Two-loop Yukawa corrections to double Higgs production

Theoretical Particle Physics Seminar ETH/UZH | Oktober 18, 2022

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in collaboration with Joshua Davies, Go Mishima, Matthias Steinhauser and Hantian Zhang | Oktober 18, 2022

Contents

Higgs Self Coupling

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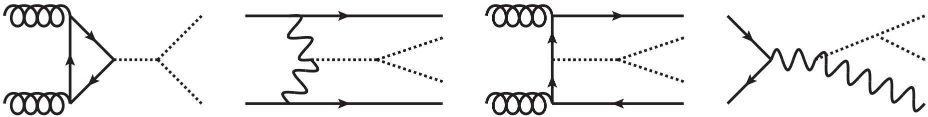
- Standard Model Higgs potential:

$$\boxed{V(H) = \frac{1}{2}m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4}, \text{ where } \lambda = m_H^2/(2v^2) \approx 0.13.$$

- Want to measure λ, to determine if $V(H)$ is consistent with nature.
  - Challenging! Cross-section $\approx 10^{-3} \times H$ prod.
  - $-3.3 < \lambda/\lambda_{SM} < 8.5$ [CMS '21]
- λ appears in various production channels:



- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- H-strahlung

Gluon Fusion

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- Leading order (1 loop) partonic amplitude:

$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

  - $\mathcal{F}_{tri}$ contains the dependence on λ at LO
- Form factors:
  - LO: known exactly [Glover, van der Bij '88]
  - Beyond LO... no fully-exact (analytic) results to date
    - QCD: numerical evaluation, expansion in various kinematic limits
    - EW: first steps: this work (HE) [Davies, Mishima, Schönwald, Steinhauser, Zhang '22]
    - (see also HTL considerations) [Mühlleitner,Schlenk,Spira '22]

$gg \to HH$ Beyond LO

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NLO QCD:

- large-m_t — [Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13]
- numeric — [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]
- large-m_t + threshold exp. Padé — [Gröber, Maier, Rauh '17]
- high-energy expansion — [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion — [Bonciani, Degrassi, Giardino, Gröber '18]

NNLO QCD:

- large-m_t virtuals — [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15, Davies; Steinhauser '19]
- HTL+numeric real ("FTapprox") — [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals — [Davies, Herren, Mishima, Steinhauser '19 '21]

N3LO QCD:

- Wilson coefficient C_{HH} — [Spira '16; Gerlach, Herren, Steinhauser '18]
- HTL — [Chen, Li, Shao, Wang '19]

$gg \to HH$ Beyond LO

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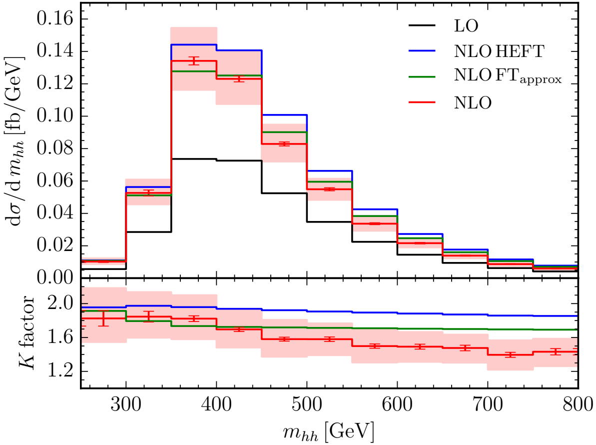


[Borowka, Greiner, Heinrich, Jones, Kerner '16]

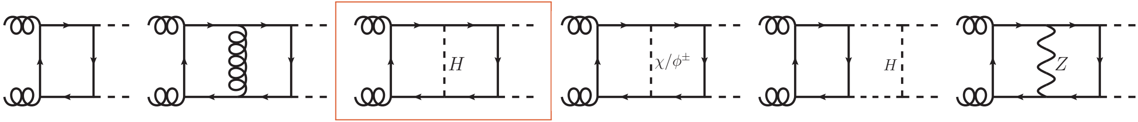
Total cross section (14TeV):

	σ_{LO}	σ_{NLO}	σ_{NNLO}
B-i HTL	–	$38.32^{+18.1\%}_{-14.9\%}$	$39.58^{+1.4\%}_{-4.7\%}$
FTapprox	–	$34.25^{+14.7\%}_{-13.2\%}$	$36.69^{+2.1\%}_{-4.9\%}$
Full	$19.85^{+27.6\%}_{-20.5\%}$	$32.88^{+13.5\%}_{-12.5\%}$	–

Electroweak Corrections

As we investigate NNLO QCD and beyond, we should consider NLO EW:

$$\mathcal{M} \sim \alpha_s \alpha_t \Big(A_1 + \alpha_s A_2 + \alpha_t A_3 + \alpha_{t,\lambda,gauge} A_4 + \mathcal{O}(\alpha_s^2, \alpha_t^2, \ldots) \Big)$$



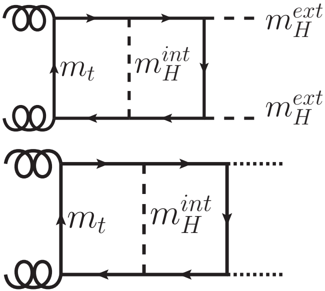
There are more scales to deal with, compared to the QCD contribution,

- start with $\alpha_s \alpha_t^2$ diagrams with internally propagating Higgs:
  - expansion parameter not small $\alpha_t = \alpha m_t^2 / (2 s_W^2 m_W^2) \sim \alpha_s / 2$
  - only planar integrals in this subset

High-Energy Expansion

High-Energy Expansion

- The full diagrams depend on a lot of variables:
  - ϵ, s, t, m_t, m_h
  - complete analytic solution is out of reach
- First, expand around $m_H^{ext} = 0$ (as for QCD):
  - expand amplitude integrals with `LiteRed` [Lee '14]
- Unlike for QCD the scale "m_H^{int}" remains, from the propagator:
  - complicates the IBP reduction
  - Master Integrals with this many scales are difficult.
- We expand in this scale also, and propose two ways to do it:
  - A: $s, |t| \gg m_t^2 \gg m_H^{int\,2} \sim m_H^{ext\,2}$,
  - B: $s, |t| \gg m_t^2 \sim m_H^{int\,2} \gg m_H^{ext\,2}$.



