{F}_B = q\bar{v}\times\bar{B} = qvB\,(-\hat{x})$$



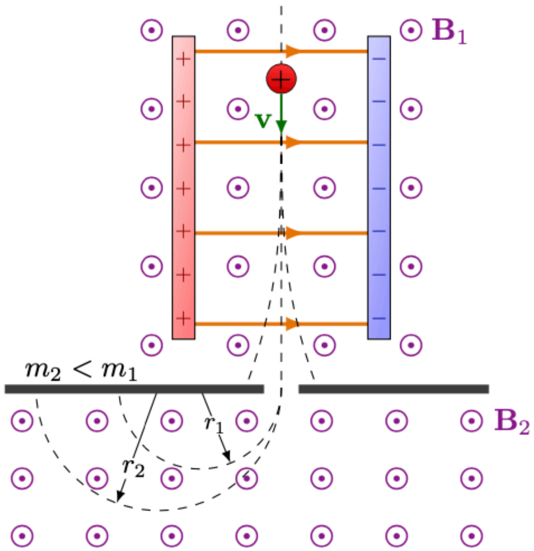
we require $\sum F_x = 0 = \bar{F}_E + \bar{F}_B = qE - qvB = 0$

$$\cancel{q}E = \cancel{q}vB$$

$$v = \frac{E}{B} \Rightarrow \bar{v} = \frac{E}{B}(-\hat{y})$$

At this velocity there is no net force. No deflection.

Mass spectrometer



(b) Mass spectrometer.

source of positive ions with different masses, different velocities.

1st stage: we select ions with a speed $V = \frac{E}{B_1}$

2nd stage:

we determine the mass from the radius of curvature:

from ① $\rightarrow$ $r_1 = \frac{m_1 V}{q B_2}$ where $V = \frac{E}{B_1}$

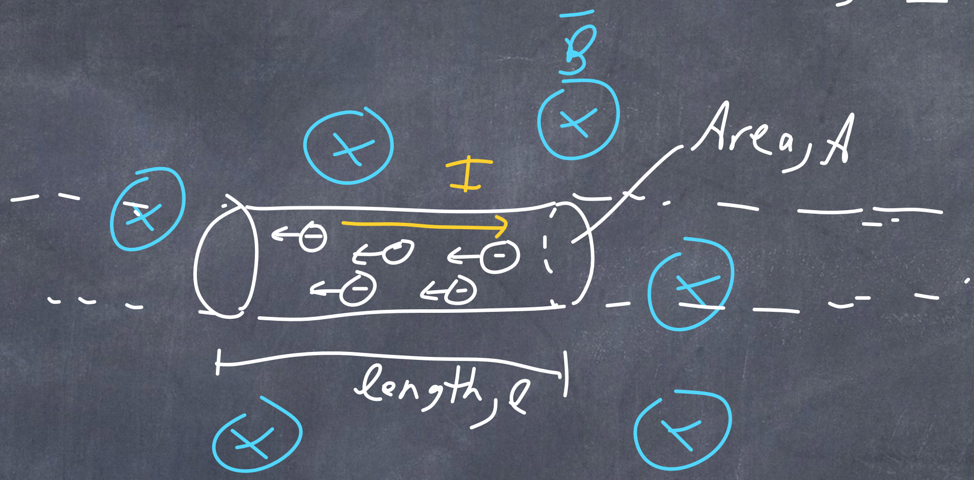
So $r_1 = \frac{m_1 \frac{E}{B_1}}{q B_2}$ $\Rightarrow$ we can calculate the mass of our isotopes.

This technique is how we discovered many stable isotopes of elements.

$\left(90\% \; {}^{20}_{10}Ne\right) + \left(10\% \; {}^{22}_{10}Ne\right)$: Neon

What if we have current of electric charges I moving $\perp$ $\bar{B}$-field?

A wire carrying electric current in a $\bar{B}$-field



Then

$$\bar{F}_{B_{TOTAL}} = (q\,\underbrace{\bar{V}_d}_{\text{drift velocity}} \times \bar{B})\left(\text{total \# of charges}\right)$$

$$\text{total \# of charges} = \underbrace{A \cdot \ell}_{\text{volume}} \cdot \underbrace{n}_{\text{\# charges/volume}}$$

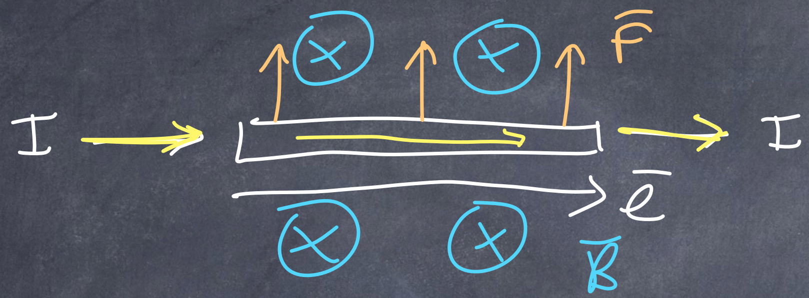
$$\bar{F}_{B_{TOT}} = q V_d B n A \ell \qquad \bar{V} \perp B \text{, so } \sin 90^\circ = 1$$