Asymptotic Expansions

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Expansion by Regions [Beneke, Smirnov '98] :

- Assign a hierarchy to the dimensionful parameters.
- Reveal all relevant scalings of the integration variables.
- Expand the integrand according to the scalings for each region.
- Integrate the expanded regions.
- Sum the contributions from all regions.

Asymptotic Expansions

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Here:

$$m_t^2 << s, |t| : \qquad \boxed{m_t^2 \sim \chi m_t^2}$$

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- Revealing all relevant regions can be a hard task.
- Automatized in the `Mathematica` package `Asy.m` [Pak, Smirnov '11] .
- Algorithm is based on the α-parameter representation of Feynman diagrams.

High-Energy Expansion "A"

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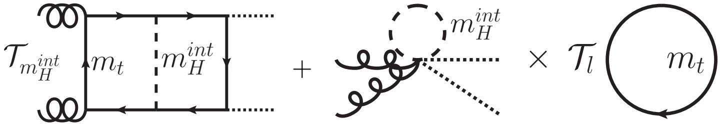
Option A: asymptotic expansion around $m_H^{int} = 0$:

- two sub-graphs:



The two-loop subgraph is a Taylor expansion of the Higgs propagator:

- results in integrals with a massless internal line, scales s, t, m_t.
- IBP reduce with `FIRE` and `Kira` [Smirnov '15] [Klappert,Lange,Maierhöfer,Usovitsch '21]
- these coincide with the QCD Master Integrals – reuse the old results [Davies,Mishima,Steinhauser,Wellmann '18,'19]

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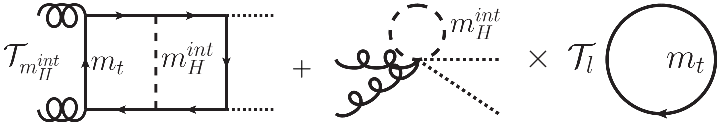

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The massive tadpoles are easily computed by `MATAD`. [Steinhauser ‘00]

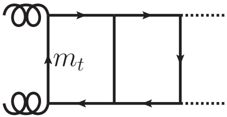
The asymptotic expansion procedure is done by `exp` and `FORM`. [Harlander,Seidelsticker,Steinhauser ‘97] [Ruijl,Ueda,Vermaseren ‘17]

We expand to quartic order: $(m_H^{int})^a\,(m_H^{ext})^b,\ \ 0 \leq (a+b) \leq 4.$

High-Energy Expansion “B”

Option B: expand around $m_H^{int} \approx m_t$,

- simple Taylor expansion, `exp` not necessary
  - much easier to implement
- IBP reduce resulting integrals, `FIRE+Kira`



Write Higgs propagator as: $\frac{1}{p^2 - m_H^2} = \frac{1}{p^2 - m_t^2(1 - [2-\delta]\delta)}$

- expand around $\delta \to 0$ where $\delta = 1 - m_H/m_t \approx 0.28$.

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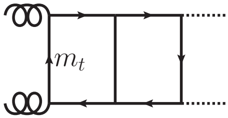

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- expand around $\delta \to 0$ where $\delta = 1 - m_H/m_t \approx 0.28$.

This yields new integral families compared to the QCD computation:

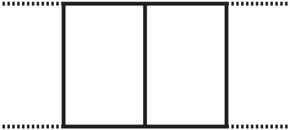
- all lines have the mass m_t,
- compute the MIs in the high-energy limit.

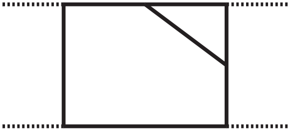
We expand to $(m_H^{ext})^4$ and δ^3.

Calculation of the Master Integrals

Calculation of the Master Integrals

Equal Mass Limit

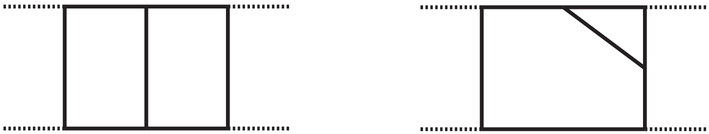




$p_1 + p_2 + p_3 + p_4 = 0$, $p_i^2 = 0$,
$(p_1 + p_2)^2 = s$, $(p_1 + p_3)^2 = t$,
$s + t + u = 0$

Calculation of the Master Integrals

Equal Mass Limit



- The integral families can be obtained by crossings from the graphs shown above.
- We reduce the scalar integrals with `Fire` [Smirnov '15] and find 140 master integrals. We make sure to reduce to a minimal set by:
  - We apply `FindRules` on all scalar integrals and run a second reduction.
  - Equating results of both reduction runs reveals non-trivial relations between master integrals of different families.
  - We run a search for master integrals with `Kira` [Klappert, Lange, Maierhöfer, Usovitsch '21] .
- We make sure to have a 'good' basis with `ImproveMasters` [Smirnov '20] , i.e.:
  - The denominators factor in $\epsilon = (4 - d)/2$ and the kinematics.
  - We get rid of spurious poles in ϵ, so that we have to calculate only to $\mathcal{O}(\epsilon^0)$.

Calculation of the Master Integrals

How to solve the master integrals?

- Full solution of the master integrals is still very complicated:
  - Solutions depend on 3 scales: s, t, m_t.
  - The master integrals have up to 7 massive internal lines.
  - The solutions have two thresholds at $\sqrt{s} = 2m_t$ and $\sqrt{s} = 3m_t$.
- However: Analytic solutions possible in the high energy region $m_t^2 \ll s, |t|$.

In the following:

- How to obtain a deep expansion utilizing the differential equations?
- How to obtain boundary conditions to solve the differential equations?
- How well does the approximation work?

Calculation of the Master Integrals

Deep Expansion

- Establish a system of differential equations for the master integrals in the variable m_t.

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- Establish a system of differential equations for the master integrals in the variable m_t.
- Compute an expansion around $m_t = 0$ by:

Calculation of the Master Integrals
Deep Expansion

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- Establish a system of differential equations for the master integrals in the variable m_t.
- Compute an expansion around $m_t = 0$ by:
  - Inserting an ansatz for the master integrals into the differential equation.

$$M_n(\epsilon, m_t \to 0) = \sum_{i=-2}^{\infty} \sum_{j=0}^{j_{\max}} \sum_{k=0}^{i+4} c_{ijk}^{(n)} \, \epsilon^i \, m_t^j \, \ln^k(m_t)$$

Calculation of the Master Integrals

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  - Compare coefficients in ϵ and m_t to establish a linear system of equations for the $c_{ijk}^{(n)}$.

Calculation of the Master Integrals
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  - Compare coefficients in ϵ and m_t to establish a linear system of equations for the $c_{ijk}^{(n)}$.
  - Solve the linear system in terms of a small number of boundary constants using `Kira` and `FireFly`.

[Klappert, Klein, Lange '19,'20]

Calculation of the Master Integrals
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  - Solve the linear system in terms of a small number of boundary constants using `Kira` and `FireFly`. [Klappert, Klein, Lange '19,'20]
- Compute boundary values for $m_t \to 0$ and obtain an analytic expansion.

$\Rightarrow$ **Why not utilize the differential equation in s or t?**

Calculation of the Master Integrals

Differential Equation in t

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- We can always put one scale to unity, we choose $s \equiv 1$.
- We can use the differential equation in t in a similar manner.
- Boundary conditions are then only needed in the limit $m_t, |t| \to 0$.
- However, calculating the boundaries in the limit $m_t \to 0$ with full dependence on t turns out to be not harder than in the double limit $m_t, |t| \to 0$.

Calculation of the Master Integrals

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- However, calculating the boundaries in the limit $m_t \to 0$ with full dependence on t turns out to be not harder than in the double limit $m_t, |t| \to 0$.

$\Rightarrow$ **No benefit in utilizing the differential equation in t.**

Calculation of the Master Integrals

Boundary Conditions

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How to obtain the boundary values?

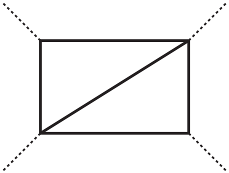
- We start with the α representation of the diagram:

$$I_n = \int_0^\infty \left(\prod_{i=1}^{n} d\alpha_i \frac{\alpha_i^{\delta_i}}{\Gamma(1+\delta_i)} \right) \mathcal{U}^{-d/2} e^{-\mathcal{F}/\mathcal{U}},$$

  with the Symanzik polynomials $\mathcal{U}$ and $\mathcal{F}$.
- We use expansion-by-regions [Beneke, Smirnov '98] and reveal the different regions with `Asy.m` [Pak, Smirnov '11].
- High-energy limit: $s, |t| \sim \chi^0$, $m_t^2 \sim \chi$
- In total we reveal 13 regions:
  - One hard region ($m_t = 0$), where master integrals are known [Smirnov, Veretin '00; Bern, Sixon, Smirnov '05].
  - 13 'soft' regions, where α parameters scale differently in χ.
- We calculate the expansion using Mellin-Barnes techniques.

Calculation of the Master Integrals

Boundary Conditions – Mellin-Barnes Techniques



$S = -s,\ T = -t$

- Symanzik polynomials: $(\alpha_{i_1\dots i_n} = \alpha_{i_1} + \dots + \alpha_{i_n})$

$$\mathcal{U} = \alpha_{23}\alpha_{14} + \alpha_{1234}\alpha_5, \qquad \mathcal{F} = S\alpha_2\alpha_4\alpha_5 + T\alpha_1\alpha_3\alpha_5 + m_t^2\alpha_{12345}\mathcal{U}$$

- 8 soft regions contribute for $m_t \to 0$: $(m_t^2 \to \chi\, m_t^2)$

$$\alpha_i \to \chi^{v_i^{(r)}}\alpha_i, \qquad \vec{v}^{(1)} = (0,0,0,0,1), \qquad \vec{v}^{(2)} = (0,0,1,1,0), \qquad \dots$$

- After rescaling we can expand in χ, e.g.:

$$I_5^{(1)} = \int \left(\prod_{i=1}^{5} \frac{d\alpha_i \alpha_i^{\delta_i}}{\Gamma(1+\delta_i)}\right) \mathcal{U}_1^{-d/2} e^{-\mathcal{F}_1/\mathcal{U}_1} \left[1 - \chi\left(m_t^2\alpha_5 - S\frac{\alpha_2\alpha_4\alpha_{1234}(\alpha_5)^2}{(\mathcal{U}_1)^2} + \dots\right) + \dots\right]$$

with the expanded Symanzik polynomials

$$\mathcal{U}_1 = \alpha_{23}\alpha_{14}, \qquad \mathcal{F}_1 = S\alpha_2\alpha_4\alpha_5 + T\alpha_1\alpha_3\alpha_5 + m_t^2\alpha_{1234}\mathcal{U}_1$$

Calculation of Master Integrals
Boundary Conditions – Mellin-Barnes Techniques

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- Useful formula:

$$\int_0^\infty d\alpha \alpha^a e^{-A\alpha} = A^{-1-a}\Gamma(1+a),$$

$$\int_0^\infty d\alpha \alpha^a (A+B\alpha)^b = A^{1+a+b}B^{-1-a}\frac{\Gamma[1+a,-1-a-b]}{\Gamma(-b)},$$

Calculation of Master Integrals

Boundary Conditions – Mellin-Barnes Techniques

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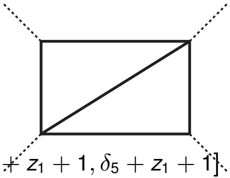
$$\int_0^\infty d\alpha \alpha^a (A+B\alpha)^b = A^{1+a+b}B^{-1-a}\frac{\Gamma[1+a,-1-a-b]}{\Gamma(-b)},$$

$$\boxed{\frac{1}{(A+B)^\lambda} = \int_{-i\infty}^{i\infty}\frac{dz}{2\pi i}\frac{B^z}{A^{\lambda+z}}\frac{\Gamma[-z,\lambda+z]}{\Gamma(\lambda)}}, \quad \text{with} \quad \Gamma[x_1,x_2,\ldots,x_n] = \Gamma(x_1)\Gamma(x_2)\ldots\Gamma(x_n)$$