Previously, we saw that $n q V_d A = I$ = current

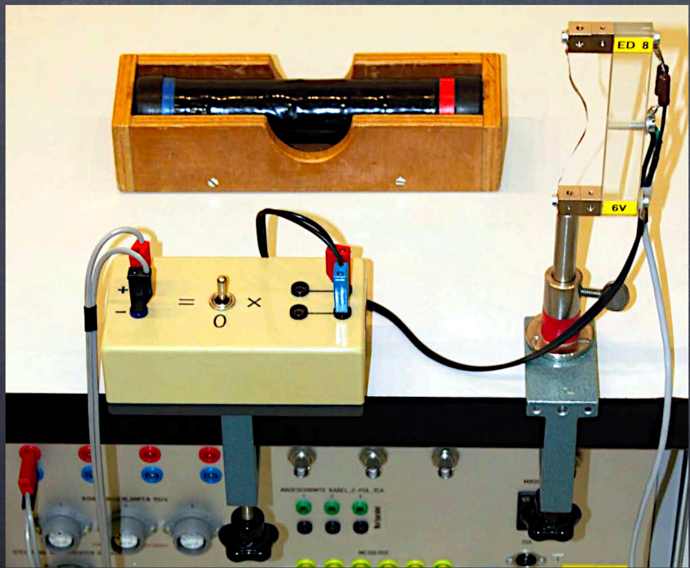
$$\boxed{\bar{F}_B = I \bar{\ell} \times \bar{B}}$$

This is the magnetic force on a straight wire with current I in a B-field.

If $\bar{\ell} \perp \bar{B}$, then $F = I\ell B = \boxed{BI\ell = F}$



The wire feels a force pointing up.

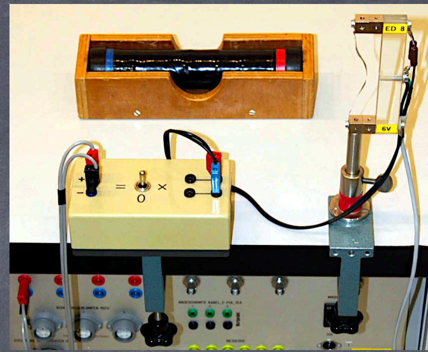


ED2

ED1

ED5

ED7



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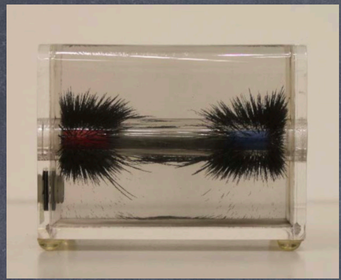

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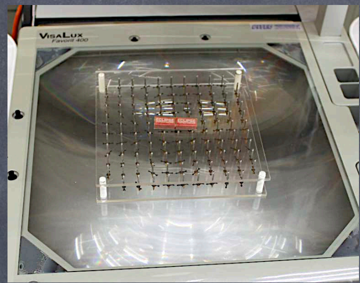

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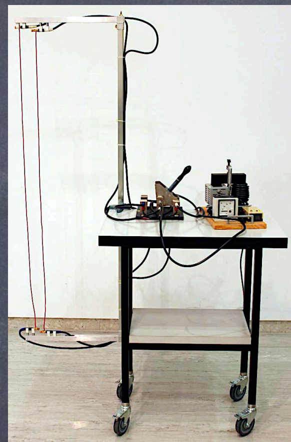

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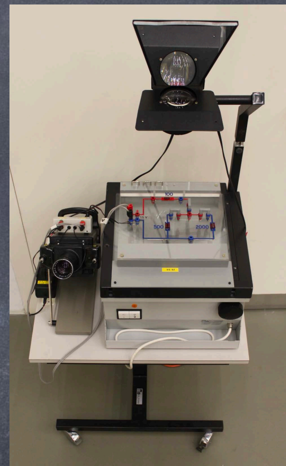

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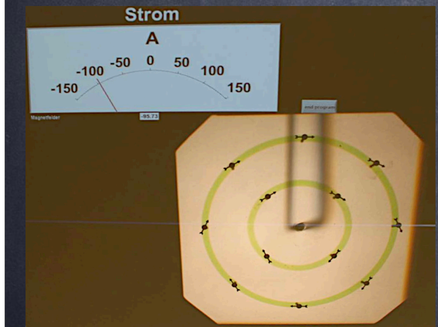

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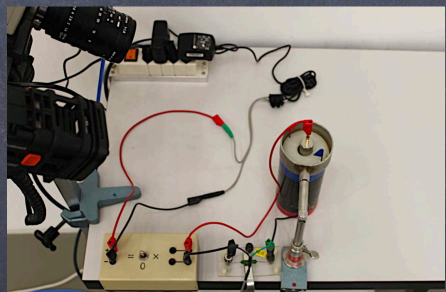

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