Calculation of Master Integrals

Boundary Conditions – Mellin-Barnes Techniques



- We can describe the expansion with one template integral:

$$T_{1,\{\delta_1,\delta_2,\delta_3,\delta_4,\delta_5\},\epsilon} = \int \left(\prod_{i=1}^{5} \frac{d\alpha_i \alpha_i^{\delta_i}}{\Gamma(1+\delta_i)} \right) \mathcal{U}_1^{-d/2} e^{-\mathcal{F}_1/\mathcal{U}_1}$$

$$= \frac{(m_t^2)^{-\delta_{1234}-2\epsilon}}{S^{\delta_5+1}} \int \frac{dz_1}{2\pi i} \left(\frac{S}{T}\right)^{z_1} \frac{\Gamma[\delta_{23}+\epsilon, \delta_{14}+\epsilon, \delta_2-\delta_5-z_1, -z_1, \delta_4-\delta_5-z_1, \delta_1+z_1+1, \delta_3+z_1+1, \delta_5+z_1+1]}{\Gamma[\delta_1+1, \delta_2+1, \delta_3+1, \delta_4+1, \delta_5+1, \delta_{23}-\delta_5+1, \delta_{14}-\delta_5+1]}$$

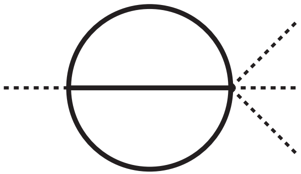
- We find up to 3-dimensional Mellin-Barnes integrals.
- The analytic continuation in δ_i and ϵ can be performed with `MB.m` [Czakon '05] .
- The sum of all regions has to be free of poles in δ_i.

$\Rightarrow$ **How to perform Mellin-Barnes integrals systematically?**

Mellin-Barnes Integrals
Example

- We find:

$$I_3 = m_t^{-4\epsilon+2} \int \frac{dz_1}{2\pi i} \frac{\Gamma[-z_1, z_1 - \epsilon + 2, -z_1 + \epsilon - 1, z_1 + 1, z_1 + 1, z_1 + \epsilon]}{\Gamma[2 - \epsilon, 2z_1 + 2]}$$



- We use `MB.m` for the analytic continuation in ϵ:

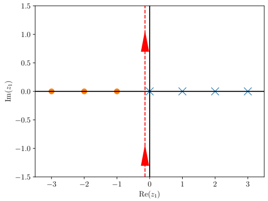
$$I_3 = m_t^{-4\epsilon+2} e^{-2\epsilon\gamma_E} \left(-\frac{3}{2\epsilon^2} - \frac{9}{2\epsilon} - \frac{21}{2} - \frac{5\pi^2}{12} + I^{(MB)} + \mathcal{O}(\epsilon). \right)$$

- With the remaining integral:

$$I^{(MB)} = \int_{-1/7-i\infty}^{-1/7+i\infty} \frac{dz_1}{2\pi i} \frac{\Gamma[-z_1 - 1, -z_1, z_1, z_1 + 1, z_1 + 1, z_1 + 2]}{\Gamma(2z_1 + 2)}$$

Mellin-Barnes Integrals
Example

- We can close the contour to the right and sum the residues at $z_1 = 0, 1, 2, \ldots$:



$$I^{(MB)} = \int_{-1/7-i\infty}^{-1/7+i\infty} \frac{dz_1}{2\pi i} \frac{\Gamma[-z_1-1, -z_1, z_1, z_1+1, z_1+1, z_1+2]}{\Gamma(2z_1+2)}$$

$$= 4 + \frac{\pi^2}{6} + 2\sum_{k=0}^{\infty} \binom{2k+1}{k}^{-1} \frac{(4k^2+8k+3)[S_1(k)-S_1(2k)] - 4(k+1)}{(2k+1)(2k+2)(2k+3)^2}$$

- Summation over residue sum can be done analytically with `HarmonicSums` [Ablinger et al. '10-], `Sigma` and `EvaluateMultiSums` [Schneider '07-].
- The (inverse) binomial sums we encounter sum to special constants, e.g.:

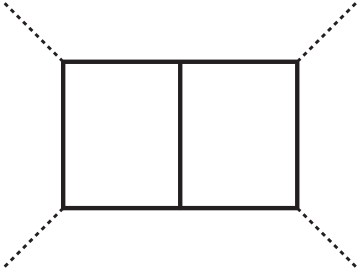
$$\sum_{k=0}^{\infty} \xi^k \binom{2k+1}{k}^{-1} \frac{1}{3+2k} = \frac{2}{\xi\sqrt{(4-\xi)\xi}} \int_0^{\xi} dt\, \sqrt{(4-t)t} - 1 \overset{\xi\to 1}{=} \frac{4\pi}{3\sqrt{3}} - 2$$

Mellin-Barnes Integrals
Example

- Most complicated boundary condition: $G_4(1,1,1,1,1,1,1,-1,-1)$
- The irreducible numerators can be handled by starting from the topology with all 9 lines.
- We end up with a large number of Mellin-Barnes integrals:

one-dimensional	two-dimensional	three-dimensional
2003	515	14

- Taking residues and summation can be automatized.



Boundary Conditions – Pitfalls

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- During our calculations we find terms like:

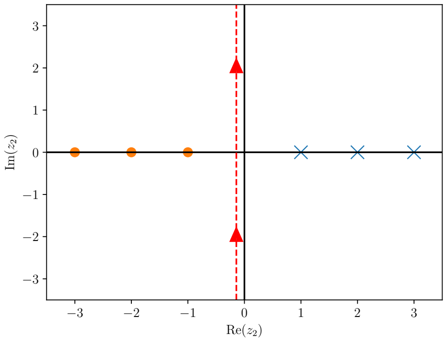
$$I = \int_{-1/7-i\infty}^{-1/7+i\infty} dz_2 \, \frac{z_2^8 \Gamma^2(-z_2)\Gamma^2(z_2)}{(z_2+1)^3(z_2+2)^3}$$

- Naive residue sum gives:

$$I = -\sum_{k=0}^{\infty} \frac{3k^5(4+3k)}{(1+k)^4(2+k)^4} = -18\zeta_3 - \frac{3\pi^2}{3} - \frac{21\pi^4}{10} + 240,$$

not in agreement with numerical evaluation.

- Problem: integral does not fall off fast enough for $|z_2| \to \infty$.



Boundary Conditions – Pitfalls

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- Problem: integral does not fall off fast enough for $|z_2| \to \infty$.
- We can solve this problem with regularization:

$$I = \int_{-1/7-i\infty}^{-1/7+i\infty} dz_2\, \xi^{z_2} \frac{z_2^8 \Gamma^2(-z_2)\Gamma^2(z_2)}{(z_2+1)^3(z_2+2)^3} = -\sum_{k=0}^{\infty} \xi^k \left(\frac{3k^5(4+3k)}{(1+k)^4(2+k)^4} + \frac{k^6}{(1+k)^3(2+k)^3}\ln(\xi) \right)$$

$$= \sum_{k=0}^{\infty} \xi^k \left(\frac{3k^5(4+3k)}{(1+k)^4(2+k)^4} + \left[1 - \frac{(2+3k)(4+12k+15k^2+9k^3+3k^4)}{(1+k)^3(2+k)^3} \right] \ln(\xi) \right)$$

$$\overset{\xi\to 1}{=} -18\zeta_3 - \frac{3\pi^2}{3} - \frac{21\pi^4}{10} + 240 + 1$$

- Alternative approach: high precision numerical evaluation in combination with `PSLQ` [Ferguson, Bailey '92] .

Master Integrals Results

Master Integrals Results

- We obtain analytic expressions of all 140 master integrals up to $\mathcal{O}(m_t^{120})$.
- The final result can be expressed via harmonic polylogarithms [Remiddi, Vermaseren '99]

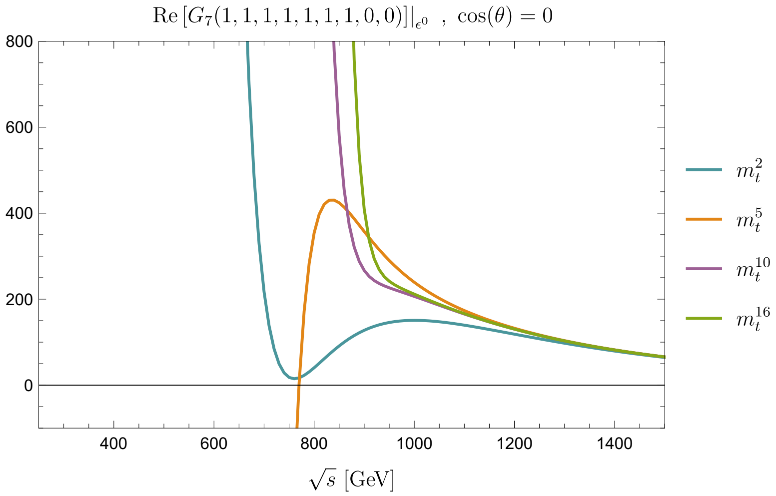
$$H_0(-t/s), H_1(-t/s), H_{0,1}(-t/s), H_{0,0,1}(-t/s), H_{0,1,1}(-t/s), H_{0,0,0,1}(-t/s), H_{0,0,1,1}(-t/s), H_{0,1,1,1}(-t/s)$$

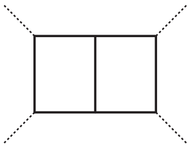
and transcendental numbers

$$\pi,\ \ln(3),\ \sqrt{3},\ \zeta_2,\ \zeta_3,\ \psi^{(1)}(1/3),\ \mathrm{Im}\left[\mathrm{Li}_3(i/\sqrt{3})\right]\ .$$

- We also extended the calculation of the master integrals with massless internal line up to $\mathcal{O}(m_t^{120})$.

Master Integrals Results





$$\cos(\theta) = \frac{s + 2t - 2m_h^2}{s\sqrt{1 - 4m_h^2/s}}$$

- Fixed order m_t expansions diverge at $\sqrt{s} \sim 1000\,\mathrm{GeV}$.
- The Padé approximation extends the range of validity.

Padé-Improved High-Energy Expansion

The master integrals for both methods are computed as an expansion in $m_t \ll s, |t|$.

The expansions diverge for $\sqrt{s} \sim 750\text{GeV}$ ("A"), $\sqrt{s} \sim 1000\text{GeV}$ ("B").

The situation can be improved using Padé Approximants:

- Approximate a function using a rational polynomial

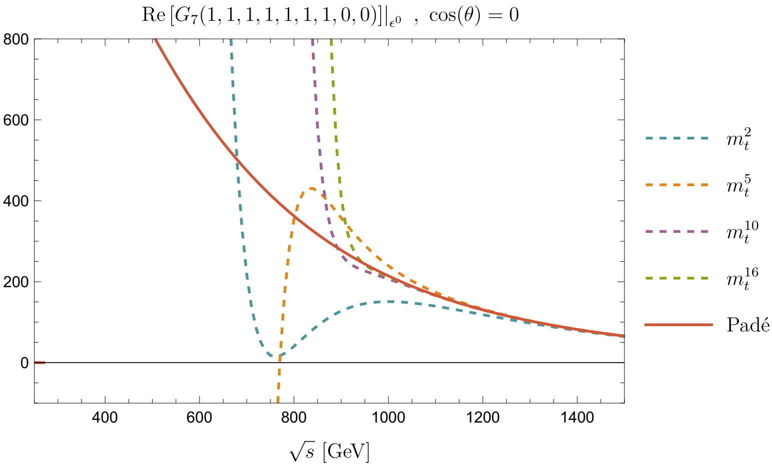
$$f(x) \approx \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m},$$

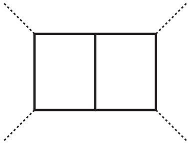
where a_i, b_j coefficients are fixed by the series coefficients of $f(x)$.

We compute a set of various Padé Approximants:

- combine to give a central value and error estimates
- a deeper input expansion $\Rightarrow$ larger $n + m$ $\Rightarrow$ smaller error
- here, m_t^{120} expansion allows for very high-order Padé Approximants

Master Integrals Results
Padé Improvement





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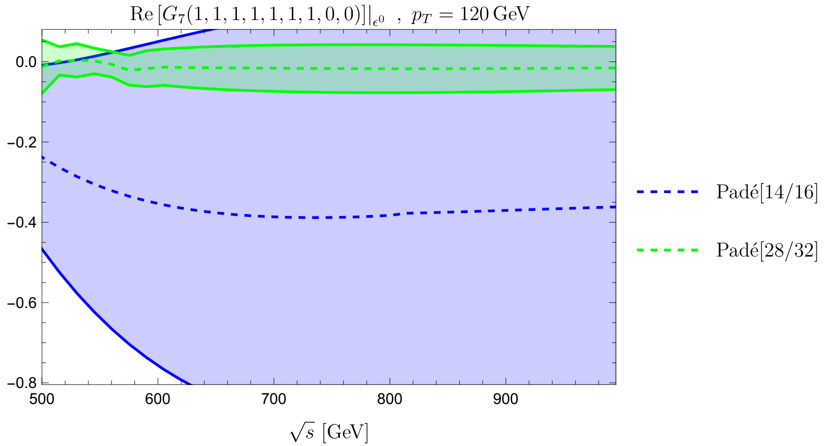
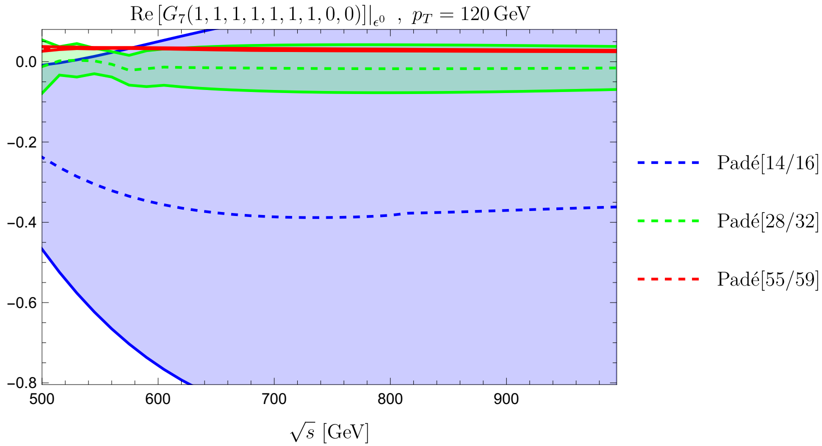


$$p_T^2 = \frac{tu - m_h^4}{s}$$

- Lower order Padé approximantions cannot reach low values of p_T.
- For QCD corrections expansions up to m_t^{32} were available: $p_T \gtrsim 150\,\mathrm{GeV}$
- With expansions up to m_t^{120} we reach: $p_T \gtrsim 120\,\mathrm{GeV}$.
- Error estimate from Padé approximations is reliable.

Padé Improvement



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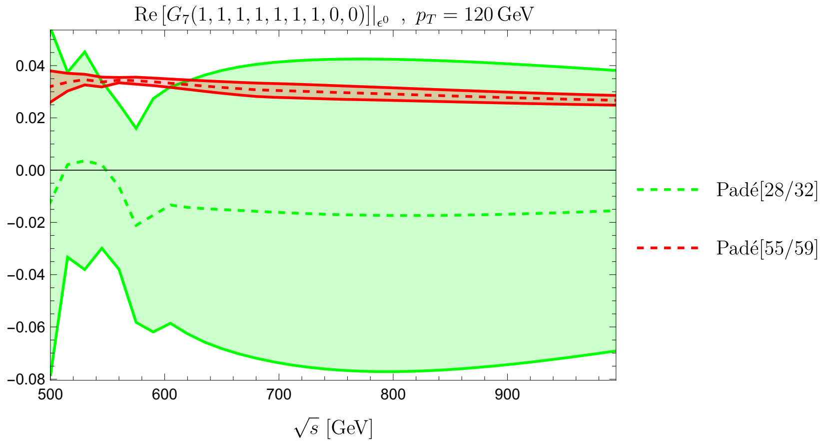

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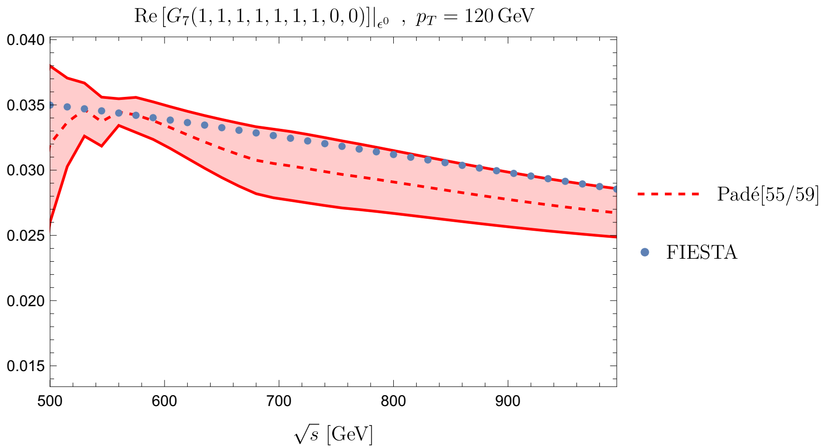

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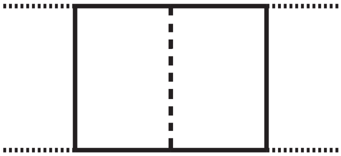
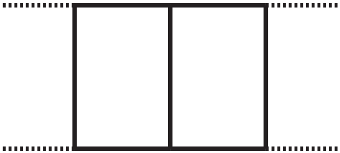
Comparison to the $m_H \to 0$ Expansion



Approach A:

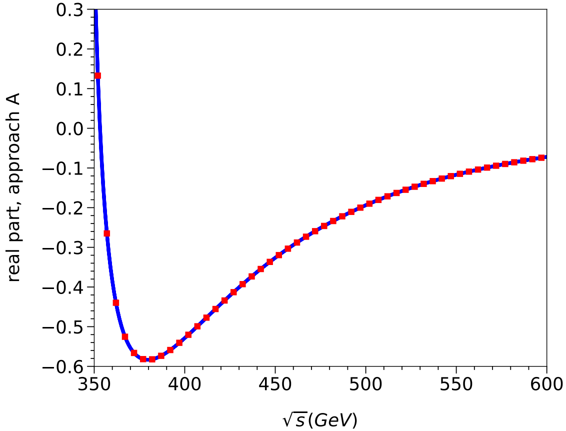
- middle line massless $m_H^{\text{int}} \approx 0$
- calculated in the context of QCD corrections
  [Davies, Mishima, Steinhauser, Wellmann '18, '19]

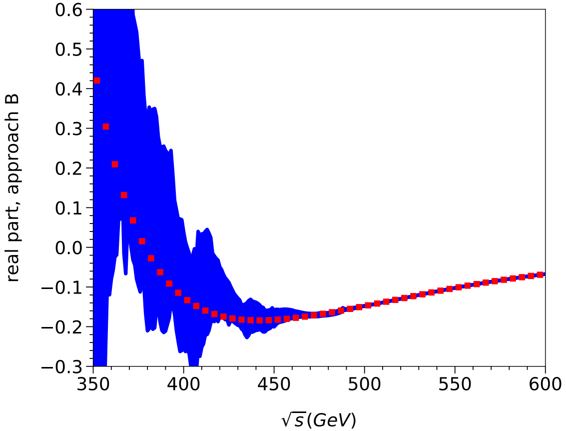


Approach B:

- middle line massive $m_H^{\text{int}} \approx m_t$

Comparison with Approach A

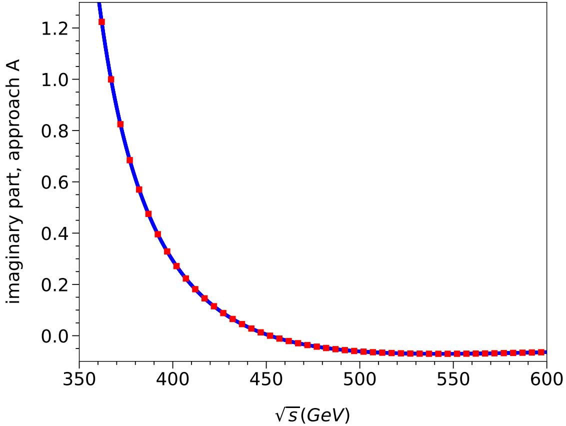


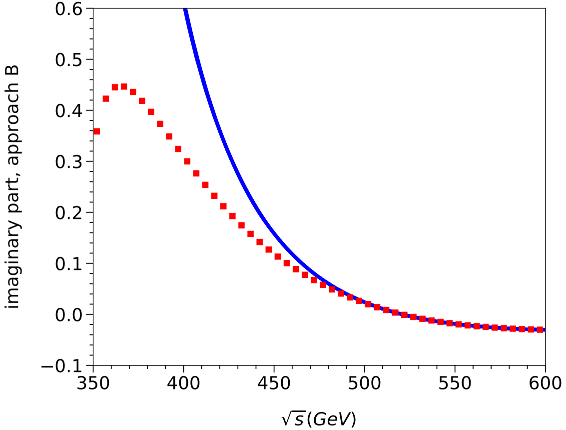


Approach A: threshold at $\sqrt{s} = 2m_t = 346\,\text{GeV}$

Approach B: threshold at $\sqrt{s} = 3m_t = 519\,\text{GeV}$

Comparison with Approach A





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Approach B: threshold at $\sqrt{s} = 3m_t = 519\,\text{GeV}$

Form Factor Results

Renormalization

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The form factors require UV renormalization (they are IR finite):

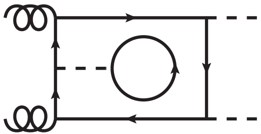
- $\overline{\text{MS}}$ renormalization of the top quark mass,

$$m_t^0 \to \overline{m}_t \left[1 + \frac{\alpha_t}{\pi} \frac{1}{\epsilon} \left(\frac{3}{16} + \frac{N_C}{2} \frac{\overline{m}_t^2}{m_H^2} \right) \right]$$

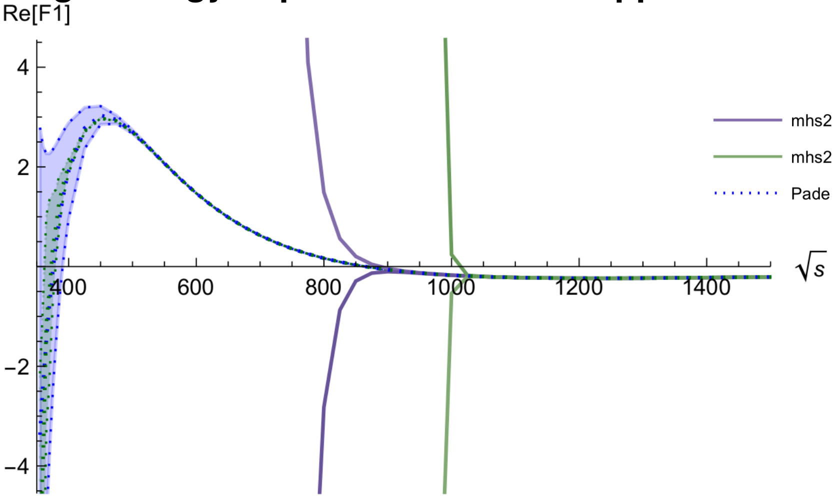
- LO has no δ expansion, so NLO δ terms must already be finite ✓

The second term in $(\cdots)$ renormalizes the tadpole diagrams,

- computed, but not included in the following plots.



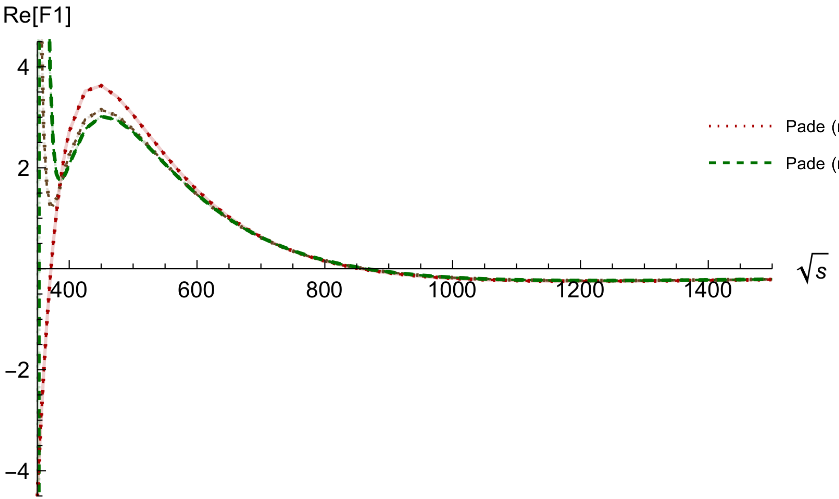
High-Energy Expansion and Padé Approximation



Re(F_{box1}), fixed $\cos\theta = 0$, expansion "B" (to $(m_H^2)^2\delta^3(m_t^2)^{\{15,16,56,57\}}$):

- m_t expansion diverges (strongly) around $\sqrt{s} \sim 1000$GeV

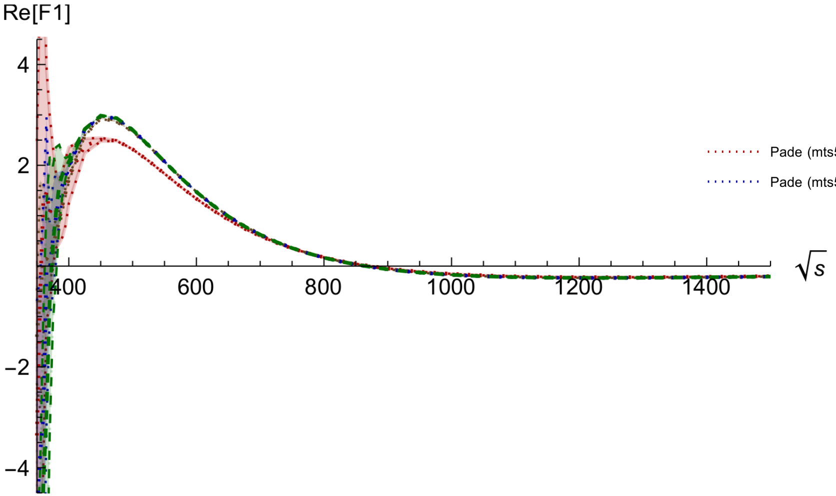
Convergence of Asymptotic Expansion (“A”)



Re(F_{box1}), fixed $\cos\theta = 0$, expansion “A” Padé (to $(m_H^2)^{\{0,1,2\}}$):

- $(m_H^2)^1$ and $(m_H^2)^2$ terms differ by at most 5% for $\sqrt{s} \geq 400$GeV

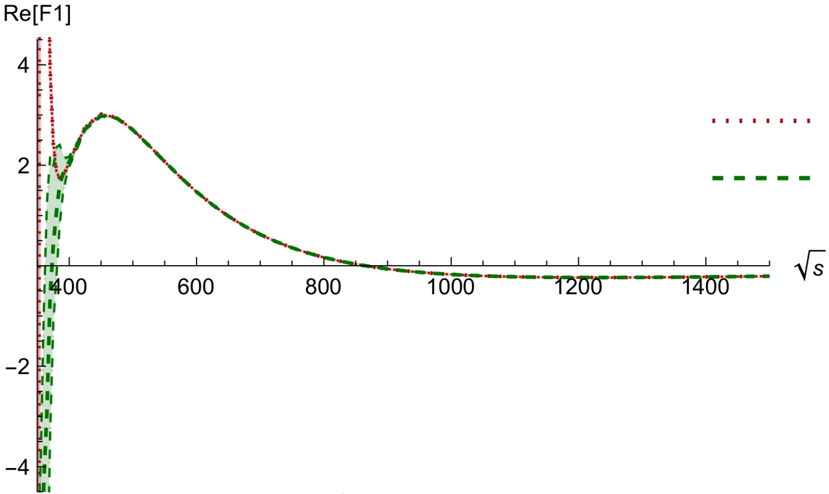
Convergence of δ Expansion (“B”)



Re(F_{box1}), fixed $\cos\theta = 0$, expansion “B” Padé (to $(m_H^2)^2\delta^{\{0,1,2,3\}}$):

- δ^2 and δ^3 terms differ by at most 0.5% for $\sqrt{s} \geq$ 400GeV

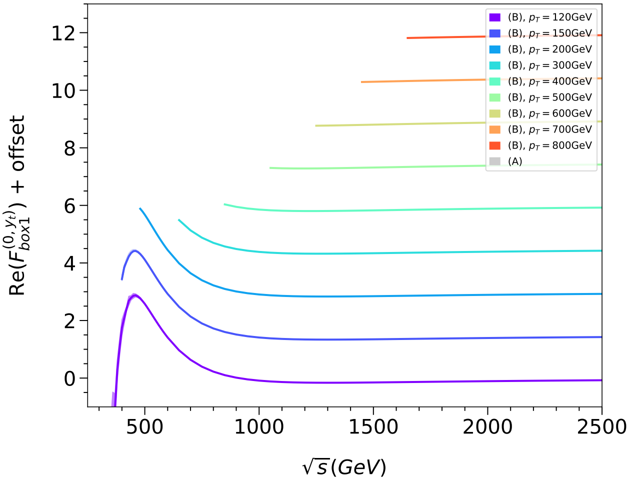
Comparison of “A”, “B” Expansions



$\mathrm{Re}(F_{box1})$, fixed $\cos\theta = 0$, best “A” and “B” Padé

- “A”, “B” differ by at most 2% for $\sqrt{s} \geq 400$GeV,
- 0.1% for $\sqrt{s} \geq 500$GeV

Form Factors at Fixed p_T



Expansions “A” and “B” agree for p_T values as small as 120 GeV.

- deep expansions of the MIs required, for small Padé errors

Conclusion

Conclusions:

First step towards electroweak corrections to double Higgs production:

- more difficult than the QCD contribution (extra internal scale)
- expansion allows us to compute them

High-energy expansion:

- Padé-based approximation to improve expansion
- good description of (partial) form factors for $p_T \gtrsim 120$GeV
- two different expansion methods, which give equivalent results
- deeper exp. of MIs compared to QCD papers $\rightarrow$ better Padé

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Outlook:

- Apply calculation strategy to the full electroweak corrections.
  $\Rightarrow$ This will include also non-planar sectors.
- Explore complementary expansions to cover the whole kinematic range.